The Impact of Mathematical Modeling on Student Learning and Attitudes

Nicola Wethall

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Sunshine Campbell, M.Ed., Member of the Faculty

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Abstract

This study addresses the following research questions: How might mathematical modeling impact student learning among a diverse group of high school students who have been traditionally underserved? How might modeling affect these students' attitudes toward learning mathematics? Participants in the study included 77 11^{th} and 12^{th} grade students enrolled in three sections of a transitional math course. The course was designed to support low-achieving students prior to enrollment in a regular Algebra 2 course. The study took place over an eight-week period. Data was collected using mixed methods: pre/post surveys, classroom videotapes, and individual interviews with six students. Data was analyzed using the five strands of mathematical proficiency. Research findings included: 1. Students recognize a positive impact of the mathematical modeling process and how it relates to their learning. 2. Students are more willing and able to try new problems and take risks with the types of mathematical processes they attempt. 3. Transfer among mathematical concepts, new problems, and contextual situations can occur, but this transfer requires guidance from an instructor to become a flexible process.

Key words: mathematical modeling, low-achieving students, minority status, low socio-economic status

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Chapter 1: Introduction and Literature Review

Motivation for the Study

American high schools are not serving all students equitably (Ladson-Billings, 1997; Lewis, 2003; Ogbu, 1987; Schoenfeld, 2002). This can be seen through achievement gaps and graduation rates of students reported in the National Assessment for Educational Progress. Students with the lowest socioeconomic status have the lowest average NAEP test scores. Additionally, African American and Hispanic students have lower test scores than white and Asian American students (AAUW, 2008). NAEP results also suggest African American students are more likely to receive mathematics instruction focusing on basic fact computation while their white counterparts are more likely to receive instruction that goes beyond computational skills, with an emphasis on reasoning to solve unique problems (Lubienski, 2002).

These results are mirrored in the high school where I teach. In particular, students with low SES and minority status are significantly overrepresented in courses designed to provide mathematics remediation or additional support. The National Council of Teachers of Mathematics (2000) suggests all students should have access to high-quality, engaging mathematics instruction. A recommended pathway toward this goal is to provide all students with opportunities to represent and understand quantitative relationships while using mathematical models. This NCTM recommendation paired with mathematics performance among low achieving students leads to the following research questions:

- How might mathematical modeling impact student learning among a diverse group of high school students who have been traditionally underserved?
- How might modeling affect these students' attitudes toward learning mathematics?

How Students Learn

All new learning involves transfer from previous learning experiences (National Research Council, 1999). Current brain research suggests that students gain access to new information by connecting it to prior knowledge or existing neuronal networks. As students grapple with complex problems and test their ideas these neuronal networks are extended and strengthened. Although practice is an essential step in developing efficient processes throughout the brain, direct instruction alone does not typically result in deep learning. This is because a teacher telling/direct instruction model merely requires students to take in new information and attempt to relate to that information. It does not necessarily require students to engage their frontal cortex or to complete the learning cycle in the brain by formulating hypotheses or testing out new theories (Zull, 2002). However, when students are engaged in problem solving, such as problems involving mathematical modeling, they have opportunities to make connections across mathematical areas in addition to learning about applications outside of the math classroom (NCTM, 2000).

How Students Learn *in Mathematics*

In this study, I will take a socio-cultural approach which stems from a Vygotskian perspective. This standpoint asserts that an individual's learning is affected by the classroom or the outside world (Cobb, 1994). Prior knowledge and context will allow students access to the mathematical modeling problems because of the design and nature of the problems. Their learning will occur as they receive assistance from classmates and their teacher while they are in their zone of proximal development (Fuson, Hufferd-Ackles, & Sherin, 2004). This perspective situates learning through the lens of participation rather than acquisition of knowledge alone (Sfard, 1998). It is participation in these classroom activities that provide the social context for students' mathematical development (Bowers & Cobb, 1999).

In addition to a discussion of *how* students learn, it is critical to address *what* is important for students to learn in mathematics. The five strands of mathematical proficiency capture this quite well: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (National Research Council, 2001). Conceptual understanding refers to students' comprehension of mathematical concepts, operations, and relations. Procedural fluency relates to students' skill in carrying out procedures efficiently. Arguments have existed for decades regarding which of these two aspects of student learning is more important and how teachers' instructional practices may provide support (Hiebert, 1986; Schoenfeld, 2004). Current research on how the brain learns may support the notion that they are equally important provided that some level of conceptual understanding is reached first (Zull, 2002). However, the National Research Council argues conceptual and procedural knowledge are only two of the five aspects accounting for mathematical proficiency. A third, strategic competence, relates to students' ability to formulate, represent, and solve mathematical problems. A fourth, adaptive reasoning, addresses students' capacity for logical thought. Finally, productive disposition deals with students' vision of mathematics as sensible and useful in addition to self efficacy. Although these five aspects do not address the *content* of mathematics, they are, "interwoven and interdependent in the development of proficiency in mathematics" (National Research Council, 2001, pp. 116).

The NCTM Standards and recommendations from the National Research Council have been supported by research and available to the public for many years. Nevertheless, these suggestions for mathematical reform have not been accepted or implemented consistently (Schoenfeld, 2004). In addition to the heated political debates over reform versus traditionally taught mathematics, the TIMSS 1999 video study provides clear evidence that U.S. math teachers reinforced attention to low-level mathematical skills and procedural knowledge. Math teachers from higher achieving countries balanced their attention among challenging content, procedural skill, and conceptual understanding (Gallimore et al., 2005). This balance seems more in line with the five strands of mathematical proficiency than the primary focus on procedural knowledge that is so predominant in American teaching.

Low-Achieving Students

Given the diversity of cultural experiences brought to the mathematics classroom, it is curious that mathematics continues to be taught in a way which is clearly consistent with white middle-class cultural experiences focusing primarily on the demands of “efficiency, consensus, abstraction, and rationality” (Ladson-Billings, 1997, p. 699). Thus, most existing math cultures exclude groups such as African Americans and children living in poverty because they do not necessarily conform to the white middle-class cultural experiences. For example, African American culture tends to value artistic and physical expressions such as music, poetry, and dance, which differs from the values of school mathematics culture. Educational opportunities are lost when students are not prompted to access their prior knowledge and address how it supports or conflicts with school mathematics. This form of exclusion is generally unrecognized within the perspective of math as culturally neutral (Bishop, 1988).

The lowest performers in American schools are often students of color and students with low socio-economic status (AAUW, 2008; Lewis, 2003). The social construction of race is unintentionally taught in our school system. It is used to categorize individuals regardless of any race or ethnicity a student may choose to identify with. Many educators claim to treat all students equally, explaining they really do not see differences. However, ignoring racial differences intensifies the problem of unequal outcomes, discrimination, and oppression because we are marked by categories which lower our status relative to others (Johnson, 2006). This presumably nonexistent categorization substantially reduces students' resources, including access to challenging mathematics. For example, results of the 1999 TIMMS showed that teachers distinguish themselves from other countries by using computational strategies with low-achieving students. Overall U.S. teachers use conceptual teaching strategies at about the international average (Baker, Desimone, Smith, & Ueno, 2005). Unfortunately, these conceptual strategies are not accessible to low-achieving students, a subgroup that is heavily populated by students of color and low SES.

Attempts have been made to address these inequities through the implementation of reform curriculum that is aligned with NCTM Standards. Data show that students using reform curriculum show no significant differences on basic skills than students using traditional curriculum. In fact, students using reform curriculum consistently outperform students using traditional curriculum on problem solving (Fey et al, 2000; Hirsch & Schoen, 2003; Schoenfeld, 2002). In fact, evidence shows that reform curriculum can narrow the achievement gap between white students and underrepresented minorities in addition to the gap between low and high SES students (Schoenfeld, 2002). Regrettably, even effective reform math curriculum has been rejected by school districts and communities because of philosophical differences between traditionalists and reformers about what is best for students to learn (Schoenfeld, 2004). This means that a potential solution for low-achieving students in mathematics classes may have been lost.

