A UNIFIED FIELD THEORY OF ELECTRODYNAMICS AND GRAVITATION

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INTRODUCTION

The theoretical work that is presented here is an idealized construction utilizing the best experimental values of the fundamental physical constants (1). The proton (p^+) and the electron (e^-) being lasting natural examples of the cause of mass will serve as the starting point of this theory. These entities will be referred to as "particles" with the understanding that they are some rotating charge distribution with accompanying magnetic and gravitational fields. These "particles" and the muon or mu meson (μ^-) will be considered one at a time as if they exist in an otherwise empty universe. This can be approximated in a real-world sense if the particle in question is sufficiently far removed from perturbing fields and masses that such effects can be neglected in the calculations to be outlined. Such is the context of this theoretical "thought experiment" which will be developed into a proposed laboratory experiment. Throughout this paper the MKSA system of units will be employed. All vector quantities will appear in **bold** face type.

When considering the three aforementioned atomic particles these will be referred to as "microscopic cases" for obvious reasons. The "spin" angular momentum of the three particles will be taken as "spin ½" ($\hbar \div 2$) in the empty universe context where a reference magnetic field is not allowed. Numerical and dimensional values for all quantities discussed can be found in the table of values at the end of this paper. Values given in the empty universe context cannot be measured directly, they are to be considered as theoretical.

This theory has as its parameters electric charge which is denoted by (e) in the microscopic cses and by (Q) in the macroscopic cases, which can include experimental situations. The magnetic flux density is denoted by (B), the rate of rotation by (f) in complete cycles per second and radius (R). Since it is easier to monitor voltage (V) than charge, the appropriate substitutions will be encountered.

DEFINITION OF VALUES

To start off the Planck frequency relationship gives:

1) $f=mc^2 \div h$

Where (m) is the observed mass or "natural mass" of the particle, (c) is the speed of light, (h) is Planck's Constant and (f) is the asserted rate of spin of the particle charge distribution. This rotation is not directly measurable because the charge distributions of the three particles of intrest are theorized to be axially symmetric and because an act of detection requires a spin 1 ($\hbar$) photon. This is valid only in the microscopic cases.

Next a value of and definition for the particle radius is required. A value that has proven useful is the Compton Wavelength (λ_c) divided by 2π which I call the "lightspeed radius" (R_c). This radius is where the rotation has an angular velocity equal to that of light and is given by:

2) $R_c=\lambda_c \div 2\pi = c \div (2\pi f) = h \div (2\pi cm)$

The radius (R_c) is the limit of the equatorial size of the particle being considered if we assume that the elements of mass rotate with the charge distribution. The last term of the equality here is for error analysis using the covariance matrix given in ref. (1). An abbrieviated version of this matrix is given at the end of this paper. The equations for and the results of the error analysis can be found in the table of values.

So we have a rotating charge distribution, which is unknown, and which gives rise to a magnetic field. The magnetic moment ($\mathfrak{M}$) which is the energy per unit magnetic flux density and the following well-known relationship for such is:

3) $\mathfrak{M}_t = e\hbar \div (2m) = eh \div (4\pi m)$

Here ($\mathfrak{M}_t$) is the theoretical magnetic moment and ($\mathfrak{M}_B$) is the Bohr magneton, ($\mathfrak{M}_N$) is the nuclear magneton and ($\mathfrak{M}_{\mathfrak{m}}$) is the theoretical muon magneton. It is these theoretical values that will be used in the empty universe context. In order to obtain the experimentally observed magnetic moment one must include the "magnetic moment anomaly" (A):

4) $\mathfrak{M}_p = A\mathfrak{M}_t = ehA \div (4\pi m)$

Here ($\mathfrak{M}_p$) is the experimentally observed particle magnetic moment. While these values for (A) will not be used in the theory they appear in the table of values should the reader be intrested.
The theorized magnetic flux density (B) at a pole of rotation and at a distance (R_c) from the center of the particle is:

5) $B = mc^2 \div \mathfrak{M}_t = 4\pi m^2c^2 \div (eh)$

This scalar form for (B) is all that needs to be known in the microscopic cases. Of course deriving a vector equation for (**B**) from an unknown rotating charge distribution is impossible.
A second, and equivalent, way of defining (B) makes use of the magnetic flux quantum (Φ_0) which is given by:

6) $\Phi_0 = h \div (2e)$

(B) is then given by:

7) $B = 2\Phi_0 \div (\pi R_c^2) = h \div (\pi e R_c^2) = 4\pi m^2 c^2 \div (eh)$

A third equivalent way of defining (B) for the microscopic cases can be found by first considering what the magnetic field in a macroscopic case is due to a uniformly charged rotating sperical shell (2):

8) $\mathbf{B} = \mu_0 Q f R^2 \div (6r^3)\ [2\cos\theta\mathbf{r} + \sin\theta\boldsymbol{\theta}] \qquad (r \geq R)$

This describes the magnetic field in sperical coordinates where (θ) is the angular distance from the axis of rotation, ($\boldsymbol{\theta}$) is a unit vector in a (θ) direction, (R) is the radius of the sphere, ($\mathbf{r}$) is a unit vector in a radial direction, (Q) is the total charge in Coulombs, (f) is the number of rotations per second and (μ_0) is the magnetic permeability of free space. This (**B**) field is independent of the longitudinal (ϕ) coordinates. At one of the poles of rotation, where the axis of rotation intersects the surface of the sphere, we find that:

9) $\mathbf{B} = \mu_0 Q f \div (3R)\ [\mathbf{r}] \qquad r = R\ ;\ \theta = 0$

Returning to the microscopic cases one finds that eq. 9) is not valid but some trial-and-error experimentation led to the following relationship:

10) $B = \mu_0 e f \div (\alpha R_c) = 4\pi m^2 c^2 \div (eh) \qquad r = R_c\ ;\ \theta = 0$

The substitution of "alpha" (α) or the "fine structure constant" for the (3) in eq. 9) does work out numerically. The dimensionless factor alpha is given by:

11) $\alpha=\mu_0 ce^2\div(2h)=7.29735308\times10^{-3}$ 11a) $\alpha^{-1}=137.0359895$

Equation 10) could provide a clue to finding the charge distributions of the proton, electron or muon all of which obey that relationship.

The attentive reader may have noticed that the empirical starting point of what has been done up to now is the knowledge of mass (m) or the equivalent energy. It will be stated here that in order to theoretically derive the mass of an atomic particle, the charge distribution must be known. More specifically the component electric field intensity vectors (**E**) and the magnetic flux density vectors (**B**) must be known and integrated over the volume of the charge distribution in a manner that will be outlined when we get to the derivation of the Newtonian Constant of Gravitation.

Before going on it will be emphasized that this theory is at present only capable of addressing single charged particles in relative isolation. In the case of the "weak boson" and the "quarks", of which three various kinds mutually associated comprise the proton and neutron, it will at this time be necessary to isolate and study these elusive entities.

