Questioning Practices and Students' Mathematical Justifications

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Abstract

Higher level thinking skills, like justification, are needed for students to become mathematically proficient. When students are able to provide a mathematical justification for their strategy or answer, they demonstrate a deeper understanding of the math concepts. Teachers can support students in their justifications by asking questions that press for a justification. This research study examined the relationship between teacher questioning practices and students' mathematical justifications in a fifth-grade classroom. Data included tests and quizzes, the teacher/researcher's journal, students' math journals, and video recordings of students working on group math tasks. Analysis of the data revealed four findings. First, I became aware of the distinction between two types of justification: justification of an answer and justification of a strategy. Second, students were more successful at providing verbal justifications than written justifications. Third, many types of questions prompted students to provide justifications, but questions that pressed students to justify prompted students to provide justifications more than any other type of questioning. Fourth, through my questioning practices, two sociomathematical norms emerged in my classroom. Students began to anticipate my questions and they began to question each other. Recommendations for further research include an examination of how teachers can encourage students to provide both verbal and written justification.

Keywords: justification, questioning, group worthy tasks, high press questioning

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Chapter One: Introduction and Literature Review

Importance of Communication and High-Level Thinking

Historically our nation's schools have focused on the importance of procedural knowledge when it comes to mathematics education (Boaler, & Staples, 2008; Fraivillig, Murphy, & Fuson, 1999; Kazemi, & Stipek, 2001; Kilpatrick, Swafford, Findell, 2001; Stein, Grover, Henningsen, 1996; Walshaw, & Anthony, 2008) If students were able to follow a set of rules and/or procedures to get to a correct answer, they were considered mathematically competent. However, research has contradicted this belief. More recent studies have found that in order for students to become mathematically competent, they need to be able to use higher level reasoning when it comes to mathematical concepts. NCTM (National Council of Teachers of Mathematics, 2000) states that all students should have the opportunity to learn mathematics concepts in depth. The deeper the level of thinking, the greater the level of conceptual understanding students will attain.

Kilpatrick, Swafford, and Findell (2001) argues that in order for students to be mathematically proficient, the five strands of mathematical proficiency need to be present: conceptual understanding, procedural fluency, strategic competence, productive disposition and adaptive reasoning. Although all five strands are important, some get more attention than others. In traditional math instruction the focus on procedures (procedural fluency) has received more attention than other strands. In order to help students understand mathematical concepts more deeply, it is critical students have the opportunity to develop adaptive reasoning – or the ability to discuss and communicate mathematical ideas. Adaptive reasoning is the capacity for logical thought, reflection, explanation, and justification (Kilpatrick, Swafford, & Findell; 2001). Adaptive reasoning is a key component because students are able to connect mathematical ideas, generalize their findings, and explain why something works instead of just focusing on getting the correct answer. This promotes a deeper understanding of mathematical concepts (Kilpatrick, Swafford, & Findell; 2001).

One of the ways students develop adaptive reasoning is through collaboration that can take place in a math-talk learning classroom (Hufferd-Ackles & Sherin, 2004). Mathematical discourse happens when students use adaptive reasoning in the classroom with their peers by making predictions, asking questioning, explaining, reasoning, justifying, and even debating the work. The focus of my study is on adaptive reasoning, specifically the importance of students communicating their justifications of mathematical concepts.

Justification

Yackel & Cobb (1996) explain that what counts as an accurate or valid justification in a math classroom is determined by the classroom norms that are present. In other words, a justification in one classroom may look different than a justification in another classroom. Yackel & Cobb explain that students will initially justify by explaining why specific procedures work within the context of a problem they solved or using numbers to support their justification. Gradually, and with the support of modeling, they are able to start rationalizing why a certain procedure will *always* work with similar problems; this is considered generalization.

One important aspect of justification is when students make meaning from their own ways of solving problems. Once students can justify their own work, they may even start to justify other people's reasoning or methods. All of these types of justifications promote higher levels of cognitive demand (Yackel & Cobb 1996; Henningsen & Stein 1997). It is critical to have students hear each other's thinking because often students will be able to explain how something works in a "kid-friendly" way that may be easier for other students to understand. Therefore, students will not only be able to justify their own work, but the work of their peers as well. This allows students to gain insight into multiple ways of thinking about problems. Students may then access higher order thinking around the mathematics.

NCTM places emphasis on higher order thinking across the grade level standards. When students justify their work, they have to think deeply and critically about the mathematical concepts they are working with (NCTM, 2000). In their study of how students problem solve and justify, Glass & Maher (2004) found that when they are given problems that require explanation and justification of their answers, students developed a better sense of the mathematics being used. Requiring students to justify provides an avenue for higher level thinking. Kazemi (1998) found that students could be pressed for a deeper level of thinking; to do this, they needed to be pressed to justify their reasoning behind the mathematics. Kazemi's study found that when teachers only focused on the explanations for procedures, students did not justify their thinking and the level of cognitive demand remained low.

Using justifications promotes a deeper understanding of mathematical concepts; educators that encourage students to justify their thinking increase student success in math (Boaler, Staples, 2008; Weber, Radu, Mueller, Power, & Maher, 2010). Boaler and Staples (2008) found that when teachers used higher cognitive thinking skills, like justification, their students' achievement in math increased. In interviews, students recognized the importance of learning thinking skills like justifying. Students stated that providing a justification was important because they felt it helped them gain a deeper mathematical understanding of the concepts.

Boaler and Staples study not only showed that higher level thinking skills like justification increased students' conceptual understanding and success in math, but it was also important to the success of all the learners. The school where Boaler and Staples did their study had a very diverse population. Despite the challenges of a diverse learning community, the requirement of higher level thinking skills supported students at the school to become successful. Justification deepens students' understanding of mathematical concepts.

All Students Are Capable of Success

When looking broadly at students' math achievement on standardized tests, it appears that certain populations of students do well in the area of mathematics, while others do not (OSPI, 2011). Historically marginalized students (i.e. students who come from low socioeconomic status (SES) families, students of a minority ethnicity, students that speak a language other than English.) tend to perform at a lower level in mathematics than students from higher SES background. Weber, Radu, Mueller, Powell and Maher (2010) make the point that historically marginalized students do not lack the cognitive skills and mathematical background to learn mathematics. Instead, teachers often lower the cognitive demand of tasks, especially for students who tend to struggle in math (Zohar, Degani, Vaaknin, 2001). When this takes place students usually gain a basic understanding of the math, but may not fully understand why it is important or how it works. Educators need to provide these students with opportunities to participate in mathematical activities instead of lowering the cognitive demand. In other words, all students are capable of high level cognitively demanding work. We, as educators, need to incorporate teaching and learning strategies that will ensure participation and understanding for all students, especially historically marginalized students. When classroom norms like justification are in place, all students do better (Boaler, 2002; Boaler & Staples, 2008; Zohar, Webb & Farivar, 1994; Yehudit, 2003). Several studies have found the achievement gap (the difference between low and high-achievers success) was narrowed when students exhibited higher level thinking skills like justification. Therefore, all students are capable of high level thinking and benefit from being taught and held accountable to justify their thinking. In order to require students to start successfully using these practices teachers need to model mathematical justification and question their students as a way to provide them opportunities to practice justification.

Questioning and Modeling Expected Behaviors

Teachers cannot simply require students to use justification skills in their classroom and expect them to automatically start using these skills. Studies have shown that when these skills are brought into the classroom without any explicit teaching, students do not benefit (Boaler, 2002; Boaler & Staples, 2008; Murrell, 1994; Swing, Stoiber & Peterson 1988). Therefore it is important that teachers explicitly teach their students how to use justification techniques in the classroom. In order to require students to start successfully using these techniques teachers need to model appropriate strategies and question their students.