In addition to lost opportunities in the regular mathematics classroom, low-income students, African American students, Latino students, and other minority students are often clustered in low-ability mathematics classes (Guiton & Oakes, 1995a; Ladson-Billings, 1997). These classes are less demanding and offer even fewer opportunities. The TIMMS indicates the U.S. is the country with the most tracking and the strongest link between achievement and socioeconomic status. Students who are ability-grouped into higher tracks often have access to better curriculum, more prepared teachers, and higher levels of mathematics during high school. Students who are ability-grouped into lower tracks have less access to resources. They also report losing motivation, receiving low-level work, and receiving lower expectations from teachers. The lower tracks clearly reduce their chances for success in life (Boaler, 2009; Guiton & Oakes, 1995b).

School faculties describe tracking as a way to meet the needs of student populations and the workforce. They claim that opportunities for students to take higher track courses are structured as open, fair, and merit-based. However, the result is a disproportionate representation based upon students' racial, ethnic, and social class (Guiton & Oakes, 1995b). Once low-achieving students are enrolled in the lower-track courses they are confronted with few teachers who believe it is important for them to develop higher order thinking (Degani et al, 2000). Growth in student achievement is significantly lower in these courses than in college-preparatory classes. It is not student ability that accounts for these differences; it is increased rigorous content coverage in the college-preparatory classes that account for achievement advantages (Gamoran, Porter, Smithson, and White, 1997). Achievement in transition courses, such as the Mathematical Modeling Course in this study, has been found to fall between the level of achievement gains found in low and high-level tracked courses.

What is Mathematical Modeling?

Mathematical modeling is one of many practices students can engage in to move beyond computational skills. NCTM (2000, p. 303) explains:

> Modeling involves identifying and selecting relevant features of a real-world situation, representing those features symbolically, analyzing and reasoning about the model and the characteristics of the situation, and considering the accuracy and limitations of the model.

This process requires students to think about a problem, pose a question, and use mathematics to solve it. Mathematical modeling is also consistent with how the brain works through the learning cycle, which was described above. This is because of the level of engagement and processes used while solving these types of problems. However, it is critical not to overly contextualize the mathematical modeling problems (National Research Council, 1999). This may reduce transfer to new problem situations if students relate solutions to particular problems without making generalizations about classes of problems. This means students may have access to math problems as they relate prior knowledge to the context or representations involved. Yet they need help making connections among mathematical modeling contexts, applications to other contexts, and mathematical generalizations. Using a modeling approach to problem solving requires multiple cycles of interpretations and re-interpretation as students' solutions evolve (Doerr & English, 2003).

Context is most often used by teachers as a way to provide relevance to students while focusing on the goal of deriving a mathematical formula. A more effective use of context involves allowing the context to be treated as a way to support, reflect, and expand on students' interests even when the context does not help solve the problem. Context becomes important as it plays an affective role in connecting students to mathematics and vice versa, offering them new opportunities to learn mathematics (Chapman, 2006; Connor & Zbiek, 2006). It is not necessary that context be restricted to real-life scenarios. In fact, it is not the models themselves that make growth in mathematical understanding possible; it is the students' modeling activities (Van Den Heuvel-Panhuizen, 2003).

Benefits of mathematical modeling have been cited in many recent research articles. The modeling work motivates students to engage in mathematics in three ways: the real-world situations appeal to some learners, students want to continue to study mathematics, and students want to learn new mathematics. Students make connections among mathematics and develop deeper conceptual understanding (Boaler, 2001; Connor and Zbiek, 2006). Modeling problems allow for many different approaches with a built-in requirement for communication. Initial attempts at verbalization lead to refinement of mathematical understandings (Doerr & English, 2003). Modeling can lead students to use criteria for evaluating algebraic representations (Izak, 2003). Students using a modeling approach have been found to outperform students using a traditional curriculum on problems that require formulating algebraic models of quantitative relationships and interpreting results of algebraic calculations (Fey et al, 2000). Finally, students participating in a curriculum focused on mathematical modeling see the relationship between school mathematics and mathematics used in the real world. They have opportunities to practice mathematics they will use outside of the classroom (Boaler, 2001).

Classroom Structures that Support a Modeling Approach

Attempts at improving instruction and achievement through a student-centered, constructivist approach may have a positive impact by addressing students' prior knowledge (Taylor, 1996). However, these attempts by reform-minded educators cannot be successful unless the cultures of school communities are transformed to treat mathematics as a tool for scientific inquiry rather than a myth of cold reason. This treatment of mathematics as a set of facts to be disseminated and practiced contributes to minority students' distrust of educators as it conflicts with cognitive, communication, and interaction style (Ogbu, 1987). To address the need for relevance of mathematics in students' lives, educators must consider real world solutions in addition to the real world problems presented to students (Boaler, 2002). Teachers can help students to build deeper understanding and more elaborate schemes through persistence and willingness to discuss the reasonableness of ideas and solutions (Francisco & Maher, 2005).

When classroom interactions support a community of inquiry the teacher and students set up a Zone of Proximal Development using scaffolding, peer collaboration, and the interweaving of spontaneous and theoretical concepts (Goos, 2004). This process allows students to gain entry into new mathematical discussions as the teacher asks questions that help them to access prior knowledge and withholds judgment regarding their responses. Students have achieved the highest gains in mathematical reasoning when metacognitive training, such as the reflective work described above, is paired with cooperative learning (Kramarski & Mevarech, 2003). In designing this cooperative group work, learning tasks should require conceptual reasoning rather than learning to apply rules (Cohen, 1994). In these groups students collectively build their solutions by sharing, comparing, and critically evaluating their ideas and strategies as they come up with them. The students develop ownership of the process of doing mathematics and of the resulting forms of the knowledge they build together (Francisco & Maher, 2005).

Whether instruction takes the form of cooperative learning groups or whole class discussion, teachers must become aware of status characteristics within the classroom and intervene using strategies such as multiple ability treatments and assigning competence to low-status students (Cohen & Lotan, 1995). In addition to these strategies, educators can positively impact achievement by collaborating with colleagues to carefully design instruction around a unifying theme and by providing challenging work that is of high cognitive demand for all students.

Chapter 2: Methods and Analysis

Setting

This study took place in a rural town with a population of about 30,000. The high school had a population of approximately 1,600 students. Half of the students were members of military families because this small town supports a naval base. In this district 36.7% of students were considered to be of low socioeconomic status as determined by free and reduced lunch participation. It is possible that the actual percentage of students who were eligible for free and reduced lunch was larger since high school students are less likely to report this status. The racial backgrounds of students in this district were: 20.8% made up of American Indian/Alaskan Native, Pacific Islander, Black, or Hispanic while 79.2% of students are White or Asian. These statistics were reported on the state superintendent's website. I chose to categorize students into these two groups because they align with research on the relationship between ethnic/racial backgrounds and low-achieving students.

Political Climate

Reform curriculum had been implemented in the elementary and middle schools in this district for several years. However, a recent attempt to adopt reform curriculum at the high school level that is aligned with NCTM Standards and current brain research was met with criticism from several vocal community members, school board members, and members of the math department. Concerns were raised over negative media reports of reform curricula and the fear of dropping standardized test scores. It is unclear what percentage of the community actually resisted adoption of reform curriculum. A traditional curriculum was chosen for the high school as a result. In addition, the reform math curriculum at the lower grades was replaced with traditional curriculum. These outcomes regarding curriculum choice and textbook adoption also impacted teachers' pedagogical decisions in that many teachers continued or began using a direct instruction approach.