FORCE AND MASS

Now that the preliminary values for the microscopic cases have been derived it is necessary to set forth some additional concepts so that these electrodynamic values can be related to mass. First of all mass manifests a gravitational field which exerts force on another mass according to:

12) $\mathbf{F}=m\mathbf{g}$

This is just Newton's Second Law with (m) being the mass, (**g**) is the acceleration vector and (**F**) the resultant force vector on the mass at the point in question. The force between two masses is given by:

13) $\mathbf{F}=GMm \div r^2\,[\mathbf{r}]$

Here (G) is the Newtonian Constant of Gravitation, (M) and (m) are the two masses, (r) is the distance and (**r**) is the direction between the centers of mass. Now it will be theorized that a single spherical mass manifests a "self force" or a "single body force" which is the product of the mass times the accelation of gravity at the surface of the mass. For the acceleration we have:

14) $\mathbf{g}=Gm \div R^2\,[-\mathbf{r}]$

Where (R) is the radius of the sphere. Note that the unit vector is in the negative radial direction. This acceleration is certainly real although vectorially the force of gravity taken over the surface of the sphere will sum to zero. For this reason the single body force must be a scalar quantity which can be thought of as a "bookkeeping" tool of how much of a total gravitational field we happen to be considering. The single body force is obtained by combining eq. 12) and eq. 14):

15) $F=-(Gm^2 \div R^2)$

With the single body force concept set forth an electrodynamic analogy is proposed for the microscopic cases which was originally found by trial-and error:

16) $F_x=-(eBfR_c)$

Here (F_x) is the theorized electrodynamic single body force due to a rotating charge distribution. This is essentially a special case of the Lorentz Force Law. Since this refers to natural mass the force is by definition negative, (see eq. 68). For now eq. 16) will be used for the electrodynamic single body force in order to give the reader a realistic sense of the chronological developement of this theory and to avoid the use of dimensionless constants in eq. 18).
Not surprisingly if one computes eq. 15) and eq. 16) for some atomic particle it is found that they do not agree numerically. In general (F_x) is tens of orders of magnitude larger than (F). In keeping a measure of respect for established theory, (F) will be retained as the gravitational microscopic single body force. Solving eq. 15) for microscopic case particle mass gives:

17) $m=\sqrt{-FR_c^2 \div G}$

Equation 16) can be used in eq. 17) by using a case-specific G value. By combining eq. 15) and eq. 16) and solving for this new (G) value we have:

18) $G_x = GF_x \div F = eBfR_c^3 \div m^2$

Here (G_x) is the "electrodynamic gravitational variable" that can be used to combine eq. 16) and eq. 17) to give the correct mass:

19) $m = \sqrt{eBfR_c^3 G_x^{-1}}$

Clearly this is a logical circle and this led me to search for other equations for (G_x). It proved possible to find five other equations, by trial-and-error. This trial-and-error process involves first finding a dimensionally valid equation and, if necessary, then taking the dimensional equation and finding the appropriate dimensionless constants so as to make the equation valid. The criteria is that a valid equation must be numerically and dimensionally equivalent to eq. 18). They are:

20) $G_x = 16\pi^2 f^3 R_c^3 \div (eB)$ 21) $G_x = \alpha^2 c^4 BfR_c^3 \div (eV^2)$

21a) $G_x = 16\pi^2 \alpha^2 BfR_c^5 \div (\mu_0^2 e^3)$ 22) $G_x = c^2 R_c \div (\pi m)$

23) $G_x = 4\pi f^2 R_c^3 \div (m)$ 24) $G_x = hc \div (2\pi^2 m^2)$

In eq. 21) the electrostatic potential (V) is given by:

25) $V = e \div (4\pi\epsilon_0 R_c)$

This follows from Gauss's Law for a uniformly charged sperical shell where (ϵ_0) is the permittivity of free space. Applying eq. 25) to eq. 21) gives eq. 21a). The potential in eq. 25) is less than the expected potential by a factor of (α). For example if the "classical electron radius" (R_e) is used in eq. 25) one obtains the correct electron potential as follows:

26) $R_e = \alpha R_c = \alpha h \div (2\pi c m_e) = \alpha^3 \div (4\pi R_\infty) = 2.1879409 \times 10^{-15}$ meter

Where (R_∞) is the Rydberg Constant. Using this classical radius gives:

27) $V_e = e \div (4\pi \epsilon_0 R_e) = 5.1099906 \times 10^5$ Volts

A substitution of eq. 20) into eq. 19) gives:

28) $m = eB \div (4\pi f)$

And a substitution of eq. 21) into eq. 19) gives:

29) $m = eV \div (\alpha c^2)$

Where (V) is defined by eq. 25).

We may write a potential energy equation for (G_x) as follows:

30) $U(r) = G_x m^2 \int_\infty^{R_c} r^{-2} dr = -(G_x m^2 \div R_c) = -(mc^2 \div \pi)$ 30a) $U(\infty) = 0$

This shows that while G_x may have enormous values in the micrscopic cases they are not so large as to violate the conservation of mass-energy.

SOME PARLLELS

This theory has a relatioship to the Planck Mass (m_P) which is:

31) $m_P=\sqrt{hc\div(2\pi G)}=2.17671\times10^{-8}$ kg 31a) $m_P\equiv1.22105\times10^{28}$ eV

One may take G_x using eq. 24) and solve for mass giving:

32) $m=\sqrt{hc\div(2\pi^2G_x)}$

This solution is valid for the microscopic cases that we have been considering. The spin or angular momentum of an isolated atomic particle can be related to the spin of a graviton. Some trial-and-error work established the following:

33) $S_p=\frac{1}{2}\hbar=h\div(4\pi)=\pi f m R_c^2$

Where (S_p) denotes particle spin. This relationship may provide a clue towards finding the mass distributions of the proton, muon and electron which all obey it. Remember that (R_c) is the limiting equatorial size of an atomic particle if the mass rotates with the charge distribution owing to relativistic effects. Another trial-and-error proceeding in which the "quantum of gravitation" or a "spin 2" graviton (S_g) was sought for gave:

34) $S_g=2\hbar=h\div\pi=4\pi f m R_c^2=eBR_c^2=\mu_0 ce^2\div(2\pi\alpha)$ 34a) $h=\mu_0 ce^2\div(2\alpha)$

One may take eq. 28) and solve for (f) to describe the change of frequency of a photon undergoing an energy state transition under the influence of an externally applied magnetic field (B_A) giving:

35) $\Delta f = 0,\ \pm eB_A \div (4\pi m_e)$

This is one possible case of the Zeeman effect.

The preceeding concepts of Planck Mass, spin and Zeeman effect have been included to help illustrate possible connections between this theory and already establised concepts of physics. Now that these preliminaries have been set forth we may ask what is the specific cause of mass? The aforementioned constraints of knowing the (**E**) and (**B**) fields adjacent to the charge distribution in the microscopic cases makes a truly theoretical derivation of such masses impossible at this time. Since the charge distributions of the three microscopic cases are unknown two macroscopic cases will be proposed. The first is a hypothetical case and the second is a practically sized experimental case. For both of these macroscopic cases the (**E**) and (**B**) fields are readily calculated and specific, as well as measurable, changes of mass are theorized to occur.