Many students do not have experience with justifying their work, especially in math class. Because the focus in math tends to be on getting the right answer, students pay little attention to the reasoning behind why their answer makes sense. Research has shown that in order for students to successfully use justification techniques, the teachers need to model the expected behavior (Goos, 2004; Henningsen, Stein, 1997; Hufferd-Ackles, Sherin, 2004; Khisty, Chval, 2002; Webb, Farivar, 1994). This allows students who are not familiar and/or comfortable with this type of classroom norm to develop an understanding of what is expected through teacher or student modeling. The importance of modeling is made clear in Webb & Farivar's (1994) study. They found that one teacher modeled the expected behavior and procedure for her students. Students in this class were able to successfully provide justifications, which lead to a deeper understanding of the math concepts. Another teacher provided the justifications rather than allowing her students to reason through the mathematics. As a result, her students did not do as well as the students whose teacher modeled what a valid justification should be and required her students provide a justification (Webb, N. M, & Farivar, S. 1994). It is important that teachers actually model the expected behavior/classroom norm and not just explain it to their students.

In order for teachers to have students start using justification in their math classes, modeling is an important classroom skill to learn. Teacher questioning is one way to provide students an opportunity to justify. Through teacher questioning, students are able to learn what counts as a mathematical justification in their math classroom. Stein, Grover & Henningsen's (1996) study showed the importance of questioning. In this study, the researchers set out to investigate how and why the cognitive demand of tasks is often lowered from the set-up stage to the implementation stage. They found that during the set-up stage, teachers changed a task to match their goals based on their teaching practices and knowledge of the students. Although the tasks may start out as a potentially high cognitive demanding, during implementation the teacher may inadvertently change some aspects of the task. Reasons may include: the classroom norms, the teacher's instructional habits and dispositions, and the students learning habits and dispositions. Stein, et al (1996) found that many tasks considered high cognitive demand in the set up stage (which often called for students to justify their work) often changed to lower cognitive demand tasks when they were implemented because the teacher took over the difficult parts of the problems prematurely without giving students the opportunity to think more deeply about the concepts being discussed. However, when the cognitive level of the tasks remained high from the set-up stage to the implementation stage it was often due to the teacher's continued press for explanations and justifications through the use of questioning strategies. A clear message was sent to the students: explanations and justifications were a critical part of the classroom norms.

In order for students to learn to reason, explain, and justify, teachers need to ask students questions that provoke high level thinking. One study found that a student's success was due to the internal idea that math should make sense (Maher, Martino; 1996). This student wanted to understand why a math strategy worked; the student wanted a justification to help make sense of the math. With support from the teacher's questioning practices, the student was able to provide clear justifications. Many students don't know how to justify their work automatically. Teachers can questions their students to support the development of their justifications and increase their understanding of the mathematical concepts (Fraivillig, Murphy, & Fuson, 1999; Kazemi & Franke, 2004;

Maher, & Martino, 1996; Morrone, Harkness, D'Ambrosio & Caulfield, 2004). Teachers can question students in a way that prompts them to naturally provide a justification.

Types of Questions

Questioning students to promote justifications and a deeper understanding of math concepts is critical. There are different types of questions that promote a deeper understanding of math concepts and encourage students to provide justifications. Herbal-Eisenmann & Breyfogles (2005) label differentiated questioning by cognitive demand as either funneling or focusing questions. Funneling questions lead students down a pre-determined path, which may not leave room for the student to develop their own ways of thinking. Teachers tend to use this type of question to lead students down the path that makes the most sense to the teacher. This does not promote justifications and higher level thinking. Focusing questions require that the teacher listen to the student's thinking and based on their thinking, the teacher will guide the students in the direction they are already headed. This type of questioning helps the students develop their own ways of thinking about math problems. Focusing questions tend to promote justification and higher level thinking because the questioning is grounded in the students' own sense making. Teachers need to listen to student thinking and based on what they are discerning, ask students questions that will allow them to further develop their thinking.

The types of questions being asked are critical for students' ability to explain and justify. Parks (2010) found that when students were asked a question like "Why," they often interpreted that question as an indication that they were wrong about their solution. In traditional direct-instruction classrooms teachers rarely ask questions to guide students' thinking, and may only ask questions if the student is wrong. If teachers are inconsistent in their questioning, students may interpret a question like “Why” as indicating they are wrong. Therefore it is important that teachers make asking questions a classroom norm.

Mewborn & Huberty (1999) point out simply asking questions to get students to provide reasoning and justification is not enough. Teachers must ask questions that provoke deep level thinking, but teachers should also follow up those questions to clarify and extend thinking for everyone. Teachers can often misinterpret a student’s justification. Therefore, it’s important for the teacher to clarify their students’ justifications. This is both beneficial for the teacher to gain a deeper knowledge of students’ understanding, but it is also helpful to the other students in the class to hear a student’s reasoning restated another way.

When students hear the ideas and explanations of other students, they may gain a better understanding of the math, which can be a very powerful way to provide equity in the classroom, because all students are gaining access to another way to think about the mathematics (Boaler & Staples, 2008). As educators we know that not all students will think about the problem in the same way or at the same level. It is critical that we allow multiple perspectives in the classroom so more students will gain access to the mathematics.

Questioning provides opportunities for justification, which in turn can prompt students to question their group mates to justify their work. Maher (2005) found that when teachers were questioning students to promote higher level thinking and require justifications, students also started to use the same questioning techniques in their cooperative groups. Essentially, the frequent use of questions every day in the classroom modeled the type of questions that were important for students. Because of this, the students started to naturally question each other in their cooperative groups using the same types of questions the teacher used. Students were asking each other higher level thinking questions that promoted justifications.

Through my review of the educational research on justification and my own classroom experiences, I have found a relationship between the types of questions asked and the level of justification students provide. The research showed benefits of students' conceptual understanding when they provided justification. Using certain questioning techniques, like focusing questions, can potentially promote an equitable learning environment and help close the achievement gap. Through my research project, I developed a deeper understanding of the connection between questioning and justification through my action research project. The question I investigated is as follows: What types of questions promote students' to provide mathematical justification?

Limitations

A common limitation in the research I reviewed was the small number of studies that showed the teacher's perspective. The studies I found had researchers enter the classroom and observed the teacher working with students. There was little or no explanation of what the teacher was experiencing while questioning his/her students. Because I am both the researcher and the teacher, I hope to add a teacher's perspective to the conversation around questioning practices and justification in the mathematical education community.

Another limitation I found was the teachers' mathematical background knowledge in elementary settings. Although I read some research where the teacher being studied had mathematical knowledge and experience, I did not find many studies in the primary setting. I have taught middle school mathematics and am qualified to teach high school mathematics. The students I worked with this year were fifth-graders. Drawing on my extensive math knowledge and experience, I was able to provide rich problems for my students to explore.

Entering My Research

Although I have only taught for three years, I have already noticed that students lack deep understanding of mathematical concepts. I observed that students often try to arrive at solutions by going through steps to solve the problem without any rationale behind their work. When I ask students questions about their work, they often become confused about why they solved the problem in a particular way. This made me realize that my students did not have an understanding of the procedures they were attempting to apply in the problem. If students are blindly going through procedures to solve problems, they are not developing a deep conceptual understanding of the mathematics.

Reviewing the educational research on mathematical justification has shown me that if I require students to provide justifications and explicitly teach them how to justify through modeling and questioning strategies, this type of thinking may eventually become a classroom norm. I believe all students are capable of giving justifications. Based on the types of questions I've asked my students, they were able to provide thoughtful and original justifications. Other times, they simply regurgitated what I expected them to say, which made me realize they did not understand the procedure but were merely trying to tell me what they thought I wanted to hear. Admittedly, I do not always ask the types of questions that have the potential to get my students to think deeply about the math. This is something I wanted to work on in my own teaching practice. That is why I focused on questioning practices that promote higher level thinking through mathematical justification. In this study I addressed the following question: What type of questions promote students to provide mathematical justification?

Chapter 2: Methods

Setting and Participants

The district in which this study takes place is a mid-size suburban district. Hemlock Elementary[1] school is a fairly diverse school where 44% of the students are Black, Asian, or Latino. About 40% of the school's population receives free and reduced lunch (OSPI, 2011).