Participants

The participants included 77 juniors and seniors in three sections of a math course that I taught. This course was designed for students who have not met standard on the mathematics portion of our state high school proficiency exam and did not perform well in both Algebra 1 and Geometry. The class was intended to support students so they could meet standards on the test rather than allowing them to struggle in Algebra 2 and possibly not graduating high school. The classes included 52% of students with low SES and 50.8% of students with minority status other than Asian. This was an overrepresentation of typically underserved students who were placed in a lower level transition math course. These statistics are not meant to single out this particular school, rather to demonstrate an issue of equity that persists in schools across the country.

This was the first year that the course was redesigned as a result of the new Algebra 2 high school graduation requirement. The result is that the new course gave students access to Algebra 2 standards, allowing seniors a better chance of graduating on time and juniors the opportunity to take a college preparatory Algebra 2 course during the following school year. The textbook chosen for this intervention course was better aligned with current research in mathematics education than the more traditional material chosen for students at other levels. In particular, the textbook was aligned with the five strands of mathematical proficiency and focused on mathematical modeling. I chose to study these students in this course because they were representative of students whose academic needs were not being met in our school system as they progressed through our traditional mathematics curriculum.

It is critical to note that students' prior experiences in mathematics courses may have impacted their attitudes during the study. Math teachers in this high school predominantly used a direct instruction approach that does not incorporate the use of mathematical models. Many teachers allowed students to work together on assignments during class time but did not facilitate collaborative group work. My classroom was distinct in that I trained students to work together in cooperative learning groups which utilized group roles and norms from the beginning of the school year. In addition, I used an inquiry approach to instruction with an attempt to make my classroom a student-centered environment. Because of this many students experienced a drastic change in pedagogy during the course of this study. However, their past experience may have influenced their perception of what math is and what types of math are valuable. Attitudes may have changed but the change might not have been solely attributable to the math modeling. Other attitudes may not have changed at all within the short period of time allotted for this study as some students' previous experiences may have caused them to be disinterested and disengaged.

Data Collection and Analysis

The study was conducted using a mixed methods approach. The use of mixed methods allowed me to gather both quantitative and qualitative results (Mertens, 2010). In particular, the quantitative data allowed me to assess whether any changes in attitudes occurred across all students and support these findings through statistical analysis. Through a qualitative data collection approach I was also able to gain a deeper understanding of student perspectives and experiences by analyzing dialogues from student to student or student to teacher interactions. The study included three sources of data. The data collection occurred in sequential form as some sources of data (survey results and videotaped lessons) impacted the choice of subjects in another source of data (interviews).

Survey

The first source of data was a survey given to all students at the beginning of the school year. The survey was developed and used by Jo Boaler (2008) in the Railside Study to assess student attitudes about mathematics. Similarly, I used this survey to analyze students' thinking about the importance of math in their lives, how they believe school math is related to math outside the classroom, and what makes students successful in mathematics. After about eight weeks of classroom instruction the students took the survey again. I looked for any changes in the survey results by matching each student's initial and final responses. Many of the questions included response choices that fit a numerical scale. For these questions I performed a matched-pairs t-test to identify any significant changes in attitudes. For some of the questions that did not fit a numerical scale, I looked at proportions of students who responded a particular way. To identify any significant changes in overall responses to these questions I conducted a 1-proportion z-test. Through these analyses I was able to detect any trends in changing student attitudes over the course of the study and attempt to support the findings statistically. The survey also asked open-ended questions about good and bad math lessons, what is interesting to students in mathematics, and what helps them learn. These questions provided feedback that addressed an impact on student learning as some students compared lessons using mathematical modeling to other approaches.

Video

The second source of data was video analysis of classroom interactions. I videotaped each class once per week over the course of eight weeks. The videotapes captured whole class discussion and small group interactions. I used the first two videos in each class to determine if there was a particular group I wanted to focus on during small group work for the remainder of the study. The analysis of these videos included transcription and coding to identify any evidence of student growth in each strand of mathematical proficiency. The dialogue and behaviors drew out complexities in student attitudes and learning that could not be revealed in a survey.

Interviews

The final source of data came from interviews conducted with six students, two from each of the three classes. These students were chosen based upon changes in attitudes and learning that became apparent through analysis of the surveys and videotaped class sessions. These semi-structured interviews lasted approximately fifteen minutes. Planned questions included:

- Describe yourself as a student in math class.
- What do you think causes a student to be successful when studying math?
- What are the things in our class that help you to learn?
- What are the things in our class that get in the way of your learning?
- How does mathematical modeling affect your learning?
- What mathematics do you think is important to learn?

Students were also asked to elaborate on their responses when necessary. Their textbook was available for reference during the interviews. Responses were audio taped, transcribed, and coded like the videotapes. The purpose of the interviews was to gain greater insight into students' perspectives about what and how they are learning. In order to reduce bias in student responses I did not conduct the interviews. Instead, one other math teacher at the high school conducted all interviews.

Limitations

One area of concern in this study is that I was the classroom teacher and was closely connected with the participants. Although this may have allowed me greater insight into students' thinking, it might also contribute to inferences made that are not supported in the data. To avoid this pitfall, I needed to act as a stranger when analyzing data. My role as a researcher analyzing the data is distinct from my role a teacher participating in classroom interactions. I have improved the validity of my findings by having my research advisor and colleagues independently analyze critical pieces of data to be sure these individuals came to the same conclusions that I did.

A second area of concern is that I conducted inferential statistics which require random sampling and approximately normal distributions to be generalizable. Due to the structure of public schools, I worked with the sample of students available in my classroom rather than choose a completely randomized sample, which would have insured the normality of the distribution. However, the sample size of students was sufficiently large enough to allow for this statistical analysis. This is because the sampling distribution of all samples this size would be approximately normal. I cannot suggest that my results are generalizable to the entire population of students who are experiencing the inequities I am addressing. Nevertheless, results of my study may be transferable to similar groups of students in similar communities with similar prior experiences in mathematics. That is, my study is limited but may still provide evidence of impacts on student learning and attitudes.

A final concern is that the use of mathematical modeling may not be the only new instructional technique that was experienced by these students in this setting. Many of them have had little experience working in structured cooperative learning groups or working in a math classroom that focused on inquiry. This made it impossible to completely separate the effects of mathematical modeling, cooperative learning, and an inquiry approach. However, classroom structures such as cooperative learning and an inquiry approach are well-developed teaching strategies used in my classroom. I believe they supported the use of mathematical models in a way that makes these models more effective. It was not appropriate to abandon these effective teaching strategies for the sake of developing a pure teaching experiment that reduces the effect of confounding variables. In analyzing the data I needed to find convincing evidence that the modeling itself positively impacted learning and attitudes. I will also admit that my results do not stand alone on the merits of mathematical modeling. In order to transfer any findings to other students, teachers must consider using classroom structures and/or cooperative learning to support the study of mathematical models.

Conclusion

The variables described above as limitations will have a confounded relationship with the effects of mathematical modeling. For this reason, I will be unable to determine *to what degree* mathematical modeling impacts learning and attitudes. However, the three sources of data in this study are each intended to address the research questions about *how* mathematical modeling impacts learning and attitudes and to add to the current body of research in this area. Trends were identified in each data source and supported by results of the other sources to improve reliability. Limitations will be considered when describing the implications of this study.