THE MKSA TEST CASE AND G

All of my efforts to derive (G) from proton and electron values and the fundamental physical constants have not been sucessful. So instead an attempt was made to invoke a hypothetical macroscopic case with values based on unit values in the MKSA system of units. This is termed the "MKSA test case". This proposal should be considered a "thought experiment" since the required parameters could not be practically realized to the level of precision that is to be discussed here, namely to one part per billion or better. The advantage that we have here is that the (**E**) and (**B**) fields can easily be specified. We will define all values for the MKSA test case to be positive. The MKSA test case is a spherical magnet with a radius of one meter (R_1) and a magnetic flux density of one Tesla (B_1) immediately adjacent to the surface of the sphere. The ($\mathbf{B}_1$) field is, in general, discontinuous at the surface itself. The sphere is to spin at a rate of one rotation per second (f_1). The sphere is located in space far removed from sources of interfering fields, that is to say in it's own universe, and it is charged to one Coulomb (Q_1). The electrostatic potential on the surface is then:

36) $V=Q_1 \div (4\pi\epsilon_0 R_1)=8.987551788\times10^9$ Volts

The corresponding electric field intensity at the surface is then:

37) $\mathbf{E}=Q_1 \div (4\pi\epsilon_0 R_1{}^2)\,[\mathbf{r}]=8.987551788\times10^9$ Volts/meter

The mass of the magnet when it is not charged and at rest does not need to be known here. A source of error here is that the magnetic field produced by the rotating spherical shell of charge (Q_1) is:

38) $B_Q=\mu_0 Q_1 f_1 \div (3R_1)=4.18879\times10^{-7}$ Tesla $(r=R_1;\ \theta=0)$

It is required that the magnetic flux density over the surface be exactly one Tesla. Clealy such a magnet cannot be practically fabricated. So the magnetic field over the surface is then:

39) $\mathbf{B}_1=B_1[2\cos\theta\mathbf{r}+\sin\theta\boldsymbol{\theta}]=1$ Tesla $(r \geqq R_1)$

Here the ($\geqq$) is defined to be just infinietesimally greater than R_1 so as to avoid the (**B**) field discontinuity at the surface itself. We can say that the charge is also at this location or that it covers the surface of the sphere.

It is now theorized that a qualitative attribute of the phenomenon of mass is that the vectorial force of gravity developed at an element of charge (δQ) is a function of the sine of the angle between the (**E**) and (**B**) fields and that it is radially directed:

40) $\mathbf{F} \sim \{(\mathbf{E} \times \mathbf{B})\mathbf{r}\}$

This can be expressed as the derivative of the single body force in spherical coordinates:

41) $d\mathbf{F}_1 = \epsilon_0 f_1 R_1^3 \mathbf{E}\mathbf{B}_1 \mathbf{r} \sin\theta d\theta d\phi$

Integrating over the surface of the sphere we have:

42) $F_1 = \epsilon_0 f_1 R_1^3 \mathbf{E}\mathbf{B}_1 \int_0^{2\pi} \int_0^{\pi} \sin\theta d\theta d\phi = 4\pi \epsilon_0 f_1 R_1^3 E B_1 = 1$ Newton

Here (E) and (B) are scalars, with the proper dimensional units, that have the same values as their vectorial counterparts that are given in eq. 37) and eq. 39). Equation 37) may be substituted into eq. 42) to yield:

43) $F_x = F_1 = Q_1 B_1 f_1 R_1 = \pm 1$ Newton

If this is a real single body gravitational force then the change of mass in the MKSA test case would be:

44) $m_x = \pm\sqrt{Q_1 B_1 f_1 R_1^3 G^{-1}} = \pm 1.22420 \times 10^5$ kg

This "artificially generated mass" is theorized to have both positive and negative solutions. In the microscopic cases only the positive solution can occur because a rotating charge distribution by itself can produce a magnetic field in only one "direction" whereas here we are free to set the magnetic field in either "direction". If we set the magnets' field so as to be aiding that produced by the rotating charge then positive mass is expected and negative mass if the magnets' field is opposing that of the rotating charge, (See fig. 1). This may be decided by taking the product (QBf) or (VBf) as shown in fig. 1. These have been constructed so that the microscopic cases or "natural mass" is always positive and the single body force will in such cases be negative Additionally it is theorized that taking single values (Q), (V), (B) and (f) to any power, positive or negative will not alter its sign but multiplying or dividing these quantities with each other, regardless of their individual powers, shall proceed in the normal mathematical convention. Finally in eqs. 44), 53) and 55) if the product ($QBfR^3G^{-1}$) is positive then the artificial mass will be positive and if this product is negative then the artificial mass is also negative. One could think of this as a selection rule of whether to take the positive or negative square root of the product which is obviously (m_x^2). Since this system of sign manipulation will seem strange to most readers a table of all possible outcomes appears with the spin-field-mass diagram. Of course a small-scale replica of the MKSA test case could easily refute or confirm this and shortly one will be proposed.

It will be theorized that in the microscopic cases one uses (G_x) to compute the particle mass (m) and in the macroscopic cases it is theorized that (G) is used to compute the artificially generated mass. All (G_x) values are positive and (G) is retained as positive by convention.

The proposed value of (G), if it proves to be true, resolves it into other fundamental physical constants with substantially better precision. With the MKSA test case preliminaries outlined we are now ready to find a value for (G). First we recall the (G_X) relations that are eq. 20) and eq. 21) and re-iterate them for the MKSA test case where these values are referred to in general as (G_M):

45) $G_M = 16\pi^2 f_1{}^3 R_1{}^3 \div (Q_1 B_1) = 1.579136704\times10^2\ (m^3kg^{-1}s^{-2})$

46) $G_M = \alpha^2 c^4 B_1 f_1 R_1{}^3 \div (Q_1 V^2)$ 46a) $G_M = 16\pi^2 \alpha^2 \epsilon_0{}^2 c^4 B_1 f_1 R_1{}^5 \div Q_1{}^3$

$G_M = 5.325136197\times10^9\ (m^3kg^{-1}s^{-2})$

Eq. 46) results from the substitution of (V^2) where (V) is given by eq. 36). Equation 45) is squared and divided by eq. 46a) which gives:

47) $G_M = 16\pi^2 f_1{}^5 Q_1 R_1 \div (\alpha^2 \epsilon_0{}^2 c^4 B_1{}^3)$ 47a) $G_M = 16\pi^2 \mu_0{}^2 f_1{}^5 Q_1 R_1 \div (\alpha^2 B_1{}^3)$

$G_M = 4.682833708\times10^{-6}\ (m^3kg^{-1}s^{-2})$

Equations 47) and 47a) are valid microscopic case expressions for (G_X) when such values are used therein. Eq. 47a) results from the definition of the speed of light, eq. 48), using eq. 48a):

48) $c = (\sqrt{\epsilon_0 \mu_0})^{-1}$ 48a) $\mu_0{}^2 = (\epsilon_0{}^2 c^4)^{-1}$

In the MKSA test case an additional step is necessary to get a value that approximates (G). Taking eq. 47) and applying a dimensionless factor, eq.50), gives:

49) $G_1=[(16\pi^2 f_1{}^5 Q_1 R_1)\div(\alpha^2\epsilon_0{}^2 c^4 B_1{}^3)\times(\alpha\div512)]=\pi^2 f_1{}^5 Q_1 R_1\div(32\alpha\epsilon_0{}^2 c^4 B_1{}^3)$

$G_1=\pi^2\mu_0{}^2 f_1{}^5 Q_1 R_1\div(32\alpha B_1{}^3)=6.67427559\times10^{-11}\ (m^3kg^{-1}s^{-2})$