At the end of last year I was involuntarily displaced due to budget cuts. I found out late in August where I was going to be teaching. This made it difficult to prepare for the grade level I would be teaching. About two weeks before school started I was hired at Hemlock Elementary school in a fifth grade position.

Hemlock's fifth graders state test scores were average for the states test scores. About 60% of Hemlock's fifth graders passed the state standardized math test. However they were almost 10% below the state average on the reading test. About 58% of Hemlock's fifth grade students passed the state reading test, compared to 67% for the state.

My class had 28 students: 16 girls and 12 boys. A third of my students receive free lunch. Only 51% of my students passed the state standardized math test in fourth grade. This is below the school average and the state average. Of the students that passed, a third of them passed by only one or two points. 22% of my students passed the state standardized test with above average scores.

Because my study is focused on how questioning practices can promote all students to provide justifications, I asked all my students to participate in this study. One student elected not to participate in the study. Of the students who chose to participate,

[1] All names of participants and places have been given pseudonyms to protect their identity.

four were pulled from my class during math time. Because these students were not in class to participate during the majority of the time I collected data, I chose not to include these students in my study. I was able to collect data on twenty-three of my students, 14 girls and 9 boys.

Before I started to collect data, I introduced justification to my students. I talked about what it meant and how to provide mathematical justifications. I also talked about the importance of providing a mathematical justification.

Data Collection

I collected data for this study using a mixed methods approach, incorporating both qualitative and quantitative data (Mertens, 2009). I collected data over a period of five weeks starting at the beginning of the school year. I gave pre and posttests, short quizzes, cooperative groupwork tasks, and I kept a reflective journal. After the five week data collection period, I also collected student work from their math journals. These approaches allowed a triangulation of data, which helped to ensure the validity of my study (Mertens, 2009). Table 2.1 shows the timeline I used for my data collection.

Table 2.1 Timeline of data collection

Week 1	Week 2	Week 3	Week 4	Week 5
• Group task 1 • Pretest • Journal	• Group task 2 • Quiz 1 • Journal	• Group task 3 • Journal	• Group task 4 • Quiz 2 • Journal	• Group task 5 • Posttest • Journal

Pre and post-tests. I gave my students a pretest within the first two weeks of school. The pretest was given to every fifth grader in the district and covered one of the state math standards for fifth grade, 5.1 multi-digit division (OSPI, 2011). Students learned how to divide large whole numbers in a variety of different ways. I used the participant's scores to assess their current ability to solve division problems. I was also able to identify how they justified their work with story problems at the beginning of the study. I used the justification rubric (Appendix A) to assess the level of justification provided by the participants. At the end of the five weeks, I gave a posttest and compared their scores to the pretest. The pre and posttest had similar questions. In order to address posttest sensitization (Mertens, 2010) the numbers used in the tests were changed.

Quizzes. On weeks two and four of the study, I gave a short quiz. The quiz consisted of two to four questions that required students to solve division problems. This data allowed me to see how students were progressing with the concept of division. I used the data to see where students had misconceptions. Through my analysis I was able to identify students' misconceptions which allowed me to create a plan to address the misconceptions. I also used the scores on the quizzes to determine if they were ready to advance on to more complex division problems.

Audio and video recordings. In order to see how my questions encourage students' justifications I observed my students working on weekly cooperative group tasks. Students worked together on the tasks in groups of four. I set up a video camera to record students' interactions with each other and myself. Videotaping the group work allowed me to see how students reacted to my questions and allowed me to reflect on the types of questions I asked. I used an external microphone to ensure I could clearly hear what each student was saying. I had two cameras set up on two different groups each time a task was given. I rotated through all the groups, and once all groups had been videotaped at least once, I randomly selected groups to be videotaped for the last couple of tasks.

Journal. During the course of the five weeks, I kept a journal with me during math time. Every day I recorded interactions I had with my students or interactions my students had with each other. I recorded how students responded to my questions and generally how they were doing with the math concepts. I also noted if students asked each other to provide justifications or modeled my questions. I carried the journal with me during math time to ensure I could write down any questions I asked, or students asked each other. I found it was difficult to write down events as they occurred. After math, students went to recess for 15 minutes and during this time I was often able to sit down and reflect more deeply about any interactions that occurred during math that day.

Student journals. Students had spiral notebooks they used as their math journals. During tasks students would do the majority of their work in their journals before transferring it to a presentation paper or poster. Students' journals were not graded or required to be turned in. Students just used the journals to record their work to help them solve problems that would be collected on another sheet of paper. Initially, I hadn't planned on collecting their journals, however as I was analyzing data I wanted to see if students began to provide mathematical justifications in their math journals. After the five weeks had passed, I collected students' math journals and made copies of their work on the five group-worthy tasks I videotaped. Students knew their journals would not be graded, so I was able to see how students provided mathematical justifications on their own across the five weeks of the study. I could also see all students' work on the problems and compare the videos of the groups I video recorded with their journals.

Data Analysis

Tests and quizzes. Tests and quizzes were taken individually. I did not help my students in any way, so I could not collect data on my questioning techniques. I was interested in students' written justifications which were analyzed using the justification rubric (Appendix A). In order to analyze the data I collected from the pre and posttests, I compared each student's individual scores to see if there was an increase in students' understanding of long division problems. I also calculated a class average to compare the class's progress. I compared students' justifications from the quiz they took on the second week and compared that to the quiz they took on the fourth week. I analyzed the quiz and test data and looked for patterns and improvement in their justifications.

Recordings and journal. I transcribed the video and audio recordings and coded the data using an open coding technique (Hubbard & Power, 2003) looking for types of questions I asked, student questions, and types of student justifications. I analyzed the writing from my journal and coded it in a similar way, looking at how my questions prompted students to provide justifications. As I analyze the codes I categorized any themes that emerged. I compared themes from both the recordings and my journal.

Student journals. I collected students' math journals and photocopied five group-worthy tasks from when students were asked to provide mathematical justifications in class. I analyzed their journals and coded for any justifications students provided. I then coded how often students provided mathematical justifications and analyzed whether more students provided justifications over the course of the five weeks.

Limitations

Working with students to develop deeper thinking skills is a complex undertaking. Even though the focus of my study was how questioning practices guide students' justifications, there were other factors at work. Some of those factors included: the classroom norms I set up, the fact that I modeled what a good justification looks like, time I spent teaching students how to work successfully in groups, and developing the types of problems I gave my students. I used cooperative group strategies to help my students work together successfully. Therefore, students may have provided justification because of their group and not from my questioning practices.

Another area of concern is that I was the classroom teacher and may have had biases regarding my students' successes. As a classroom teacher, I may have inferred into students' thinking, and I could not always support certain ideas I inferred with data. In order to validate my claims, I took detailed notes and referred to the codes and themes when writing the findings section of this paper. I also used a justification rubric when I graded any tasks, quizzes, or tests. This helped limit my biases during the analysis phase.

Time is another area of concern. I collected data in a period of five weeks. This was a very short amount of time to collect data to analyze an exploratory study. In order to address this concern, I collected a variety of different types of data to triangulate the data. This allowed me to analyze patterns over a short period of time.

The pre and posttests had similar questions that may have affected the internal validity of the study. To ensure that students did not memorize the question format, I gave quizzes to see their progress on providing justifications. Giving quizzes allowed me to collect data on problems that differed from the pre and posttest. I also used triangulation through my observations of their group tasks.

I used a variety of methods to collect data on my question: what type of questions promotes students to provide mathematical justification? I gave quizzes, pre and posttests to compare the classes' progress. I recorded groupworthy tasks to collect data on the types of questions I asked and how they prompted students to give mathematical justifications. Finally I used both my personal journal and students mathematical journals to assess how my questioning practices were affecting how students used justification. As I analyzed my data, I coded for similar themes as they emerged. As I evaluated the themes, I developed a deeper understanding of justification and the types of questions I asked and how they promoted my students to provide justifications.