Chapter 3: Description of Participants and Research Findings

The Classroom Setting

A total of 62 students participated in my three Advanced Algebra Prep classes throughout the eight-week period of data collection. Another five students dropped out of school, four students switched to other classes, and six students joined our classes late. These frequent changes impacted classroom dynamics and required students to be flexible with changes among their group members. In addition, the participants in this study included juniors and seniors from varied mathematical backgrounds. Some students passed Algebra 1 and Geometry, but struggled in both. Among these students, some were placed in a remedial course called Segmented Mathematics during their junior year and then this course during their senior year. Others were placed into this course during their junior year. Other students failed and retook either Algebra 1 or Geometry before registering for this class as a senior. Finally, some of the students in this course have already taken Algebra 2 and either failed or struggled so much that it was recommended they take this course which is meant to be taken *prior* to Algebra 2.

The one thing all of my students have in common is that they did not meet standard on our state's high school proficiency exam for mathematics. This fact had an enormous negative impact on student attitudes that was apparent on the first day of school. Many students entered the classroom making comments about being in the "math class for stupid people" or they tried to make light of their poor performance in math. Assigning competence is one of the ways that I try to combat these notions and empower students. When a student or group figures out a complicated problem I give them credit as being experts and send others to them for advice. Or I call attention to a students' progress on a solution strategy and ask how we can take that idea further. In addition, assigning competence has the potential to improve students' attitudes about learning mathematics and get them more engaged in the learning activities. This elevation in status is something that many of my students have not experienced in previous math courses. Subsequently, it may be difficult to separate the positive effects of assigning competence from the mathematical modeling process being addressed in this study. This is because both assigning competence and the use of mathematical modeling may impact attitudes and learning to some degree.

Another aspect of our daily classroom routine involves the extensive use of cooperative learning groups. While I think this is an integral part of mathematical modeling, the level of interdependence among group members goes beyond the mathematical modeling process. In general, a curriculum which provides instruction through mathematical modeling would require students to think about mathematics in the context of some problem (possibly collecting data), devise a plan, then use mathematics to represent and solve that problem. Once again the effects of modeling become interwoven with the support provided by cooperative learning groups and the inquiry method I use to facilitate these groups.

Finally, our new school-wide redo policy provides another classroom structure to support student success. Because I believe in this initiative I encourage all of my students to come to after school tutoring for extra help when working on incomplete or missing assignments. I also allow my students to retake assessments after they've had an opportunity to address the learning targets in which they did not meet standard. This creates a classroom atmosphere in which students who have traditionally been unsuccessful in math begin to value learning and take pride in their achievement.

Analysis of Data

The following sections provide evidence of how two of the five strands of mathematical proficiency developed throughout the study. This analysis leads to research findings regarding impact on student attitudes, students' willingness to take risks in problem solving, and students' ability to transfer mathematical knowledge. I have chosen to focus on these two strands because the initial coding of data presented the strongest evidence of growth in the areas of productive disposition, or attitudes toward mathematics, and strategic competency, or formulating and devising strategies. This is not to say that there was no growth in the other three areas. However, this study aims to dig deeper into the most notable changes that occurred as a result of mathematical modeling. All pre and post survey responses, classroom video transcriptions, and interview transcriptions were coded using the five strands. Some interactions among students solving mathematics problems or answering interview questions fit into multiple categories or strands. Anecdotal evidence is provided in addition to the three main types of data collection because some powerful interactions were not caught on videotape. Interactions and interviews in this data analysis only include students who are from low SES backgrounds and/or students who have self-ascribed minority status.

Productive Disposition. This is described as "Seeing mathematics as sensible, useful, and doable – if you work at it – and being willing to do the work" (National Research Council, 2001, p. 9).

On the second day of school students were given a survey to assess their individual attitudes about learning mathematics. I was surprised by the high number of students who reported that school mathematics was in fact like mathematics used in the real world. This led me to believe that although most of my students were not experiencing success in high school mathematics, many of them may regard mathematics as useful. However, I have considered the possibility that even by the second day of school my students knew what I valued as a mathematics teacher and some of them may have responded in such a way to please me. In either case, the post surveys, which matched and compared students' previous responses to their responses at the end of the study, show an increase in the *number of students* who responded favorably to this question. However, the proportion of increase is *not statistically significant to show evidence of a change* in their beliefs overall.

Throughout the eight-week period I witnessed a distinct change in a student named Marie. She is a student who has failed previous math courses and is making up math credits through the alternative school in addition to attending our class. One of the early investigations required students to determine models to represent relationships between weights applied to a spring and the amount of stretch on the spring. Each of these relationships involved graphs, tables, equations, and verbal models of linear functions determined from data collected by students. During a subsequent lesson on multiple representations of linear relationships Marie demonstrated that she is developing confidence in her ability to solve problems and work together with other students on mathematics. This assignment required students to transfer knowledge from the context of stretching springs to other contextual problems and to math problems out of context. In the following excerpt Marie guides group members:

> *Marie*: Do you feel sick? (Connor motions that he is not) OK, so did you get 3a?
>
> *Connor*: Yes?
>
> *Ken*: Come on, it's Connor… (Ken's facial expression suggests that Connor is not capable of solving the math problem.)
>
> *Marie*: Yeah, but he needs to *get it*. He needs to *process the steps*. So write down your equation first. What is it you're solving for?
>
> *Connor*: x. So I'm going to multiply?
>
> *Marie*: Yeah. You might need your calculator. And then you're going to divide by 12. (Marie watches Connor try to work out the problem.) OK. So now 3b.

This interaction offered Marie an opportunity to show that she was able to successfully apply her knowledge of solving equations from the spring context to new situations. She was shy about working with other students in the first few weeks of school because she didn't believe in herself as a mathematics student. As she worked through many math modeling lessons Marie began to offer her opinion confidently, demonstrating that she was beginning to see mathematics as *doable* for her. This was also an opportunity for Marie to demonstrate that she believes solving math problems is *useful*. During the interaction she responded to Ken who was criticizing another student's mathematical ability. In doing so Marie advocated for her peer by showing that she valued helping Connor learn a process rather than allowing him to copy from a group member. Marie's facial expression, words, and tone revealed that she wanted Connor to make sense of the mathematics. Marie showed evidence of her productive disposition as her self-confidence improved and as she held her peers accountable for learning mathematics.

At the end of the eight weeks Marie was asked to participate in an interview given by another math teacher. The first question was for Marie to describe herself as a student in math class:

> *Marie*: I used to be not very good at math. I've always consistently failed in math class. But I try to learn as much as I can without… I ask questions and… (Marie pauses)
>
> *Teacher*: So you said you *used* to be.
>
> *Marie*: Yeah.
>
> *Teacher*: Has something changed?

Marie: Well, this math class is a lot more helpful than my… than other math classes I've taken… So this year, I don't know if it's my attitude towards school or if it's the way I'm being taught.

Marie did not clearly state why a change occurred. However, she made it clear that she is trying to learn and that *something* in this math class is helping her to learn. She pointed out that this is a new experience for her as a math student and that the math modeling class is positively impacting her as a student. Later in the interview she explained:

Marie: I like the investigations. I like, I like the experiments that we do in the book. They're really helpful to me. Especially since I'm like a hands on learner. So when we do the investigations it's really easy for me to do them and figure it out… I don't know why.

Teacher: Well, when you're investigating do you make connections with what you're doing and what you already know?

Marie: Sometimes. And I like that they're kind of realistic situations. They're not just talking about building skyscrapers and stuff. Like who knows that at our age?

Teacher: OK. Can you give me an example of something realistic?

Marie: Well, I've always had an interest in forensic science and right now we're learning about the relation between your head length and how tall you are. So

> that's kind of interesting because I've always kind of been into forensic science. And like other experiments like the Slinky experiment – we all know what Slinkys look like. So just like talking about the strength of them and the distance that they stretch, it's just more realistic than talking about building a skyscraper or building a football field.