The error of (G_1) is the same as the error of (α) which is 45 parts per billion. The value of (G_1) will be kept positive by definition. The dimensionless factor used in eq. 49) was found by trial-and-error:

50) $\alpha\div512=1.425264273\times10^{-5}$ 50a) $512\div\alpha=70162.42662$

50b) $\sqrt{512\div\alpha}=264.8819107$

This factor likely reflects the difference of field configurations between the MKSA test case charged sphere in isolation and an atomic particle in isolation. If eq. 49) were used for (G_x) with electron values the following would result:

51) $G_t=\pi^2\mu_0{}^2 f^5 e R_c\div(32\alpha B^3)=1.728479247\times10^{29}\ (m^3kg^{-1}s^{-2})$

Here (G_t) is a theorized electrodynamic gravitational variable. This gives an erroneous electron "mass" of:

52) $m_t=\sqrt{eBfR_cG_t{}^{-1}}=2.4129126\times10^{-29}$ kg

This "theoretical mass" (m_t) differs by the factor given by eq. 50b). This factor is the theorized difference of mass between a charged rotating sphere in isolation and an atomic particle of the same parameters (e,B,f,R_c), were it possible to have a macroscopic uniformly charged rotating spherical magnet be of microscopic size.

Now we revisit eq. 44) with the value (G_1):

53) $m_x = \pm\sqrt{Q_1 B_1 f_1 R_1^3 G_1^{-1}} = \pm 4B_1^2 R_1 \sqrt{2\alpha} \div (\pi\mu_0 f_1^2) = \pm 1.224046545 \times 10^5$ kg

This is theorized to be valid only for the MKSA test case. While there are several ways to write (G_1), upon solving for the artificially generated mass (f_1^2) appears in the denominator. However, in the microscopic cases we see that mass is directly proportional to (f). In eq. 53) if (f) were to become zero then (m_x) would become ($\pm\infty$) which is clearly an unacceptable result! For this reason it will be theorized that a constant (G) value holds for all microscopic cases and that (G_1) will be proposed as the required value in MKSA units.

A TEST

Since experiment is the ultimate test of theory a set of convenient laboratory values are proposed:

$Q=1.15\times10^{-7}$ Coulomb V=13500 Volts $C=8.48\times10^{-12}$ Farad

B=0.5 Tesla m=0.5 kg (magnet rest mass) $m_x=\pm1.06$ kg

f=80 cy/sec $R=R_i=0.0254$ meter (sphere radius)

$R_o=0.0381$ meter (cavity radius)

This is diagrammed in fig. 2. The relatioship between (V) and (Q) per the idealized fig. 2a. is:

54) $V=Q(R_o-R_i)\div(4\pi\epsilon_0 R_i R_o)$ 54a) $Q=4\pi\epsilon_0 R_i R_o V\div(R_o-R_i)$

The practical experimental setup is diagrammed in fig 2b. Since this introduces complications in the field geometry we will pusue the idealized version in order to keep the math reasonable. This will cost some accuracy as far as the artificially generated mass is concerned. Rewriting eq. 53) gives:

55) $m_x = \sqrt{QBfRG_1^{-1}} = \pm[(8R^2 \div \mu_0) \times \sqrt{(2a\epsilon_0 B_1^3 BfR_0 V \div (\pi f_1^5 Q_1 R_1 (R_0 - R)))}]$

$m_x \cong \pm 1.06$ kg

55a) $E_x = m_x c^2 \cong \pm 9.53 \times 10^{16}$ Joules

55b) $m_T = m_x + m \cong +1.56$ kg; -0.56 kg

55c) $G_x = QBfR^3 \div m^2 \cong 3.015 \times 10^{-10}$ $(m^3 kg^{-1} s^{-2})$

55d) $G_1 = +|QBFR^3 \div m_x^2|$

The observed total mass of the sphere (m_T) then has two possible solutions, and the change of mass in the experiment itself is simply (m_x). Eq. 55c) is a macroscopic electrodynamic gravitational variable that is defined in the same way as the microscopic case (G_x) values are if the rest mass of the magnet is known.

At this point one may wonder whether this mass correctly predicts experiment. Shortly a calculation of the electromagnetic energy in the fields of this experimental test case and the MKSA test case will be performed that will cast doubt on these large (E_x), and (m_x), values. Before doing that it is theorized that if this scenario is true then this energy will come from space itself when negative mass is generated and will go back to space when positive mass is generated. The volume of the space affected will include the sphere.

In general the electromagnetic energy stored in the fields in a given volume of space is given by:

56) $W_{EB} = \frac{1}{2}\int [\epsilon_0 E^2 + \mu_0^{-1} B^2] d\tau$

The term ($\int d\tau$) signifies an integration over the volume of space of intrest. Since we have already defined the (**E**) and (**B**) fields for the MKSA test case these fields for the experimental test case will now be defined:

57) $\mathbf{E}=Q\div(4\pi\epsilon_0 R_i R_0)\,[\mathbf{r}]=1.063\times10^6$ Volts/meter (at R_i)

58) $\mathbf{B}=BR_i^3\div(2r^3)\,[2\cos\theta\mathbf{r}+\sin\theta\boldsymbol{\theta}]=0.5$ Tesla ($r\geqq R_i$)

58a) $B_Q=\mu_0 Qf\div(3R)=1.52\times10^{-10}$ Tesla ($r=R$; $\theta=0$)

As in eq. 39) the ($\geqq$) sign in eq. 58) is to be read as (r) is infinitesimally greater than (R_i). Equation 58a) gives the magnetic field due to the rotating charge alone as in eq. 38).

In the following it will be assumed that energy can be equated with an equivalent positive or negative mass. For the energy in the electric field we have:

59) $$W_E=(\epsilon_0\div2)\int_0^{2\pi}\int_0^{\pi}\int_{R_i}^{R_0}[Q^2\div(16\pi^2\epsilon_0^2r^2)]\sin\theta\,dr\,d\theta\,d\phi$$

$$W_E=[Q^2(R_0-R_i)\div(8\pi\epsilon_0R_iR_0)]\cong7.73\times10^{-4}\text{ Joule}$$

For the energy in the magnetic field we have the following:

60a) $$W_B=(2\mu_0)^{-1}\int_0^{2\pi}\int_0^{\pi}\int_{R_i}^{R_0}(B^2R_i^6\div r^4)\sin\theta\cos^2\theta\,dr\,d\theta\,d\phi$$

$$=[2\pi B^2R_i^3(R_0^3-R_i^3)\div(9\mu_0R_0^3)]\cong1.60\text{ Joule}$$

60b) $+(2\mu_0)^{-1}\int_0^{2\pi}\int_0^{\pi}\int_{R_i}^{R_o}(B^2R_i^6\div r^4)\sin^2\theta\cos\theta\,dr\,d\theta\,d\phi=0$ Joule

60c) $+(2\mu_0)^{-1}\int_0^{2\pi}\int_0^{\pi}\int_{R_i}^{R_o}(B^2R_i^6\div r^4)\sin^3\theta\,dr\,d\theta\,d\phi$

$=[\pi B^2R_i^3(R_o^3-R_i^3)\div(9\mu_0R_o^3)]\cong 0.8$ Joule

60d) $W_B=[\pi B^2R_i^3(R_o^3-R_i^3)\div(3\mu_0R_o^3)]\cong 2.4$ Joule

60e) $W_{EB}=\pm(W_E+W_B)\cong\pm 2.400773$ Joule

60f) $m_x=\pm[(W_E+W_B)\div c^2]\cong\pm 2.67\times 10^{-17}$ kg

Such a small mass is undetectable by any concievable laboratory balance upon which the experiment would be performed.