Chapter 3: Research Findings

This research project examined the relationship between questioning strategies and students' mathematical justifications. In order to analyze this question, I collected data from a variety of sources. I gave my students quizzes, pretests and posttest. I videotaped groups as they engaged in group worthy tasks, wrote in my journal every day, and analyzed student work from their math journals. Through my analysis I found four major themes. First, I found that I asked students to justify in two different ways. The first was a justification of an answer and the second was a justification of a procedure/strategy. Second, I found that students were more successful at providing verbal justifications than written justifications. Third, I found that many types of questions I asked prompted the students to give a justification; however higher level thinking questions prompted students to give justifications more often than lower level thinking questions. The fourth theme was an unanticipated outcome of my study. The questioning strategies I implemented created a sociomathematical norm in my classroom (Kazemi & Stipek, 2001; Yackle & Cobb, 1996). Specifically, students started anticipating my questions and began providing mathematical justifications before I could ask a question. Students also began to question each other rather than telling each other how to solve a problem.

In this chapter, I will discuss my findings from the data I collected. First, I will describe the school setting and the teaching practices I set up and used. Then, I will discuss my findings around the two types of justifications and how students were able to justify both verbally and in writing. Finally, I will describe the different types of questions I used and how those questions promoted students to provide justifications.

The School Environment

I was hired at Hemlock Elementary School to teach 5^{th} grade. This was my first experience as a 5^{th} grade teacher. Aside from teaching a new grade at an unfamiliar school, there were some additional challenges that affected my ability to focus on being a teacher/researcher. I had to focus my attention on all subject areas, not just mathematics and I did not have access to any math manipulatives, which greatly impacted my instruction. Especially in that students' justifications were limited to verbal and written justifications. Students were not able to justify their thinking with physical models. Although this impacted my ability to offer my students resources to help them develop their justifications, the students were still able to justify their thinking verbally and in writing.

Although I did not have access to math manipulatives, I did make use of the document camera and a projector in my classroom. I used my projector and document camera almost every day in math. I would often project the task on the overhead screen and model problems with the document camera. I would also have students share their work under the document camera so the whole class could easily see each other's work. I often had students show their work. I gave my students group-worthy tasks (Lotan, 2003) at least once a week.[2] These tasks provided students with additional opportunities to use justification and reasoning to solve problems.

During group tasks, I required that students worked together to come up with a solution and/or strategy to solve a problem. Students were often required to make a final

[2] In order for tasks to be considered group-worthy, Lotan explains they need to have five critical components. The task needs to be open-ended, have multiple entry points, deal with intellectually important content, require individual accountability, and have clear evaluation of the group's final product.

product after they worked together and talked about their strategies. Once students had time to talk and collaborate, they would then agree on how to present their work on their final copy. I required that all students in the group be able to explain their work. I often approached a group and randomly called on a student to explain their group's work. This held each student accountable for understanding and being able to explain the mathematics they were learning. Once students completed their final product, I often called either the whole group or an individual from the group to present their findings. When students presented their groups' work, they were required to share their group's strategy, even if it wasn't originally their own strategy.

In order for students to work together effectively and equitably, I had to teach them some important skills. Starting the first day of school I began preparing students to work together cooperatively and collaboratively in groups. I believe group work and collaboration are key components of any classroom. When students collaborate, they are able to share ideas, strategies, ask questions, and develop a deeper understanding of the concepts. Most students to not possess these skills naturally and therefore, they needed to be taught these skills. During the first two weeks of school, I worked with my students to develop the skills needed in order to work together in a group. We discussed how to use the school principles to guide their behavior (responsibility, fairness, etc.) while working in groups. I used groupworthy tasks to get students talking about the importance of collaborating. We also had discussions about how to treat each other with respect. During the first full week of school, my students had gained some knowledge of how to successfully work in a group, so I started collecting data.

Findings

The following sections provide insight into the different types of justification and questioning strategies I encountered during my research. The analysis leads to research findings regarding the impact of the relationship between the types of questions being asked and the types of justification students provided. I will also shed light on the effect my questioning practices had on the sociomathematical norms in my classroom.

Justification. Through my analysis of the classroom videos, my journal, and student work from quizzes and math journals, I found I was questioning for two distinct types of justification. I either focused on justification of the *answer*, or a justification of the *strategy*. When students provided a justification of their answer, they explained why their answer made sense. When they provided a justification of their strategy, they explained why a certain procedure worked mathematically. During the fourth week of the study, I gave my students the following problem: "All the students in the school are going on a field trip. There are 1,421 students. A bus can hold 72 students. How many buses does the school need for all of the students to go?" An example of justification of the *answer* would be: The school would need 20 buses because when you divide 1421 by 72 you get 19 with a remainder of 53. However, all the students have to go on the field trip, so you would need one more bus for the 53 students. An example of justification of the *strategy* would be: I couldn't multiple 72 by 1,000 or 100 and subtract the product from 1421 because it would be too much. But I can multiple 72 by 10 and get 720. So first I put the one in the tens place and subtracted 720 from 1421 and got 701. Then I multiplied 72 by 9 and got 648. I subtracted 648 from 701 and got a remainder of 53 students. So the answer to the division problem 1,421 divided by 72 is 19R53. So the school needs to order 20 buses.

As I was teaching and collecting data, I did not have a clear distinction between these two different types of justification. It was through my beginning analysis that these two different types of justifications came to the surface. As I continued to analyze my data, I focused on these two types of justifications. Students either provided these justifications verbally or in written form. I included both of these methods of providing justification in my analysis.

Verbal justifications. I analyzed students' verbal justifications by using the data I collected in my journal and the video recordings. While analyzing student responses from my personal journal, I found that during the first week, students did not justify but instead, would focus on the answer. When I asked one student how he got his answer and if he could show me, instead of showing me, he second-guessed himself. The class was working on the division problem 75 divided by 5. The following conversation was recorded in my personal journal during week one.

Teacher: How do you know the answer? Why is it 13?

Hanson: Cause13x5=75.

Teacher: Can you show me? How do you know?

Hanson: I thought in my head. 5x10=50, 5x11=55…oh wait (thinking). 14.

Teacher: Why did you change it to 14?

Hanson: 5x13=65

Teacher: How'd you know?

Hanson: no, it's 15.

Teacher: How'd you know?

Hanson: 5x14=70.

Teacher: How'd you know?

Hanson: 5x10=50, 5x11=55, 5x12=60, 5x13=65, 5x14=70, 5x15=75.

In this example, Hanson seemed more focused on telling me the answer. It wasn't until the end, when he was confident in his answer, that he finally told me how he got the answer 15. This seemed to be the case with other students as well. If I asked them how they knew, they would tell me the answer. Sometimes students would respond that they knew because they 'did all of this work.' Many students had a hard time verbalizing their justifications. During some tasks, it was clear that students knew a rule, but when I asked them to explain why the rule worked, they couldn't provide a justification.

During week two students had difficulty justifying why their strategy worked when I asked them to solve the following problems and look for patterns: 6 divided by 2, 60 divided by 2, 600 divided by 2, etc. When I asked students how they solved the problems so quickly, they said that all they did was "add the zeros". When I asked them why that works, they couldn't give me an explanation but instead, continued to give me examples of how to do it. Only one student could provide a partial justification as to why it works. When I asked Orlando in week 2 why he "added zeros", he replied: "the quotient is multiplied by 10, so the dividend is too." Then I asked him if it works every time, and he said it would. Then I proceeded to ask him why. He went on to explain that when you multiply the dividend by ten then you must also multiply the quotient by 10. He seemed to have an understanding as to why it worked, even though he still couldn't clearly justify it yet. Although I was able to record quotes in my journal on some days;

most days consisted of a reflection of what occurred during the lesson after my students had gone to recess.

In addition to capturing students' verbal justifications by recording their comments in my research journal, I was also able to capture their verbal justifications through the five videos of the group tasks students worked on. The videos allowed me to more closely examine students' justifications and provided a rich data set to examine the relationship between questioning and student justifications. During the course of the five videos, I was able to capture students' verbal justifications. I videotaped two groups, once each week, while they were working on a groupworthy task. On the third week, I had a technical malfunction and could only get audio from the video of one group. Over the five-week period, I videotaped nine groups while doing five groupworthy tasks. I was able to use this data to examine the types of justifications students were providing both during groupwork and in response to my questions. Table 3.1 describes the types of tasks students were asked to work on.