As Marie dug deeper into her own thinking about mathematics she revealed more about her productive disposition. She shared that the modeling topics we have studied so far are useful and make sense to her in addition to being connected to the prior knowledge she has constructed. Marie also pointed out that she recognized attempts at introducing mathematics through contextual problems such as building skyscrapers or football fields in previous math instruction. However, she is aware that those isolated attempts at using word problems do not necessarily lead to productive mathematics or relevance to students' lives.

An additional example of a student's productive disposition occurred during group work regarding a particular math model. Dawn is another student who has not experienced success in previous math classes. She has a positive attitude about learning in general, but does not see herself as a competent mathematics student. I guided Dawn's group in choosing the independent variable for the context of that problem. Rather than tell Dawn the answer, I asked her questions that helped her to determine the appropriate variable and a reasonable domain. She was visibly excited when she figured it out, showing a sense of ownership for her solution. In addition to observing

Dawn's joy at solving a math problem, I recognized this as an opportunity to raise her status. When another group was experiencing difficulty on the same problem I sent that group to Dawn for advice, thus assigning her competence. To my surprise, Dawn began the same inquiry method I had used to help her choose the independent variable. She started asking the students her version of the same guiding questions that I had asked her. When they said "just tell me the answer" Dawn responded "No way! I want you to *learn it* so you'll know how to do it on your test". Then she was persistent with her questions until they figured out the solution. Dawn experienced success during this math modeling investigation. She used this success to become a model for her peers, insisting that if they were willing to do the work she would continue to help them solve their math problems.

A student named Ben demonstrated a similar kind of change in beliefs during this eight week period. At the beginning of the school year Ben seemed unimpressed with the class and mathematics in general. By the second week Ben was beginning to become more engaged in his cooperative group work. During a whole class discussion Ben excitedly offered his solution regarding a modeling problem. Unfortunately for Ben, the class figured out that the solution was incorrect. Suddenly Ben looked defeated and shrunk into his seat. My response to the class was that I was so relieved to hear Ben's reasoning and the class's attempt to make sense of it. I explained that I believed making mistakes and sharing misconceptions was a crucial part of solving mathematical problems as well as other types of problems. Ben looked as though he felt validated. This was confirmed as he returned to class the next day. Ben told me that our interaction in the previous class was the highlight of his day and that he had shared the story with friends and family. He said that he was beginning to understand how making mistakes was a vital part of learning. This process of testing ideas and revising our strategies became a regular occurrence in the mathematics modeling classroom.

Most of the students in these three classes had earned D's and F's in all previous high school math classes. They had claimed to give up on learning mathematics because they believed it was not worth learning or that they were incapable of learning it. However, as the action research project progressed, students became more involved in solving mathematical modeling tasks. Many of them showed strong signs of increased confidence in their ability to learn mathematics and even showed greater interest in earning higher grades in the course.

Strategic Competency. This is described as "Being able to formulate problems mathematically and to devise strategies for solving them using concepts and procedures appropriately." (National Research Council, 2001, p. 9).

Strategic competency was addressed on the pre and post student survey by asking which statement students agree with more: "Success in math is mainly about memorization" or "Success in math is mainly about thinking for yourself" (see the Appendix). Those who tend to believe strategic competency is a valuable trait for mathematics students to possess would be expected to choose the latter. The proportion of students reporting that success was about thinking for yourself only rose from 38% on the pre student survey to 41% on the post student survey. This is an improvement, but such a small change in beliefs is not statistically significant and may have been caused by chance variation.

An example of how strategic competency appeared in classroom videotapes occurred during a lesson on inverse functions in which a constant is being divided by the independent variable ($y = k/x$). The constant can be determined by multiplying corresponding values of the independent and dependent variables. Students' first encounter with inverse functions involved an investigation relating the distance away from a wall and the apparent size of an object hanging on the wall. Their measurements created graphs of functions in the form $y = 63/x$. The follow-up activity provided new sets of data that they needed to type into their calculators. They graphed the data and analyzed the table values in order to conjecture about equations and test those equations. The following is an excerpt of one group's interaction:

> *Kally*: Now what do I do? (shows graphing calculator to Mara)
>
> *Mara*: OK, this one – you redid it? (pointing at a table of values that Kally typed into her graphing calculator)
>
> *Kally*: Yeah, I put it back in.
>
> *Mara*: OK, do you remember what I taught you? You put in the 2^{nd} then 1, times 2^{nd} 2 then enter. (shows Kally how to multiply list 1 by list 2 to find the product in list 3, essentially finding the constant by multiplying x by y values)
>
> *Kally*: Oh yeah, I remember.

Mara: And then whatever you get from your stat, it's just the number you get from the times, it's 12. So you go to y = and it's going to be 12 divided by x. And then graph and then… It's making me mad. (visibly frustrated because the graph is not showing up for this data set) Oh, it's cause you don't have your plots on. There you go. See? The line goes through all the points.

At this point Kally took the calculator back and began typing. She smiled as soon as she produced a table and graph of her own. While this was happening, Ty and Ben were trying to solve a similar problem. They hadn't come up with a plan of how to use the data to produce a function model yet:

Ty: I'm so confused and looking at the board just makes me more confused.

Ben: Yeah, I just see a bunch of numbers and letters. That's the trouble with math.

Ty: OK, so I've got this (shows calculator to Ben and Mara). Where do you go after that?

Ben: And I've got that (shows calculator to Ty and Mara).

Mara: OK, what'd you get? Go to the stat. And then look at L3.

Ben: Oh, we have to find that average. (typing L1xL2 into list 3)

At this point Ben has connected the ideas in this calculator process with the mathematical model for data he collected during the investigation relating the distance away from a wall and the apparent size of an object hanging on the wall. The only difference is that during the data collection activity he found the constant of variation by *averaging* the products of the independent and dependent values for all the data collected. In the current problem his products will all be the same, thus his average will be the number 9 which he sees repeated in L3. However, in Ty's problem the data set produces a constant number of 1.98. See below:

Ty: Yeah.

Mara: (speaking to Ty, looks at his list 3) And you just go 1.98 divided by x.

Ty: Oh, so we go to the y = and type 1.98 divided by x.

Ben: Oh, so there'll all 9. This is awesome. Oh my God. So 9's the slope, right? (Ben made an error in how he named the constant of variation by calling it slope. However, he realized that it is, in fact, a constant in the function)

Mara: Well, 9 divided by x. Yeah.

Kally: So mine would be 12 divided by x?

Mara: Yeah. (looks at Ben's graphing calculator) OK, so graph and… TA DA

Ben: Oh, that's so cool! That's perfect.

This interaction demonstrates how Mara was able to determine a method to find the inverse function using her graphing calculator and how she used that method to lead her group toward a solution. Mara developed a process that involved working through procedures on her graphing calculator while connecting the multiple representations of the graphs, tables, and equations related to these particular inverse functions. She shared strategic competency with group members. They were pleased as they began to develop understanding of the process and outcome, thus, demonstrating productive disposition.

Each of the problems described above showed how the group of students was able to transfer their understanding from the context of the investigation to math problems that arose from no particular context or scenario. The next problem they were asked to solve involved a new context about a group of senior citizens who were chartering a plane for $10,000. In this problem students would need to make a table of values representing variable rates per passenger depending upon the number of passengers who flew. Many students had a common misconception about the problem. They divided the $10,000 charter cost by 80 passengers and then decided that this resulted in a constant rate of $125 per passenger. Then they filled in a table of values for different numbers of passengers using this constant rate to represent a linear function rather than demonstrating the inverse relationship in which the rate per passenger changes. The following excerpt reveals how students overcame this misconception and worked through a plan to solve the problem:

Ty: OK. So we need this graph right here (points to table that they will fill in)

Mara: Oh.