In the MKSA test case following a similar line of reasoning and letting $(R_i=R_1)$ and $(R_o=\infty)$ we have:

61) $W_E=Q_1^2\div(8\pi\epsilon_0R_1)=4.493775894\times 10^9$ Joules

62) $W_B=\pi B_1^2R_1^3\div(3\mu_0)=8.33333333\times 10^5$ Joules

63) $m_x=\pm[(Q_1^2\div(8\pi\epsilon_0c^2R_1))+((\pi B_1^2R_1^3)\div(3\mu_0c^2))]$

$=\pm[((\mu_0Q_1^2)\div(8\pi R_1))+((\pi\epsilon_0B_1^2R_1^3))]=\pm 5.000927207\times 10^{-8}$ kg

Unfortunately this scheme of calculating the energies in the (**E**) and (**B**) fields does not take into account rotation which is central to this theory. To do this we take the Lorentz Force Law:

64) $\mathbf{F}=Q(\mathbf{E}+\mathbf{v}\times\mathbf{B})$ 64a) $\mathbf{v}=f_1R_1\sin\theta\,[\boldsymbol{\phi}]=\omega R_1\sin\theta\div 2\pi\,[\boldsymbol{\phi}]$ 64b) $f=\omega\div 2\pi$

Here ($\mathbf{v}$) denotes the velocity of the charge elements on the surface of the sphere, ($\boldsymbol{\phi}$) is a unit vector in the (ϕ) direction and (ω) is the angular velocity in radians per second. Taking the cross product we have:

65) $\mathbf{v}\times\mathrm{B}=\pm\omega\mathrm{BR}\sin\theta\div(2\pi)\,[2\cos\theta\boldsymbol{\theta}-\sin\theta\mathbf{r}]$

Writing out eq. 64) gives:

66) $\mathbf{F}=[(Q^2\div(4\pi\epsilon_0R^2))\,[\mathbf{r}]+\omega QBR\sin\theta\cos\theta\div(2\pi)\,[\theta]-\omega QBR\sin^2\theta\div(2\pi)\,[\mathbf{r}]]$

Integrating over the surface of the sphere we obtain:

67) $$\mathrm{F}=\int_0^{2\pi}\int_0^{\pi}\mathbf{F}\,d\theta\, d\phi=[(\pi Q^2\div(2\epsilon_0R^2))-(\pi\omega QBR\div2)]$$

67a) $$\mathrm{F}=[(\pi Q^2\div(2\epsilon_0R^2))-\pi^2 QBfR]$$

In eq. 67a) we see that the second term looks familiar and that the first term does not depend upon (f). Furthermore the first term takes on enormous values in the macroscopic cases giving implausible forces and masses. In light of this it is proposed that the macroscopic electrodynamic single body force can be expressed by:

68) $$\mathrm{F_x}=\int_0^{2\pi}\int_0^{\pi}Q(\mathbf{v}\times\mathrm{B})\,d\theta\, d\phi=-\pi^2 QBfR$$

As before we have that natural mass and positive artificial mass have a negative single body force and that negative artificial mass has a positive single body force. This result could make it necessary to re-evaluate (G_x) values (eqs. 18 & 20-24) and it may well make it necessary to re-evaluate the (G_x) value given by eq. 55c) but even so this would mean that, at worst, values for the macroscopic case (m_x) values are off by a factor of (π).

CONCLUSION

While this theory has some decidedly speculative aspects hopefully the reader will find something of value herein. For now it cannot yet be said that a sound theoretical means exists to calculate mass from electrodynamics. Perhaps one will be found someday. Hopefully this paper has provided some meaningful clues towards that end. In the meantime, performing the suggested experiment would be one way of assesing the validity of this theory. Should the result of this experiment be correctly predicted by eq. 55) then this would lend credibility to the derived value of G_1 and to the notion of electrodynamic mass itself.

TABLE OF VALUES

VALUE	UNITS	RELATIVE UNCERTAINTY $(\times 10^{-9})$	VARIANCE OR COVARIANCE $(\times 10^{-9})^2$
$\pi=3.141592654$	dimensionless	-	-
$c=2.99792458\times10^{8}$	(m^1s^1)	exact	-
$\epsilon_0=8.854187817\times10^{-12}$	$(m^{-3}kg^{-1}s^4A^2)$	exact	-
$\mu_0=1.25663706\times10^{-6}$	$(m^1kg^1s^{-2}A^{-2})$	exact	-
$R_\infty=1.0973731534\times10^{7}$	(m^{-1})	1.2	#
$h=6.6260755\times10^{-34}$	$(m^2kg^1s^{-1})$	598	358197
$e=1.60217733\times10^{-19}$	(s^1A^1)	303	92109
$\Phi_0=2.06783461\times10^{-15}$	$(m^2kg^1s^{-2}A^{-1})$	297	87988
$\alpha=7.29735308\times10^{-3}$	dimensionless	45	1997

ELECTRON VALUES

VALUE	UNITS	RELATIVE UNCERTAINTY $(\times 10^{-9})$	VARIANCE OR COVARIANCE $(\times 10^{-9})^2$
$m_e=9.1093897\times10^{-31}$	(kg)	591	349702
$A=1.001159652193$	dimensionless	0.01	0.0001*
$f=1.23558978\times10^{20}$	(s^{-1})	89.4	7987
$R_c=3.86159326\times10^{-13}$	(m)	89.4	7987
$\mathfrak{M}_B=9.2740154\times10^{-24}$	(m^2A^1)	335	112330
$B=8.8280110\times10^{9}$	$(kg^1s^{-2}A^{-1})$	334	111440
$V=3.7289406\times10^{3}$	$(m^2kg^1s^{-3}A^{-1})$	296	87862
$G_x=1.2127430\times10^{34}$	$(m^3kg^{-1}s^{-2})$	598	357181

#Covariance for R_∞ works out to be -1 $(\times10^{-9})^2$. ($R_\infty=\frac{1}{2}m_ec\alpha^2\div h$).

MUON VALUES

$m_\mu=1.8835327\times10^{-28}$	(kg)	609	371254
$A=1.0011659230$	dimensionless	8.4	70*
$f=2.5548075\times10^{22}$	(s^{-1})	172	29539
$R_c=1.8675948\times10^{-15}$	(m)	172	29539
$\mathfrak{M}_m=4.4852219\times10^{-26}$	(m^2A^1)	366	133882
$B=3.7742497\times10^{14}$	$(kg^1s^{-2}A^{-1})$	365	132992
$V=7.7102657\times10^{4}$	$(m^2kg^1s^{-3}A^{-1})$	331	109414
$G_x=2.8366190\times10^{29}$	$(m^3kg^{-1}s^{-2})$	615	378733
$m_\mu/m_e=206.768262$	dimensionless	147	21552

PROTON VALUES

$m_p=1.6726231\times10^{-27}$	(kg)	592	350110
$A=2.792847386$	dimensionless	23	529*
$f=2.26873158\times10^{23}$	(s^{-1})	91.6	8395
$R_c=2.10308932\times10^{-16}$	(m)	91.6	8395
$\mathfrak{M}_p=5.0507865\times10^{-27}$	(m^2A^1)	336	112738
$B=2.9763259\times10^{16}$	$(kg^1s^{-2}A^{-1})$	334	111848
$V=6.8469045\times10^{6}$	$(m^2kg^1s^{-3}A^{-1})$	297	88270
$G_x=3.5970888\times10^{27}$	$(m^3kg^{-1}s^{-2})$	598	357589
$m_p/m_e=1836.152701$	dimensionless	20.2	408

*The variances for A are included only for the purpose of comparison.