Table 3.1
Groupworthy Tasks

Week 1	Mrs. Moore noticed she added 256 miles to her car's mileage by traveling to and from school for four days. How far does Mrs. Moore live from school? Be sure to explain how you got your answer and why it makes sense.
Week 2	Using base ten blocks, please model the problem: 24/2. (If students were successfully able to model this problem, I asked them to model 24/3.)
Week 3	The teacher has 2268 stickers and 28 students. If the teacher were to give each student an equal amount of stickers, how many stickers would each student get? Please be sure to explain how you got your answer and why it makes sense.
Week 4	The school is going on a field trip. There are 1321 students in the school. A bus can hold 72 students. How many buses would the school need to get if all the students were going to go on the fieldtrip? Be sure to explain how you got your answer and why it makes sense.
Week 5	A factory made 3,850 blocks of cheese. They need to ship out the cheese; they can fit 24 blocks of cheese in a box. How many more blocks of cheese will they need to fill up the last box?

The first group-worthy task I gave did not require that students solve a problem in a particular way. Students approached this problem with a variety of methods, and most students were able to clearly justify their answer. In Cathy, Tamara, and Hanson's group they showed their work, but didn't justify their *strategy*. However, they had a clear explanation to justify their *answer*. Their group solved this problem by first dividing 256 by four because the problem stated Mrs. Moore traveled 256 miles in four days. They found that Mrs. Moore traveled 64 miles a day. Then they realized they needed to divide 64 by two because she traveled to and from work in one day. Hanson: "Thirty-two miles, because the whole day was sixty-four, but thirty-two was there and back. So four days it was thirty-two there and thirty-two back." (Week 1) Hanson clearly justified why his group's answer made sense. At the time, I was satisfied with the justification because I wasn't making a distinction between the two different types of justification. In retrospect I would have continued to press the students in this group to explain why their *strategy* worked for this problem.

During the second week's group-worthy task, I focused on asking students to justify why they used a particular strategy. In this situation, Jane was describing why 24 divided by 2 was twelve.

> Jane: So you start out with 24, two tens which equals twenty, and four ones which equals four, so twenty-four. And then you take the two ones and you divide the two ones, which equals two, and then you divide this (tens) by two which equals twelve in each section. So we divided each by two and we got the whole thing by two and got twelve. (Week 2)

In this case, Jane was justifying why her group's strategy made sense and worked. Jane's group understood that dividing by two meant that they could split the whole into two equal groups.

As I analyzed my data, I realized that students were providing both justification of their *answers* and justification of their *strategies* depending on the questions I asked. During the video analysis I found that students had given a partial or a full justification 35 times throughout the five group-worthy tasks. Of those 35 justifications, 57% were justifications that supported an answer and 43% supported a strategy.

When I looked back at the task that was given, I noticed that students provided more justifications with some tasks than they did with others. During the first two tasks, students provided mathematical justifications ten times for each task. The third task, students only provided justifications four times. The last two tasks, students provided mathematical justifications only five to six times each. Therefore, as the weeks went on, students were providing fewer mathematical justifications. As I analyzed this further I found that even though students were providing fewer mathematical justifications, the justifications they were providing were more sophisticated.

During the first three tasks students provided only a partial justification 17% of the time. However, during the last two group-worthy tasks all justifications were complete and accurate. Therefore, it seems as though students started to provide fewer but more accurate justifications as the weeks went by. Unfortunately, the level of sophistication of students' verbal justifications did not carry over to their written justifications.

Written justifications. Through the analysis of students' assessments and journal entries, I found that students' written justifications were not as complete or concise as their verbal justifications across the entire study. Instead of providing a justification, students would describe the steps they took to arrive at the answer.

All of my students took three types of assessments; a pretest before we started the unit, quizzes during weeks two and four, and a post-test in the fifth week. The pre-test and the post-test asked students to solve three story problems and required that they show their work using words, numbers, and/or pictures.

For the pre-test not one student provided a justification for the story problems. Almost all of my students didn't even attempt to solve the story problems on the pretest and just left them blank. This let me know that they didn't have strategies to solve story problems involving division. I believe they left it blank because they didn't know how to solve it, not that they didn't know how to justify their answer and/or work.

During the two quizzes, I asked that students solve division problems without any context. This meant that students could only provide justifications to support their strategy as there was no context in which to support the reasonableness of their answer. At this time, I had only questioned students to have them provide justifications; I had not modeled how students might provide a solid justification in writing. Students did not do well on the first quiz. Only 8% of my students provided an accurate justification of their strategy. For the first problem students were asked to solve a division problem: eighty-four divided by six. Lamar provided a justification of his strategy for this problem.

> Lamar: 6 goes into 8 once so I put the 1 in the tens place and subtracted 60 from 84 to leave 24. 6x4=24 so I put this (4) in the ones place to get 14. To make sure I was correct I multiplied 14x6 to get 84! (Week 2)

Lamar's justification scored a three on the Justification Rubric (Appendix A) because he clearly justified why he was subtracting the 60 and the 24. He not only justified why he subtracted these numbers, but also showed an understanding of place value when he subtracted 60 from 84 instead of 6 from 8. He would have received a score of 4 on the Justification Rubric if he did not leave me to infer that he got 60 from multiplying the one in the tens place by 6. Lamar described the steps he took to solve this problem, but he also provided a proficient mathematical justification. 70% of my students only described the steps they did to solve the problem. Gail provided only an explanation of the steps she took to solve the problem.

> Gail: First, I put the one in the tens place. Next I subtracted 84 and 6 and got 24. Then I subtracted 24 and 24 which is zero.

Gail explains the steps she took to solve this problem; however she is clearly missing justification of her strategy because she described subtracting 84-6=24, which is incorrect. Really, she subtracted 84-60=24. Because she did not provide this critical component, she only provided us with the steps she used to solve this problem and not a justification as to why it works.

As I graded the students' quizzes, I realized that they thought that providing the steps they took to solve the problem was what they thought I wanted. After this quiz, I handed it back and showed students the papers in which students did accurately justify their work. I then had a class discussion about what constituted a sound written justification. On the second quiz, 90% of the students explained the steps they used to solve the problem and did not justify their strategy. Again, I was disappointed because my students continued to consider describing the steps they took to get the answer as a justification.

The posttest justification scores were better than the quiz scores, but I believe that was due to the question differences. The posttest had questions situated in a context, which caused students to justify their *answer*. On the posttest students provided some mathematical justifications of their *answer* 81% of the time. The percentage of students providing justifications did increase from their pre-test to their posttest; however the students continued to struggle to provide written justification for their strategy. It was much easier for students to provide a verbal mathematical justification to support their answers.

As I looked through the data I collected, I realized that it would helpful to see some of the written work my students completed during the five groupworthy math tasks. I initially did not collect their written work, but had them work in their math journals and, at the end of the period, do a short presentation how their group solved their groupworthy tasks. During their presentation, I would ask questions to get students to justify their work. Typically, groups would be required to make a mini poster to combine all of their work and one student would be called up to the front of the room. All students were required to work in their math journals while also discussing and sharing their strategies with their group. I was curious to see if students started writing down justifications in their math journal, something that they knew was not going to be collected and graded. Students have an understanding that their math journals are for them to keep track of their own work, write down notes, etc. At the end of the five weeks, I collected math journals after school one day and made some copies of student work. Unfortunately, some students had chosen to not take notes on some of the tasks, and others were absent and therefore could not participate in the activities. I managed to make copies of 18-20 students' notes on each groupworthy task I recorded.

As I analyzed this data, I found that students did one or more of the following: only provided an answer, described the steps they did to solve the problem (like many students provided on their assessments), had an incomplete justification, or provided a complete justification. A complete justification was a score of a three or four on the justification rubric (Appendix A). Table 3.2, indicates that as the weeks went by, more students started providing justifications in their personal journals. Interestingly, only 10% of the students provided a mathematical justification on the first task. On the second task, 20% of students provided a mathematical justification. However, on the third task, the number of students providing justifications dropped. Then on the 4th and 5th task the number of students that provided a mathematical justification increased to almost 40%.