Ben: OK. So I'm already doing it in the calculator. (shows graphing calculator to Ty) 140, 150… So it's 10,000x (looks to group for approval). So y = 10,000x is our equation. (made an incorrect assumption that this function is linear)

Ty: (typing in calculator) So if all the people go, it's $125 per ticket.

Ben: Wait. Did you divide 10,000 by 80?

Ty: Yeah. Cause for 80 passengers it would be $10,000. So it's $125 per ticket.

Ben: Oh. OK

Teacher: (hears that students are planning on a fixed cost of $125) Oh, if all 80 people go? What if only 10 go?

Ty: OK, so then…

Ben: Then we'll figure it out.

Ty: Oh! So then we do 10,000 divided by 10 people.

Ben: So then it's y = 125x? (Ben did not process what Ty just said)

Ty: So for 10 people it's $1,000 each. Of course, I should have known that!

This interaction demonstrates strategic competency in the group's willingness to persevere through the problem and try to come up with a plan to solve it. They used the information accurately to determine a price per passenger and they made an equation that modeled the table they produced. Also, they recognized that something was wrong with their solution. However, the interaction also provides evidence that students have difficulty interpreting the new context and transferring prior knowledge of inverse functions to this context. To address their misconception, I asked students to consider the cost per person for a different number of passengers. This helped Ty get on the right track. But the other group members were still struggling with what they thought was a linear function.

Teacher: OK, so tell Ben what you just told me.

Ty: OK. So it costs $10,000 for the plane. Also, it has maximum capacity of 80 people. So for 80 people it costs $125 a ticket. But if you have only 10 people it would cost $1000 a ticket. So if you do 10,000 divided by *how ever many passengers* there is then that's how much it's gonna cost.

Ty made sense of the context in this problem and how it relates to inverse functions. Encouraging him to explain his reasoning to group members was an opportunity to raise the status of a struggling math student. There are frequent opportunities to assign competence in this manner when working through mathematical modeling problems. The group members needed to deepen their understanding of Ty's process by asking questions:

Mara: Are you sure? Because listen. The cost to charter a private airplane is $10,000.

Ty: Yeah.

Mara: That's just it.

Ty: Yeah, because there's a maximum of 80 people. So you do 10,000, that's how much it costs. So you divide that by how many people there are to see how much they have to pay per ticket.

Mara: OK (furrowed brow, doesn't seem convinced). So the equation would be what?

Ben: 125x (still making an error by creating a linear function)

Mara: That doesn't make sense to me.

Ben: $125 per person. So 125x. (hasn't processed what Ty just explained)

Kally: How do you do this? I don't have my sheet with me.

Mara: Wait, 125x? (starts typing) Are you sure? Cause it's 1250 for the first one ("the first one" corresponds to 10 people)

Ben: The first one? I thought that was 1,000.

Ty: They're $1,000 each for the first one.

Mara: Are you sure? Cause it says 1250 for the equation Ben told me. (erases 1,000) So for 10 passengers is $1,000 each is wrong?

Ty: No, wait. So you do 10,000 divided by how many passengers there are.

Mara: I know, but his equation that he's coming up with is 125x.

Ben: Yeah cause it's $125 per customer. So…

Ty: No, cause this is cost per passenger (points at y – values) So how much is it going to cost for… but how many is it going to cost for 80 per person? So you put that here. And then for 70? And then for 60?

Finally, the group followed Ty's thinking and Mara wrote the equation $y = 10{,}000/x$. She demonstrated her strategic competency when she questioned the reasonableness of Ben's equation and tested it. Together they devised strategies to solve this problem by applying procedures such as finding table values under given constraints and concepts such as determining functions to represent verbal models and table values.

Interactions like this were common throughout the eight week study. This is likely due to the investigative nature of modeling tasks, which led students to conjecture about math problems and test out their ideas with peers. Students confirmed their individual growth in this area by solving related problems on formative and summative assessments.

An example of how students established an awareness of strategic competency arose in an interview with Ben:

Teacher: OK. So, anything else that really helps students to be successful besides just examples?

Ben: Well, as long as they can kind of picture how the thing works out in their head. And find a simpler way to do something, like their own way. It's really an intuitive process. You have to find your own way. So that's the way you're always going to do it. You know, you're not always going to do it the exact same way as the teacher.

Teacher: So you have to figure out what makes sense to you?

Ben: Uh huh.

These responses show how Ben finds it valuable for students to work through their own problem solving process in order to be successful in math. When asked what about this math modeling class helps Ben to make sense of mathematics he responded:

Ben: Um because you just kind of… it's more of a hands on thing when you do the investigation. You understand where the numbers are coming from.

Ben suggested that the nature of the math modeling problems increased his access to the mathematics itself. At the same time, it allowed him to make sense of the problems so that he could devise a plan to solve them. A similar response was given in an interview with a student named CeCe:

CeCe: I think using that (math modeling) book… it makes me feel more confident in using math. Because it's set up in a way and we're taught in a way that I can understand. And in my other math classes I didn't understand much and then we'd just move on when I didn't understand it. I like how the book takes things in multiple steps and it solves every problem. Well *it* doesn't solve every problem. But it's set up in a way that I can understand it so I don't feel like the dumbest person in the class.

CeCe has hit on an important aspect of math modeling problems. There is often a degree of scaffolding early on so that students can build confidence when solving math problems. This scaffolding gives them resources to draw upon when solving problems in which the scaffolding has been removed. She also addressed the way that this math modeling course doesn't just "move on". This is because she has opportunities to revisit concepts in new problem situations or problems out of context and then draw on what she knows to solve them, in turn, demonstrating strategic competency once again.

An example of how students solved a problem involving less scaffolding occurred when a later lesson required them to determine the longest possible driveway that could be constructed under given constraints. The solution to this problem was 27 feet and could be found by recognizing that an equation to model the problem might be $y = 108/x$ or $y = 9/x$. One of the challenges of this problem is that there was more information given than what is needed to solve the problem because students were presented with values for length, width, and depth, and volume of the driveway. Another challenge is that measurements are in multiple units and multiple dimensions.

Teacher: Now how do you *know* it's inverse?

John: Because we're using multiplication of these (points to x and y columns)

The group has come to an agreement that the function is inverse by checking the products of the x and y values. They made a plan to determine a function but seemed to be having difficulty with using the function when the units of measurement do not match and when there are three columns of data rather than two. They worked through this challenge and determined equations that modeled the situation.

Teacher: OK, so when we have multiplication of x and y variables that is constant we have an inverse function. So now you're at a point where you agree on an equation. So now what do you have to find in here? (referring to the table of values)

Ken: A missing value.

Teacher: OK, so you have a graph. You have an equation. You have a table. (pauses while students think)

Marie: A depth.

John: A depth of 4 inches.

Since the group agreed on what they were looking for I stepped back and allowed them to work through the problem. Two of the students were trying to solve the problem with an equation using feet as the unit of measurement ($y = 9/x$) while the other two students were converting their equation into values using inches ($y = 108/x$). Each of them devised a plan and was working it out by comparing solutions to their equation with the table of values and their graph.

Marie: OK. She said that we could convert inches and feet.

John: The length would be 27 feet.

Ken: How did you find that?

John: Well, I converted because 4 inches equals 1/3 of a foot. I think that's how you do it.

Marie: OK, so what did you get?

John: I got 27 feet. I just cross multiplied the .333 (has 1/3 = 9/x on his paper).

Marie: 1 over 3?

John: 1 over 3 being 4 inches.