EQUATIONS FOR ERROR ANALYSIS

$f = mc^2 \div h$

$R_c = h \div (2\pi cm)$

$\mathfrak{M}_t = eh \div (4\pi m)$

$B = 4\pi m^2c^2 \div (eh)$

$V = cem \div (2\epsilon_0 h)$

$G_x = hc \div (2\pi^2 m^2)$ (microscopic cases only)

These equations can be used with the expanded covariance and correlation coefficient matrix which is given in {1}, page 27. An abbrieviated matrix which contains theory variances is given here. The variance of (K_V) is included because of its relationship to (Φ_0) and the variances of (K_Ω) and (K_V) are included because of their relationship to the ampere. The ratios (m_μ/m_e) and (m_p/m_e) are assumed to be uncorrelated to the values represented in the matrix. Variances appear on the main diagonal in **bold** face type and covariances appear above the main diagonal both in parts in $(10^9)^2$; correlation coefficients are below the main diagonal in *italics*

	α^{-1}	K_V	K_Ω	e	h	m_e
α^{-1}	**1997**	-1062	925	-3059	-4121	-127
K_V	*-0.080*	**87988**	90	89050	177038	174914
K_Ω	*0.416*	*0.006*	**2477**	-835	-744	1105
e	*-0.226*	*0.989*	*-0.055*	**92109**	181159	175042
h	*-0.154*	*0.997*	*-0.025*	*0.997*	**358197**	349956
m_e	*-0.005*	*0.997*	*0.038*	*0.975*	*0.989*	**349702**

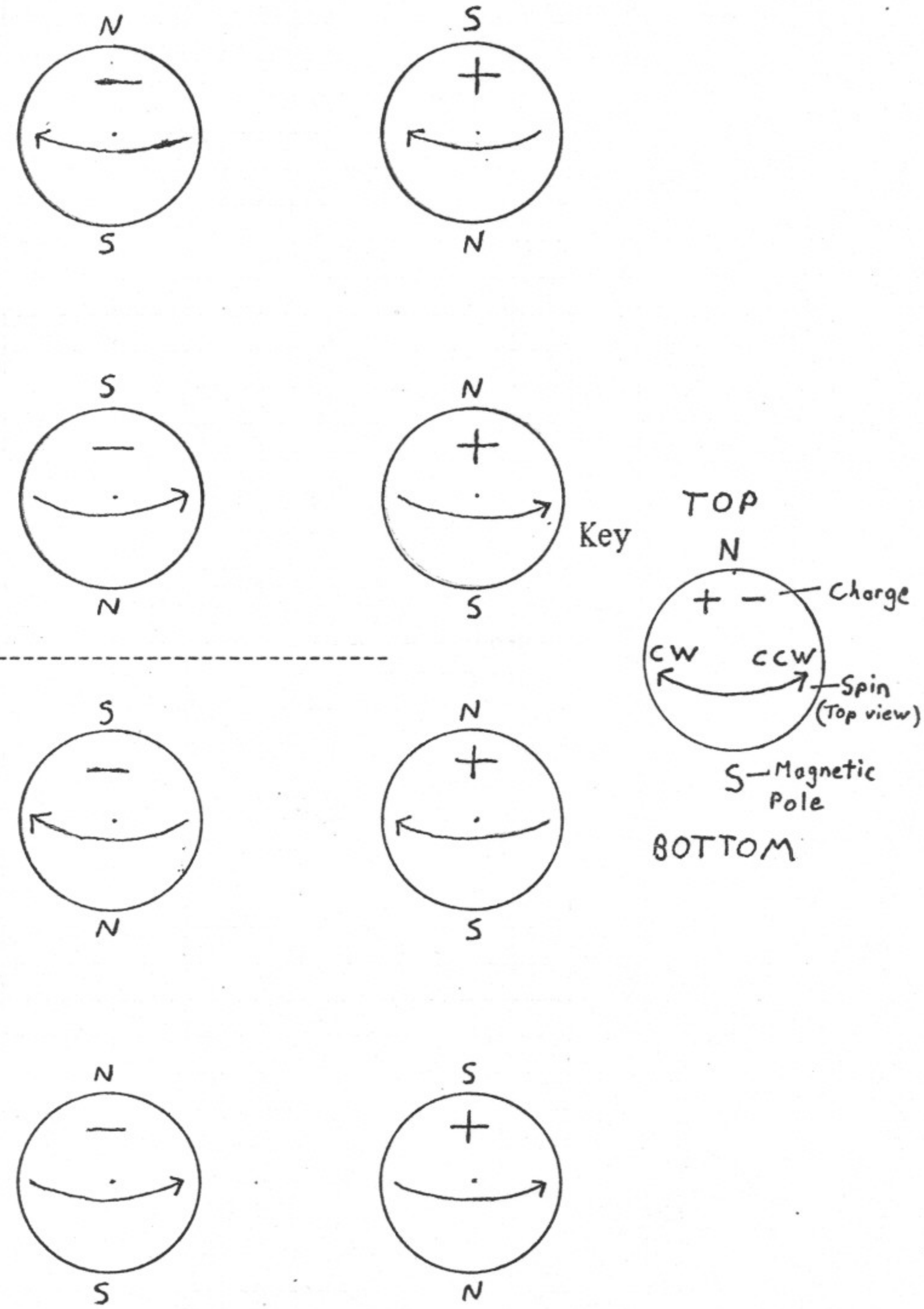
fig. 1 SPIN-FIELD-MASS-DIAGRAM

Positive Mass

(Occurs naturally)

Negative Mass

(Cannot Occur Naturally)



SPIN-FIELD-MASS-TABLE

CONVENTIONS

Q or V	B	f	sign in m_x^2
+		CCW	+
−		CW	−

Note: Q (or V): are positive (+) with proton charge and negative (−) with electron charge.

B: the orientations are as shown.

f: this is the observed rotation in the laboratory as viewed from above. CW denotes "clockwise" and CCW denotes "counterclockwise".