Table 3.2
Student Justification

Task#	described method	Provided partial justifications	provided complete justification
1	5%	10%	10%
2	5%	10%	20%
3	16%	33%	5%
4	5%	20%	35%
5	28%	33%	39%

Students knew that I was not going to collect and grade their journals. However, more students started recording their mathematical justifications and explanations in their math journals. This may be due to the fact that I would randomly call on students to present their groups' strategies in front of the class. Some students would bring their journal with them to help them explain. Perhaps because students thought they might get called up, they wanted to be better prepared by having their explanation and/or justification already written out.

In summary, students had difficulty providing written justifications on their tests and quizzes. I had not modeled how to provide a written justification until after their first quiz during week two. Even after I modeled a solid written justification, students still did not provide justifications, but rather described their procedure. Even in students' math journals, the number of students describing their procedure increased. Some students started providing written mathematical justifications more often as the weeks went by both in their journals and in their tests/quizzes. However, only a small number of students did this. This leads me to believe that having students describe their procedure may be a stepping stone in getting students to start providing written mathematical justifications.

Through my analysis of the videos, my journals, and students' journals and quizzes, I found that students tended to provide a justification of their *answer* more often than their *strategy*. Students provided sound verbal justifications more often than written justifications. When students did not provide a written justification, they often described the steps they used to solve the problem. Near the end of the five weeks, more students provided a written description of their procedures.

Once I was able to identify the different types of justification students provided – justifying an *answer* or justifying a *strategy* - and the level of sophistication of their verbal and written justifications, I turned my attention to the types of questions I asked my students and how they responded to my questions.

Questions. Through my analysis of the data, I found that I used a variety of different questioning techniques. Some questions focused on student thinking and promoted justifications, while others only focused on students' procedures and did not necessarily expose students' thinking.

In order to analyze the types of questions I asked, I could only use data that revealed my questioning practices. I was able to use my reflection journal to a certain degree because I often carried the journal around the room with me and I jotted down some of the questions I asked during class time. However, I did most of my reflective writing right after math time while the students were at recess. I did manage to write down some of the specific questions I asked, but mainly my notes consisted of how well students were providing justifications. The main source of data that I could use to analyze my questioning was the video transcripts.

The video transcripts provided me with an accurate account of all the questions I asked that were caught on video. As I analyzed the data, I was able to categorize the types of questions I asked into eight groups. Table 3.3 gives a description of each type of question I asked.

As I analyzed the questions I looked at the data from two different angles. I wanted to know the frequency of each type of question. This would provide me insight into what types of questions I typically ask. I also wanted to analyze how students responded to each type of question. I was curious to know if students only provided justifications if I asked questions that promoted justifications. The answer was somewhat surprising.

Table 3.3
Types of questions

Questions for:	Question Examples:
answer	• What did you get for an answer? • How much cheese do you need to fill the last box?
strategy or procedure	• What did you guys start off doing? • How did you get that? • Have you guys figured out a strategy?
clarify or understand student thinking	• So what you're saying is 64 miles is how far I lived from school, and 64 miles is how far I traveled in a day? • So, you're saying doubling is like multiplication for twos, and dividing by two is like subtracting by 2? • What do you mean 'a little bit more'?
labels or refer back to the question	• What does 64 represent? • What is the checking answer called in multiplication…math terms? • What does the remainder of 25 mean? • Okay, so when we look back at the problem, how many buses does the school need for all of the students to go?
explain their procedure	• Okay, could you explain? • How does your representation model the division problem? • So how did you get that?
justification for a strategy	• How did you know you needed to do division? • Why did you divide by 2? • How do you know it's the closest one without going over? • Why did you put a zero down?
justification for an answer	• How do you know that's correct? • How did you know that 12 was the answer?
generalization	• Will that always work?

As I analyzed the data, I looked at how frequently I asked each type of question. I found that I if I approached a group that wasn't talking, I tended to start my line of questioning by asking students about the procedure they were using to start the problem. I did this to give myself insight to how students began to think about the problem or how they entered the problem. If I approached a group that was already discussing something, I would stop and listen to their discussion before asking any questions. Again, I did this to gain insight into what they were working on. Table 3.4 shows how frequently I asked each type of question and the percentage of how many times students justified after I asked each type of question.

Table 3.4
Questions and justification responses

Questioning Objective	**Abbreviation**[3]	**Questions asked**	**Time each question prompted a justification**
Answer	Answer	7%	43%
Strategy or procedure	Strategy	20%	5%
Clarify or understand student thinking	Clarifying	15%	14%
Labels or original question	Label	22%	33%
Explain procedures	Explain	8%	38%
Justifying an Strategy	Justify Strategy	16%	47%
Justifying an Answer	Justify Answer	7%	57%
Generalization	Generalization	4%	25%

Label questions were asked more frequently than any other type of question. Label questions are questions that ask students to label their answer or ask students to refer back to what the question asked. I asked label questions 22% of the time. Strategy questions were the second most frequently asked question. These types of questions focused on asking students what their strategy was to solve the problem. I asked strategy questions 20% of the time. Label questions and strategy questions were not intended to get students to provide a justification. Justify Strategy questions and Justify Answer questions were questions specifically asked to get students to provide a justification. I only asked students' Justify Strategy questions 16% of the time. I asked Justify Answer questions even less frequently, only 7% of the time.

[3] For the remainder of the paper I will use the abbreviated version of each type of questions.

I asked Justify Strategy and Justify Answer questions less frequently than other types of questions. However, because I was interested in the types of questions that promoted justification, I examined how each type of question promoted students to provide a justification.

Justification questions. Even though I asked the type of questions that encouraged justifications fewer times than other questions, these types of questions produced justifications more frequently than other types of question. Justify Answer questions were asked less frequently than most other questions, but they prompted more justifications than any other questions. Even though Justify Answer questions required students provide justifications of their answer, these questions didn't always encourage students to provide these types of justifications. Only 75% of the justifications given after Justify Answer questions were justification for the answer, the other 25% of the justifications given after these questions were for the strategy.

Justify Strategy questions also encouraged students to provide justifications frequently. I asked Justify Strategy questions 16% of the time and almost half of the total number of justifications were given after I asked these types of questions. In the cases when I asked Justify Strategy questions, students only provided justifications for their strategy; they did not provide justification of their answer.

This seems to indicate that when students were asked Justify Strategy questions, they tended to respond with a justification of their strategy. However, when students were asked Justify Answer questions that required them to give a justification for their answer, students were more likely to provide a justification of their answer, but may sometimes provide a justification of their strategy. The following excerpt is from the second recorded group worthy task from week two.

Teacher: How does you representation model the division problem: 24 divided by two? (Questioning for a justification of their strategy.)

Jane: Because 12 plus 12 equals 24. (Partial justification of their answer.)

In this example I asked students to provide me with a justification of how their model represented the problem 24 divided by 2. However, instead of providing a justification of how their model represented the problem, Jane provided a partial justification of her answer.

Labeling questions. Labeling questions also provoked a fair amount of justifications. I asked labeling questions 22% of the time, which prompted students to provide a mathematical justification with 33% of their responses. Students provided a mathematical justification after these labeling questions because they were missing a key piece of information, either from the problem or the label, and couldn't clearly justify their answer without it. Therefore, when I asked a question that led students to pay closer attention to important aspects of a problem, such as the units or label, students were able to gain insight to that critical piece of information and justified their answer. During week one, I asked students to pay closer attention to what the question was asking, this prompted students' to think about the question more deeply and provide a justification. The following excerpt is from the first recorded group worthy task from week one.