John's solution was correct. However, Marie was troubled by it because her solution was not the same. She was frustrated because she tried to solve the problem by using an equation in inches and then converting her solution to feet. She found an incorrect solution because she did not properly isolate the variable. She explains below:

Marie: (calculating) Ok, I got 36.

John: 36 what?

Marie: 36 feet.

John: How did you do it?

Marie: OK, so I took this equation: 4 inches = 108 inches divided by x. And since it's divided by then I timsed it. (multiplying 4 by 12 instead of 4 by x). Which I don't know if that's correct. But it got me something better than point something something something. So now I have 432 = x. So I just divided by 12 because there are 12 inches in a foot. (John has questionable look on face, but does not interrupt)

Ken: Yeah, I'm not sure.

Connor: You're talking about your equation?

Marie: Do you understand why I wrote that?

Ken: Hmm 36 feet.

Marie, Ken, Connor, and John have each demonstrated their strategic competency during this classroom video as they related the table of values and context of the problem to inverse functions and devised a plan to solve it. They used procedural knowledge to find products of table values. They used conceptual knowledge to relate the relevant products to inverse functions and to create an appropriate inverse function to model this problem. John continued to use both conceptual and procedural knowledge as he solved his equation and checked his solution. However, Marie's procedural knowledge of the steps necessary to solve an equation with a variable in the denominator failed her. This group of students demonstrated perseverance because they did not give up on trying to understand Marie's process. They worked hard at getting her to be clear in her explanation so they could help her figure out what she did incorrectly. At this point I provided more questions to guide Marie's thinking.

> *Teacher*: (grabs scratch paper to write and looks at John's paper) Did you do this too?
>
> *John*: No, but I think I did it right. I got 27 and I'm pretty sure it's right.
>
> *Teacher*: (addressing Marie) How did you get the 108?
>
> *Marie*: 108 was the inches. I took 9 times 12.
>
> *Teacher*: Oh I see. So you're converting your whole equation into an inch equation instead. Nice. Do you guys agree with that? (nods) And now you tried something else (points at John's paper). So you're trying to get 4 inches as your output in this equation?

I could see that Marie's method would work if she solved the equation correctly, and John's method already worked. So I decided to work through the equation with them. I wrote 4 = 108/x. It took very little guidance to get Marie to fix her own mistake. In fact, I did not even tell her that she had made a mistake. I just asked questions and wrote down her responses. She found the solution herself.

> *Teacher*: So have you all tried to solve this? How do we solve this equation?
>
> *Marie*: Times by x?

Teacher: OK, so times by x on each side (shows division and multiplication by x cancelling) OK, so that makes the x go away (on the right side). And then what?

John: You've got 4x = 108.

Marie: Oh, that would have been so much easier!

Teacher: But you said it though, you said times by x. And then how do you get x by itself?

Connor: Divide, divide by 4.

John: And then you get 27. And that's what I got.

Marie: (after recording her work) It represents the length of the driveway.

This interaction demonstrated that when scaffolding was removed some students were able to work through the entire problem while others required teacher intervention to find the final solution. This intervention modeled for students how they might support each other in figuring out the thinking of their group members. Consequently, all students were able to devise a plan and apply their procedural and conceptual knowledge to work toward that solution.

Summary of Findings

Interactions among students during videotaped lessons, interviews between students and a mathematics teacher, and changes in student responses on pre and post surveys support three research findings regarding attitudes and learning among traditionally underserved students in mathematics courses. These findings were demonstrated through the analysis of students' productive disposition and strategic competency among all three types of data collected. Greatest support for the findings was revealed in the classroom videotapes and interviews. The findings are as follows:

1. Students recognize a positive impact of the mathematical modeling process and how it relates to their learning. This is demonstrated through their productive disposition, specifically their awareness of mathematics as useful, doable, and sensible.
2. Students are more willing and able to try new problems and take risks with the types of mathematical processes they attempt. This is demonstrated through their productive disposition as they show positive attitudes about trying new problems and excitement when solving these problems. Students demonstrated that they are willing to do the work of mathematical modeling. This finding is also demonstrated through their strategic competency as they develop plans to solve these problems and carry out plans using mathematical procedures and concepts.
3. Transfer among mathematical concepts, new problems, and contextual situations can occur, but this transfer requires guidance from an instructor to become a flexible process. This is demonstrated through their strategic competency and the occasional need for teacher intervention.

This analysis focused primarily on only two strands of mathematical proficiency. The remaining three strands are:

- Conceptual Understanding. "Comprehending mathematical concepts, operations, and relations – knowing what mathematical symbols, diagrams, and procedures mean."
- Procedural Fluency. "Carrying out mathematical procedures, such as adding, subtracting, multiplying, and dividing numbers flexibly, accurately, efficiently, and appropriately."
- Adaptive Reasoning. **"**Using logic to explain and justify a solution to a problem or to extend from something known to something not yet known." (National Research Council, 2001, p. 9).

Growth in the area of each of these strands was also noted in the coding of videotape transcriptions and student progress throughout the study. There was evidence of an impact on student attitudes and learning as a result of math modeling. In this short time period and through these types of data collection it was difficult to fully support findings among these other strands. The use of surveys, videotapes, and interviews tended to support conclusions related to productive disposition and strategic competency.

In addition, the evidence above is not an exhaustive list of all the interactions related to productive disposition and strategic competency. In fact, there were many more occurrences of data coded to represent growth in these areas. Because the aim of this study was to address how mathematical modeling impacts the learning and attitudes of students who are traditionally underserved, the data analyzed above is a representative sample of interactions with these students.

Chapter 4: Conclusions from Research Findings

Relating Findings of the Research Project to Findings in Literature

The data analysis in chapter 3 demonstrates evidence of growth in productive disposition and strategic competency during the course of this eight week study on mathematical modeling. The growth in these two strands of mathematical proficiency provides evidence for findings regarding positive impact on students' attitudes, students' willingness to take risks when solving mathematical problems, and students' ability to transfer mathematical knowledge to new situations. The relationship between these findings and existing research on mathematics education is described below.

Positive Impact

The classroom atmosphere captured on videotapes during this study showed students actively engaged with each other and focusing much of their energy on solving math problems. This can be partially attributed to the context of mathematical modeling problems that allowed students entry into mathematical problem solving and led them to discuss possible solution strategies with peers. This is consistent with research that shows context becomes important as it plays an affective role in connecting students to mathematics and vice versa, offering them new opportunities to learn mathematics (Chapman, 2006; Connor & Zbiek, 2006). Students also responded favorably during classroom instruction and during one-on-one interviews with regards to how the mathematical modeling process gets them engaged and helps them make connections among mathematical concepts and to careers involving mathematics. This aligns with other findings that suggest math modeling gets students engaged and interested in mathematics beyond the current mathematics course that they are studying (Boaler, 2001). In addition, students in this study showed great interest in working together to solve mathematical problems. The nature of math modeling investigations promotes and is supported by cooperative learning within a community of inquiry. Similar to findings in previous research, the instructional choices in this study offered students worthwhile problems to investigate and opportunities to collaborate with peers, leading to greater gains in learning (Cohen, 1994; Goos, 2004; Francisco & Maher, 2005; Kramarski & Mevarech, 2003)

Taking Risks

The majority of students in this study were accustomed to working on problems that had been modeled for them in a traditional setting of direct instruction. Prior to this course these students typically solved problems in which the objective was to practice procedures using different numbers than the numbers used in the teacher's example problems. The students were often considered successful in previous mathematics courses if they reached correct solutions with new practice problems. However, in this study students began to try new problems through investigations that were designed to lead them to mathematical models. In the beginning of the study students were very apprehensive and waited for direct instruction to guide them. As this study progressed I observed students attempting new problems with greater comfort. They began to use verbal models, tables, graphs, and function rules interchangeably to solve new problems. This is consistent with research on reform curriculum, which introduces mathematics in a student-centered, problem-based method similar to the mathematical modeling of this study. Such research showed evidence that students using reform curriculum consistently outperformed students using traditional curriculum on problem solving (Fey et al, 2000; Hirsch & Schoen, 2003; Schoenfeld, 2002). The focus of this study on students who are traditionally underserved is also consistent with research demonstrating that such reform curriculum can narrow the achievement gap between white students and underrepresented minorities in addition to the gap between low and high SES students (Schoenfeld, 2002).