POLARITIES

Q or V	f	B	m_x
+	+	+	+
+	+	−	−
+	−	+	−
+	−	−	+
−	+	+	−
−	+	−	+
−	−	+	+
−	−	−	−

fig. 2 EXPERIMENT CONCEPTION

fig. 2a Idealized Setup:

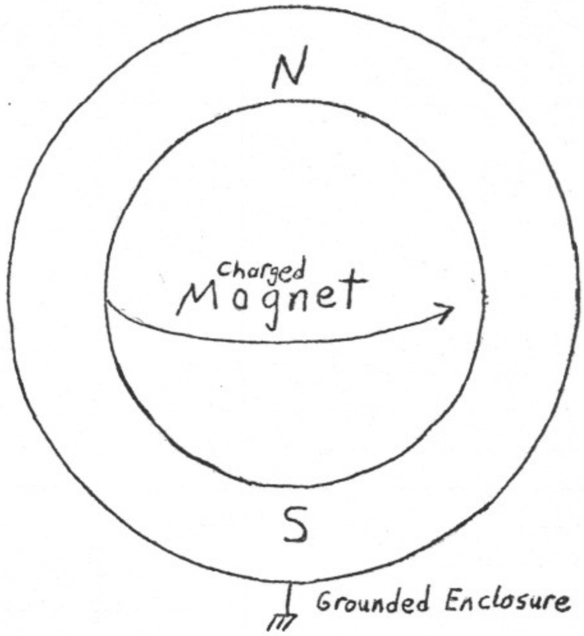
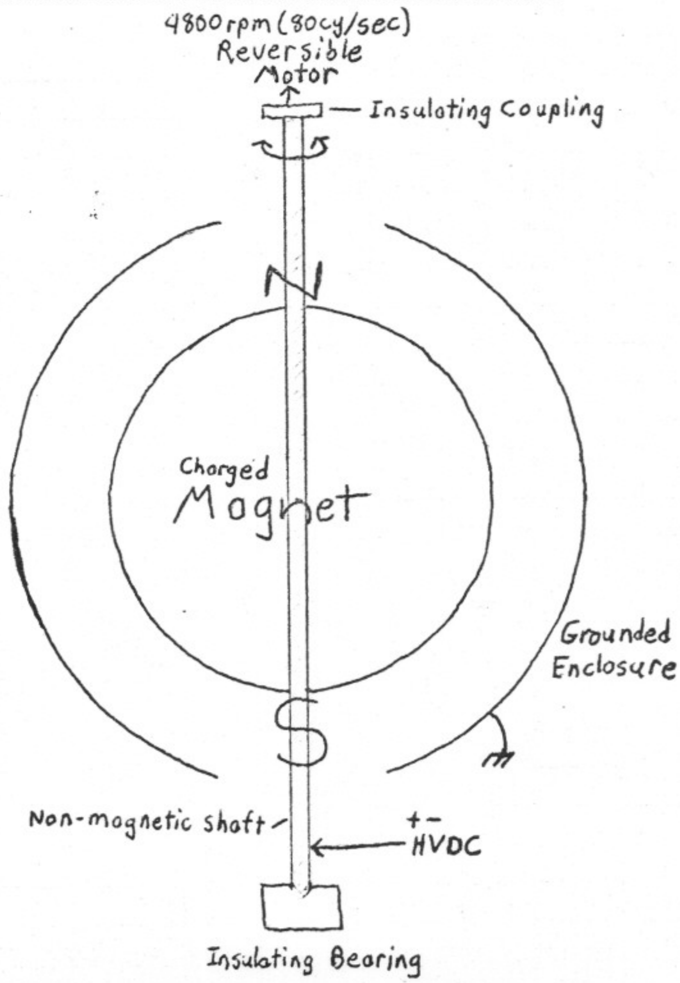


fig. 2b Practical Setup:



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Note: see erratum concerning the electron magnetic moment anomaly. The correct value appears on page 4 of this citation.

(2) Griffiths, David

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I would especially like to thank all of the people who have made contributions to all knowledge of the fundamental physical constants.

Also a special thank-you for The Evergreen State College computer services for their kind assistance with their word processing facilities.

Charles Ames Paper

2/28/87 ref 12/24/86

MKSA test case; an attempt at an empirical derivation of the Natural Gravitational Constant:

Memory X: $\frac{\alpha^2 c^4 B f R^3}{QV^2} = 5.325132371\times10^9\ Nm^2/kg^2$ Q = 1 coulomb B = 1 tesla f = 1 cy/sec R = 1 meter

Memory Y: $\frac{16\pi^2 f^3 R^3}{QB} = 1.579136704\times10^2\ Nm^2/kg^2$ $(16\pi^2)$ } Memory G: $G_n = 6.672\times10^{-11}\ Nm^2/kg^2$

$V = \frac{Q}{4\pi\epsilon_0 R} = 8.987551788\times10^9$ Volts

Memory Z: $\left(\frac{Y}{X}\right) = \frac{16\pi^2 f^3 R^3}{QB}\cdot\frac{QV^2}{\alpha^2 c^4 B f R^3} = \frac{16\pi^2 V^2 f^2}{\alpha^2 c^4 B^2} = 2.965441221\times10^{-8}$ (dimensionless)

$\rightarrow \frac{(m^4\cdot kg^2\cdot s^{-6}\cdot A^{-2}\cdot s^{-2})}{(m^4\cdot s^{-4}\cdot kg^2\cdot s^{-4}\cdot A^{-2})}$ = (dimensionless)

$\frac{YZ^{1.5}}{\pi^2} = 8.170600248\times10^{-11}\ Nm^2/kg^2$

$\frac{YZ^{1.5}}{G\pi^2} = 1.224610349$ $M_x = \sqrt{QBfRG_n^{-1}} = 1.224255267\times10^5$ kg

$\frac{\alpha^2 YZ}{\pi^4\sqrt{2}} = 6.67470513\times10^{-11}\ Nm^2/kg^2 \div G_n = 1.000405445$

$\rightarrow \frac{256\pi^3 f^5 V^2 R^3}{c^4 QB^3 \times 4\sqrt{2}} = 6.67470513\times10^{-11}\ Nm^2/kg^2 \frac{(s^{-5}\cdot m^4\cdot kg^2\cdot s^{-6}\cdot A^{-2}\cdot m^3)}{(m^4\cdot s^{-4}\cdot A\cdot s\cdot kg^3\cdot s^{-6}\cdot A^{-3})} = m^3\cdot kg^{-1}\cdot s^{-2}$ ok

$\frac{2^{(7.75)}\pi^3 f^5 V^2 R^3}{c^4 QB^3} = 6.67470513\times10^{-11}\ Nm^2/kg^2$ $(2^{10})^{31/40} = 2^{7.75}$

$\frac{\pi^3 f^5 V^2 R^3}{c^4 QB^3} = 3.100627668\times10^{-13}\ Nm^2/kg^2$

$\frac{\pi}{2}\cdot\frac{1}{\alpha}\cdot\frac{\pi^3 f^5 V^2 R^3}{c^4 QB^3} = \frac{\pi^4 f^5 V^2 R^3}{2\alpha c^4 QB^3} = 6.674277986\times10^{-11}\ Nm^2/kg^2$

$6.674277986\times10^{-11}\ Nm^2/kg^2 \div G_n = 1.000341425$

at R_c $G_{xpt} = \frac{256\pi^4 f^5 V^2 R^3}{2\alpha^2 c^4 QB^3}$

for the MKSA test case:

$\frac{\pi^4 f^5 V^2 R^3}{2\alpha c^4 QB^3} = \frac{\pi^4 f^5 R^3}{2\alpha c^4 QB^3}\cdot\overbrace{\frac{Q^2}{16\pi^2\epsilon_0^2 R^2}}^{V^2} = \frac{\pi^2 f^5 QR}{32\alpha\epsilon_0^2 c^4 B^3} = 6.674277984\times10^{-11}\ Nm^2/kg^2$