Teacher: And what is the question asking you find? Have you found that yet? (Labeling question)

Gareth: Well, kind of. She lives sixty-four miles away from school so she would have to travel sixty-four miles a day. (Justification for an answer)

Teacher: So how far do I live from school? (Answer question)

Sara: Oh…I know...because she traveled...she went to school and she went back…so we have to divide it. (Justification of strategy)

This example shows that the students were not aware of part of the question, the fact that I had to travel to school and then back. The students were aware of how many miles I traveled in a day, but did not make the connection that I needed to travel to and from school. Through Label questions, I prompted students to pay closer attention to the question. Therefore, if students were not aware of exactly what a question was asking, then they could not provide an accurate justification of their answer.

Series of questions that promoted justifications. Even though Justify Answer questions and Justify Strategy questions encouraged students to give a justification more often than other questions, these two types of questions were not the only ones that promoted justification. Answer questions also encouraged students to provide justifications. After I asked Answer questions, 43% of student responses were justifications. As I analyzed why students provided a mathematical justification after Answer questions, I found they only provided a justification after I had already asked a series of questions, some of which were engaging them in higher level thinking. It is possible students provided a justification because of questions I asked earlier in the period. During the fifth groupworthy task, students provided a justification immediately after I asked them answer questions. However, before I asked the Answer question I asked a couple of different types of questions. The following excerpt was taken from a video of a lesson during week five and shows students working on the following question: A factory made 3,850 blocks of cheese. They need to ship out the cheese; they can fit 24 blocks of cheese in a box. How many more blocks of cheese will they need to fill up the last box? Notice that the question is asking students to find the remainder and determine the amount of blocks it would take to fill up the next box (the next whole number). This question is more complex than traditional division problems which normally only ask students to find the answer to the division problem.

Ken: Shouldn't it be how many boxes? Instead, cause this doesn't make any sense. (Student questioning for understanding)

Teacher: How many blocks of cheese would they need to fill up that last box? (Clarifying question to get students to realize what the question was asking)

Ken: So they already filled up the other ones?

Teacher: yeah, assume they have already filled up the other ones.

(10 minutes of elapsed time: I walked away to let the students continue working)

Teacher: How are you doing Tamara?

Tamara: I got more than I needed.

Teacher: What do you mean you have more than you needed? (Clarifying question)

Tamara: I have more than 3,170.

Teacher: Where did that come from? (Clarifying question)

Tamara: Because 152 times 24.

(Tamara had already divided 3,170 into 24 and got the quotient 152. She was then checking her answer by multiplying 152 by 24. She knew she had made a mistake because she wasn't getting 3,170. I then prompted her to go back through her work to see where she had made her mistake. I left the group and came back later.)

Teacher: How we doing?

Walker: Good, we all got the answer.

Teacher: You all got the answer? What is the answer? (Answer question)

Walker: I said they need fourteen more blocks of cheese to fill up the last box because there are ten (cheese) remaining and we need twenty-four blocks of cheese to fill one box. (Justification of the Answer).

During this excerpt Walker had provided a justification for his group's answer after I had asked an Answer question. However, before I asked that question, I had asked a series of clarifying questions to get students to recognize what the question asked because the students had initially just focused on the division and not on the actual question. This seems to indicate that sometimes a series of questions also promoted students to provide justifications, rather than individual questions. Strategy questions would often be part of the series of questions that led students to provide mathematical justifications.

I asked strategy questions frequently; approximately 20% of my questions were Strategy questions. As I said earlier, I asked these types of question typically when I first came up to a group to gain insight into their understanding and procedure. After asking these types of questions, I followed up by asking them to provide a justification 56% of the time. If I felt that students were still missing part of the problem or a label, I would follow up the Strategy question with Label questions 33% of the time. Therefore, after I evaluated what the students did I either asked them to look for information they missed or

I would ask them to justify their work. I did this 89% of the time after I asked students what their strategy was. So although these types of questions by themselves didn't promote a justification, it led me to ask another type of question which then lead my students to provide a mathematical justification.

Explaining questions. On some occasions I asked students Explaining questions. These questions asked students to explain their procedure. When asked these types of questions, students would sometimes respond with a justification. When I asked students Explaining questions, only 8% of the time, students explained what they did, but they also provided a justification with 38% of their responses. Students did not do this right away. During the first three tasks students only added a justification with 8% of their responses. However during the last two tasks, students provided a justification with 66% of their responses when I asked them Explaining questions.

As the weeks went by, students started to provide justifications more often after I asked Explaining questions. This leads me to believe that students anticipated my questions and knew I would continue to press them for justifications. Their anticipation may have caused my students to provide justification more often toward the latter part of the study.

Questioning summary. After analyzing my questioning practices, I was able to identify which questions prompted students to provide mathematical justifications more often. Justify Answer questions and Justify Strategy questions prompted students to provide mathematical justification more frequently than other types of questions. However, they were not the only types of questions that provoked students to provide justifications. As I further analyzed the other questions I asked, I found that I asked a series of questions that led my students to provide justifications. I asked these questions in order to gain insights to students thinking and understanding. If I found students were missing key pieces of information, I asked questions that prompted students to analyze the problem further through Label questions. Often if students were not quite ready to provide a justification, I asked a series of questions to get them to continue to work and think about the problem. Finally, I found students started providing justifications more frequently after I asked them Explaining questions. I believe this is due to students anticipating our classroom norm of providing justifications.

Questioning my students in particular ways did encourage mathematical justifications, and some questions did this more effectively than others. However, questioning my students had another impact on my classroom. By asking questions, I created a classroom norm of questioning. Students started helping each other by asking questions of each other during groupwork. Instead of specifically telling or showing students how to do something, they began to ask questions to guide them through the procedures.

Sociomathematical norms. I noticed a pattern start to emerge as I analyzed my video transcriptions. By the fifth week some students automatically explained their strategy before I even asked a question. Often I would get half way through my initial greeting and students would interrupt me by explaining their method. Students appeared to be internalizing questioning as a normal part of our classroom discourse and interaction. They seemed to know that I would come to their group and ask them questions, so they didn't wait until I was completely done greeting them; they just started telling me about their work.

Students were aware of my expectations and types of questions I normally asked by the second week. During the second video observation, I found evidence of students' awareness of what I expected them to be able to tell me. Students had just finished explaining how their model showed 24 divided by 2. I then asked them to see if they could model 24 divided 3. After I walked away, the students leaned in and began working. It was difficult to see exactly what they were doing on the videotape, but I could make out the conversation. They were discussing whether they could split up the 24 in a certain way. They didn't think they could because they had accidentally grabbed twenty-five ones instead of twenty-four. While they were waiting for me to come back to their table, they continued to talk about how I was going to ask them why they did what they did.

> Teacher: Why don't you guys see if you can do 24 divided by 3 and see if you can use a representation to show that. (I walked away.)
>
> (The students grabbed more base ten blocked and leaned in working with the base ten blocks. After they finished they raised their hands and were waiting for me to come back.)
>
> Nathan: You guys, you know she's going to ask why we did this.
>
> Jane & Claire: yeah…
>
> (They did not continue to talk about why they solved the problem in the way that they did.)

This example clearly shows how students were beginning to be aware of my expectations. However, they did not continue to discuss their justification of why they solved the problem how they did. This shows that students were beginning to be aware of my expectations.

Through the course of the five weeks I was collecting data on my questioning practices, I noticed another classroom sociomathematical norm developing. Students started imitating my questioning techniques in their small groups. There were a couple of distinct instances that I was able to observe and record in my journal or capture on video. Students asked each other questions to help them solve a problem. During the fourth week Leila, who knew how to divide large numbers, was helping Haley, a student who was struggling with this concept.

(Leila tried to help Haley by asking her questions to help her arrive at an answer.)

Leila: Okay so what's 132 take away 72?

Haley: Four?

Leila: Take away 2.

Haley: Zero

(10 minutes elapsed time. Leila guided Haley through her multiplication facts.)

Leila: What's 3 times 7?

Haley: I don't know.

Leila: What's 1 times 7?

Haley: 7

Leila: What's 2 times 7?

Haley: 14

Leila: What's 3 times 7?

Haley: 21(counted on her fingers)

(Elasped time: 5 minutes. Leila tried to help Haley by rounding the numbers they were working with to make it easier.)