Transfer of Knowledge

Students explored a variety of mathematical problems and scenarios throughout this study. A major concern of the study is that some students had difficulty transferring their knowledge from one context to another. This is consistent with the National Research Council’s (1999) recommendation to not overly contextualize mathematical modeling problems because students may believe the solutions are unique to particular scenarios rather than making generalizations about classes of problems. However, results of this study demonstrated that this difficulty can be overcome with careful planning on the teacher’s part. Students showed progress in their ability to transfer knowledge when they were offered some degree of scaffolding through teacher questioning or scaffolding within the problem. Early problems within a unit typically involved a great deal of scaffolding. This method, along with the introduction of previously learned concepts within new problem situations, provided opportunities for students to interpret and re-interpret their mathematical knowledge. This is consistent with a finding by Doerr and English (2003) that it is critical to work through *cycles* of mathematical modeling problems in this way to help students generalize their ideas and apply their knowledge to new situations. The benefits of mathematical modeling suggest it is worthwhile to promote transfer through scaffolding and questioning.

Implications of Findings and Their Relevance to Future Educational Practice

The results of this action research study are particularly powerful for this group of underserved students because these findings demonstrate the impact that a reformed teaching philosophy might have on students' attitudes and learning. In particular, the mathematical modeling process prompted student interest among those whose test scores and academic status suggested otherwise. These students were often drawn into math lessons through the context of particular problems or through physical investigations that involved collection and analysis of data. The level of student engagement in this study draws into question the common treatment of low achieving students which focuses on procedural knowledge through direct instruction. In previous courses involving these instructional methods students were experiencing failure; in this study involving mathematical modeling students were beginning to experience success.

The focus of this study on productive disposition and strategic competency does not suggest that practice is unimportant. In fact, procedural fluency is one of the five strands of mathematical proficiency that this particular mathematical modeling curriculum is organized around. During the study, and throughout the course, students had opportunities to practice. However, this practice occurred after conceptual understanding was attained. Practice was also incorporated into new problem situations as concepts were cycled throughout a chapter and then revisited in future chapters. Development of procedural knowledge through practice should be a goal of any mathematics curriculum, but possibly not the goal of initial instruction.

The positive impacts of this study did not occur through observation alone. They were the result of developing a community of inquiry through a mathematical modeling approach. It is critical to recognize that the teacher is an active member in this community. Certainly, we want to offer students challenging problems, allowing them time to process and grapple with ideas. When they are unable to move forward we need to also offer the appropriate scaffolding. Finally, we need to help all students develop generalizations and engage in practice so that they have the ability to transfer knowledge to new contexts.

Questions for Future Action Research

One of the greatest limitations of this study was the length of time available to collect data. Surveys, interviews, and six weeks of videotapes certainly provided plenty of data to analyze. However, this may not have been enough time to capture significant changes in students' attitudes and learning. In future research I would like to study student progress over a longer period of time. I would like to see whether changes that I observed were persistent over time, extending beyond the topics of linear and inverse functions that were introduced early in the school year. I would also like to see if an even greater impact would be noticeable as students became more accustomed to the structure and expectations of a course such as this. Only subtle changes were noted between the pre and post surveys in this study. Although I observed changes in attitudes, I would expect it to take more than eight weeks for students to experience changes in *beliefs* about learning mathematics. An extended length of time for data collection could also allow for results on standardized tests which include items based upon state or national mathematics standards and are scheduled near the end of the course.

Future action research could also address the other three strands of mathematical proficiency: conceptual understanding, procedural fluency, and adaptive reasoning. These strands might be addressed through additional classroom observations. It might also be helpful to use additional types of data collection such as administering pre/post assessments or collecting student work to determine whether students are meeting specific learning targets. A study such as this might include a comparison group that is not receiving instruction of mathematical modeling problems. Or it might be helpful to conduct a similar study by observing another teacher's classroom rather than videotaping my own. In addition, I would like to study the effects of mathematical modeling within a regular mathematics course, not just an intervention or support course, and how modeling impacts all students.

Another area of research might include a look at the effects of mathematical modeling with or without the additional structures in our class that are designed to promote student success. I suspect that teachers would not have the same results if they were to provide mathematical modeling problems without using an inquiry approach in small group or whole class discussion. I also suspect they would not see the same results if they had not trained their students to work effectively in groups or had not continued to support that group work.

Closing Comments

The purpose of this study was to observe the impact of mathematical modeling on the attitudes and learning of traditionally underserved students. Findings of the study offered evidence of a positive impact on these particular students. However, the benefits of mathematical modeling do not undo the problems of tracking. These students had access to this mathematical modeling curriculum only because they were not successful in the regular high school curriculum at our school. Although this course offered students a great deal of support, it has become another level of tracking. Instead, we might offer all students the benefits of mathematical modeling by adapting instruction in all math courses to support these low-achieving learners and others.

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Appendix

Math Questionnaire
Your answers will be kept strictly confidential

Math Class: ______________________________ Math Teacher: ______________________________

1. How much do you agree with these statements about math: (check one box)

	strongly agree	agree	disagree	strongly disagree
Math will be really important in my future career				
Other subjects are more interesting than math				
Math is really useful in life outside school				
Math is a lot of procedures that have to be memorized				

2. Which of these statements do you agree with MORE: (check one box)

School math is based on things that happen in the world ☐
OR
School math is very different from things that happen in real life ☐

3. Which of these statements do you agree with MORE: (check one box)

Success in math is mainly about memorization ☐
OR
Success in math is mainly about thinking for yourself ☐

4. In math class, how often do you:

	never	seldom	sometimes	often	always
Try to help your classmates solve a problem?					
Try to learn things because you want to get a good grade?					
Try to learn something new even when you don't have to?					
Try to get more answers right than your classmates?					

5. How much do you agree with these statements about math:

	strongly agree	agree	disagree	strongly disagree
It is important to use the teacher's method				
It's OK to make mistakes in work				
It is important to avoid looking stupid in front of others				
Students are encouraged to try new things				
It is good to make mistakes at the board				

- Please turn over -

Math Questionnaire

Your answers will be kept strictly confidential

6. I really enjoy math class when:

	strongly agree	agree	disagree	strongly disagree
The problems make me think really hard				
I am the only one who can answer a question				
I don't have to work hard				
The whole class learns together				
I am the first one to get a question right				

7. When I try hard in math it is because:

	strongly agree	agree	disagree	strongly disagree
I want to get a good grade				
The work is interesting				
I want to learn new things				
I want my classmates to think I'm smart				

8. Put these aspects of math in order of importance – put a 1 by the most important, a 2 by the 2nd most important etc.

memorizing facts and rules	
learning to use calculators and computers	
understanding big ideas	
finishing lots of work	
helping others learn	

9. Describe an idea you thought was really <u>interesting</u> in math class:

10. Describe a <u>really good math lesson</u> you have had (in this class or any other) –saying why it was good:

11. Describe a <u>really bad math lesson</u> you have had (in this class or any other) –saying why it was bad.

12. What helps you to learn math?