$\frac{\pi^2 f^5 QR}{32\alpha\epsilon_0^2 c^4 B^3} = \frac{\pi^2 f^5 QR}{32\alpha\epsilon_0^2 B^3}\cdot\frac{\mu_0^2\epsilon_0^2}{1} = \frac{\pi^2\mu_0^2 f^5 QR}{32\alpha B^3} = 6.67427798\times10^{-11}\ Nm^2/kg^2$

$\frac{\pi^4 f^5 V^2 R^3}{2\alpha c^4 QB^3} = \frac{\pi^4 f^5 V^2 R^3}{2\alpha QB^3}\cdot\frac{\mu_0^2\epsilon_0^2}{1} = \frac{\pi^4\epsilon_0^2\mu_0^2 f^5 V^2 R^3}{2\alpha QB^3} = 6.674277982\times10^{-11}\ Nm^2/kg^2$

$\frac{\pi^4\epsilon_0^2\mu_0^2 f^5 V^2 R^3}{2\alpha QB^3} = \frac{\pi^4\epsilon_0^2\mu_0^2 f^5 R^3}{2\alpha QB^3}\cdot\frac{Q^2}{16\pi^2\epsilon_0^2 R^2} = \frac{\pi^2\mu_0^2 f^5 QR}{32\alpha B^3} = 6.67427798\times10^{-11}\ Nm^2/kg^2$

ref 9/16/85 Fine structure constant $\alpha = \frac{\mu_0 c e^2}{2h}$

ref 12/28/86 & 2/28/87

for R_{pt} : $\frac{\alpha^2 c^4 B f R^3}{eV^2} = 2.665162238 \times 10^{16}\ N\,m^2/kg^2$ $\times\left[\left(\frac{U_{p^+}}{U_{cp^+}}\right)\left(\frac{1}{\alpha}\right)\right]^2$

$1.534697955 \times 10^{14}\ m$ $\frac{16\pi^2 f^3 R^3}{eB} = 3.9037958836 \times 10^{21}\ N\,m^2/kg^2$

$\frac{256\pi^4 f^5 V^2 R^3}{\alpha^2 c^4 e B^3} = 5.718084145 \times 10^{26}\ N\,m^2/kg^2$ $\times\left[\left(\frac{U_{p^+}}{U_{cp^+}}\right)\left(\frac{1}{\alpha}\right)\right]^2$

All for MKSA test case : $\frac{\pi^2 f^5 QR}{32\alpha c^4 \epsilon_0^2 B^3} = \frac{\pi^2 f^5 QR}{32 c^4 \epsilon_0^2 B^3}\cdot\frac{2h}{\mu_0 c e^2} = \frac{\pi^2 h f^5 QR}{16 c^5 \epsilon_0^2 \mu_0 e^2 B^3} = 6.674277986 \times 10^{-11}\ N\,m^2/kg^2$ ok

$\alpha = \frac{\mu_0 c e^2}{2h}$

$= \frac{(m^2\cdot kg\cdot s^{-1}\cdot s^{-5}\cdot A\cdot s\cdot m)}{(m^5\cdot s^{-5}\cdot m^{-6}\cdot kg^{-2}\cdot s^8 A^4\cdot m\cdot kg\cdot s^{-2}\cdot A^{-2}\cdot A^2\cdot s^2\cdot kg^3\cdot s^{-6}\cdot A^{-3})}$

$\epsilon_0 = \frac{1}{\mu_0 c^2}$ $\frac{1}{\epsilon_0} = \mu_0 c^2$ $c^2 = \frac{1}{\mu_0 \epsilon_0}$

$= \frac{(m^3\cdot kg\cdot s^{-5}\cdot A)}{(kg^2\cdot s^{-3}\cdot A)} = (m^3\cdot kg^{-1}\cdot s^{-2})$

$$\frac{\pi^4 \epsilon_0^2 \mu_0^2 f^5 V^2 R^3}{2\alpha Q B^3} = \frac{\pi^4 \epsilon_0^2 \mu_0^2 f^5 V^2 R^3}{2 Q B^3}\cdot\frac{2h}{\mu_0 c e^2} = \frac{\pi^4 \epsilon_0^2 \mu_0 h f^5 V^2 R^3}{c e^2 Q B^3} = 6.674277984 \times 10^{-11}$$

$$\sqrt{QBfR^3G^{-1}} = \sqrt{QBfR^3\cdot\frac{2\alpha Q B^3}{\pi^4 \epsilon_0^2 \mu_0^2 f^5 V^2 R^3}} = \sqrt{\frac{2\alpha Q^2 B^4}{\pi^4 \epsilon_0^2 \mu_0^2 f^4 V^2}}$$

$$= \frac{QB^2\sqrt{2\alpha}}{\pi^2 \epsilon_0 \mu_0 f^2 V} = 1.224046326 \times 10^5\ kg$$

$= \frac{(A\cdot s\cdot kg^2\cdot s^{-4}\cdot A^{-2})}{(m^{-3}\cdot kg^{-1}\cdot s^4\cdot A^2\cdot m\cdot kg\cdot s^{-2}\cdot A^{-2}\cdot s^{-2}\cdot m^2\cdot kg\cdot s^{-3}\cdot A^{-1})} = \frac{(kg^2\cdot s^{-3}\cdot A^{-1})}{(kg\cdot s^{-3}\cdot A^{-1})} = kg$

$$\sqrt{QBfR^3G^{-1}} = \sqrt{QBfR^3\cdot\frac{c e^2 Q B^3}{\pi^4 \epsilon_0^2 \mu_0 h f^5 V^2 R^3}} = \sqrt{\frac{e^2 Q^2 B^4 c}{\pi^4 \epsilon_0^2 \mu_0 h f^4 V^2}} = \frac{QeB^2}{\pi^2 \epsilon_0 f V}\sqrt{\frac{c}{\mu_0 h}}$$

$= 1.224046325 \times 10^5\ kg$

$$\sqrt{QBfR^3G^{-1}} = \sqrt{QBfR^3\cdot\frac{2\alpha c^4 Q B^3}{\pi^4 f^5 V^2 R^3}} = \sqrt{\frac{2\alpha c^4 Q^2 B^4}{\pi^4 f^4 V^2}} = \frac{c^2 Q B^2}{\pi^2 f^2 V}\sqrt{2\alpha}$$

or $\frac{QB^2\sqrt{2\alpha}}{\pi^2 \mu_0 \epsilon_0 f^2 V} = 1.224046326 \times 10^5\ kg$ $= 1.224046325 \times 10^5\ kg$

$$\sqrt{QBfR^3G^{-1}} = \sqrt{QBfR^3\cdot\frac{32\alpha \epsilon_0^2 c^4 B^3}{\pi^2 f^5 QR}} = \sqrt{\frac{32\alpha \epsilon_0^2 B^4 c^4 R^2}{\pi^2 f^4}} = \frac{4\epsilon_0 c^2 B^2 R\sqrt{2\alpha}}{\pi f^2}$$

$= 1.224046325 \times 10^5\ kg$