Leila: Let's estimate this to 70. And we're rounding this to 600. How many times do you think 70 goes into 600?

Throughout the entire video Leila tried to help Haley understand the concept by asking her questions instead of showing her or telling her how to do it step by step. She even went as far as to try to simplify the problem by rounding the numbers so they were easier to work with.

Students demonstrated their ability to question each other instead of telling each other how to do the problem. Some students began to internalize questioning as an appropriate method to use when their peers did not fully understand a concept and to help other students gain an understanding of mathematics. This leads me to believe that students made a connection between how the questions I asked helped students with their understanding of the math concepts. However, students were not asking justifying questions. This may be because I did not ask these types of questions more frequently than other types of questions.

Summary of Findings

The goal of this study was to investigate how questioning promoted mathematical justifications. Through my investigation I came to the realization that I was asking students to provide two different types of justification: justification of an *answer* and justification of a *strategy*. At the time of the study, I was not making a distinction between the two types of justification. Through my study I found that students could more easily justify their answers verbally and somewhat in their writing. Students provided justifications of their strategy less often than they did for their answer. Many of my students started describing the steps they took to solve the problem, and later in the five weeks provided more accurate justifications. This led me to believe that having students describe the steps they took to solve a problem is a stepping stone for them to start providing justifications of their procedures.

Throughout the study I asked a variety of different types of questions; some questions promoted justifications while others did not. Much to my surprise, students gave justifications after many types of questions, not just questions that asked students to justify. Although questions that asked for a justification prompted students to provide them more often than other questions, they were not the only questions for which students provided a justification. Some questions that I asked more frequently, like Strategy questions, did not lead students to respond with a mathematical justification, but were in fact a stepping stone to first understand student thinking. I then followed up with a higher level thinking/high-press question (Kazemi, 1998). I believe that students started providing justifications with other types of questions because it was becoming a classroom norm.

Students started providing explanations and/or justifications on their own without being prompted by higher level thinking questions. Students started to do this because they knew I would ask them why they used a particular strategy or why their strategy worked. My questioning practices were becoming a sociomathematical norm; students started anticipating and enacting questions that pressed for justification.

Not only did students anticipate that I would ask them certain questions, they also started modeling this behavior. In their groups students started mimicking my questioning techniques. Instead of telling their peers how to do something, they would guide them through a procedure by asking questions. Most of the time, these were not justifying questions, but questions that helped their peers to get the next step.

When I reflected on how questioning practices promote justification, I found that Justify Answer questions and Justify Strategy questions frequently prompted students to provide a justification. However, other questions, and series of questions, prompted students to provide justifications. Students started anticipating the expectation that they would need to provide a justification, so they started providing explanations and justifications on their own. Finally, students started questioning their peers to help each other better understand the mathematics.

Chapter Four: Conclusion

Through this research project I investigated the relationship between teacher questioning and students' mathematical justifications. I know that if students can justify their work, they understand the mathematical concepts more deeply than if they cannot justify why the mathematics works. Kilpatrick (2001) describes justification, which is part of adaptive reasoning, as a key component for students to be mathematically proficient. In this chapter I will connect my findings with the existing literature. I will then reflect on my current teaching practices and describe the next steps I plan to take in my classroom to continue to increase the frequency and quality of students' justifications. Finally, I will suggest topics that could be researched more deeply.

Connection Between Literature Review And Study Findings

Justification. When I started collecting data I had only a broad definition of justification in mind as I collected research. It was through my data analysis that I was able to further develop my understanding of the ways in which students could justify. While I was gathering data, I asked students questions that prompted them to justify in two different ways. I would either ask students to provide a justification to support why their answer made sense, or I would ask students to justify why their strategy works. At the time I was unaware of the distinction between these two types of justification. As I looked back at the literature, I found articles that supported this finding. Lannin, Barker, & Townsend (2006) described what counts as a valid justification. The article focused on a class that was required to justify why their answer made sense. The students were creating algebraic rules for pattern blocks and they had to justify why the rule they created would work every time. This is similar to the types of justification my students were finding when I asked them to support why their answer made sense.

Swing, Stoiber & Peterson (1988) also had a similar definition of justification. The authors describe justification as deciding if the answer makes sense, but they also said that justifying corresponds to evaluating problem-solving procedures. This supports the other type of justification I was asking my students to provide, justifying their strategy or procedure.

As I conducted my research and gathered data, I was unaware of the distinction between these two types of justification. Although the types of questions I used asked students to provide both types of justification, I focused more on having students justify their strategy. I asked these types of questions more than twice as many times as I asked students to justify their answer.

Questioning. The types of questions I asked throughout my study varied from low press questions to high press questions. Kazemi (1998) defines high press questions as those that require students to justify the mathematics. Kazemi found that teachers need to press for higher cognitive demanding work from their students in order for them to justify the mathematics. Through my research I found this to be true. When I asked justifying/high press questions, students provided justification more often than when I asked non-justifying/low press questions. However, I also found that my students provided justifications after I asked low press questions like “What did you guys start off doing?” I believe that students started providing justification after I asked low press questions because providing a justification was becoming a classroom norm.

Questioning norm. A sociomathematical norm of questioning was developed in my classroom. Similar to what Mahar (2005) found, students started to anticipate my questions and they started asking questions to help each other understand the math concept. Maher found that when teachers were questioning students to promote justifications, students also started to use the same questioning techniques in their cooperative groups. Stein, et al (1996) found that because the teachers in the study continuously questioned for justifications, this sent a message to students that justifications were a critical part of the classroom norms. In my classroom, through my questioning, a message of the importance of questioning was sent to the students. However, it appeared that because I did not ask justifying questions more often than other types of questions, my students did not use justifying questions with each other. My students would guide each other down a specific path to help each other with their work. This is what Herbal-Eisenmann & Breyfogles (2005) would call funneling questions. These types of questions are considered lower cognitive demand because they do not leave room for students to develop their own ways of thinking.

By the end of the five weeks, my classroom was developing into a math-talk community. Hufferd-Ackels, Fuson, & Sherin (2004) describe one aspect of a math-talk community as including students who ask each other questions. As I reflect on where my classroom was based on their three levels of a math-talk community, my class would be at a level two for questioning. Students started to ask each other questions and they started to listen to each other's ideas. We have not yet reached level three because students have not started asking questions that would prompt others to provide a justification. This is one area I would like to continue to focus on as I continue thinking about my own teaching practice.

Implications For My Teaching Practice

As I reflected on my investigation I thought about what I learned about my own teaching practice and ways I might improve the quantity and quality of justifications students provide. In the future there are some things I would do differently.

Although many of my students were able to provide a mathematical justification because of my questioning, they were not successful in providing a mathematical justification in their writing. Many students only described the steps they took to find the answer when I asked them to write down why their strategy made sense. In the future I plan to model what a good written justification is and have the students practice writing and identifying what a valid written justification is. Hufferd-Ackles, Fuson, & Sherin (2004) describe that in order for a mathematical community to move more into giving valid justifications, the teacher needs to probe for justifications and model them. Webb & Farivar's (1994) study found that when one teacher modeled the expected behavior for her students, her students were able to provide justification when compared to the teacher that did not model for her students. I have started to probe for justifications through my questioning and now I would like to move into modeling what a valid justification looks like both orally and in writing.

Goos (2004) found in her study that in order for students to begin to offer justifications the teacher first modeled the expected behavior. To further develop my teaching practice to support my students to provide written justifications, I would like to start modeling what is a good written justification. I will model what a valid written justification includes for my students and I will also have my students identify a strong and complete justification. To practice, I plan on providing my students with written mathematical justifications, and have them rate the papers using the Justification Rubric (Appendix A). Afterwards, we will discuss which papers provided valid justifications, what the papers included that supported a clear justification, and how their work could be changed to provide a complete and valid justification. This may help them understand more deeply what a good written justification is.

Along with making sure my students are aware of what a good written justification is, I would like to change some of my own teaching practices. A goal of my study was to ask students questions to prompt them to provide mathematical justifications. I know higher level questions like high-press questions (Kazemi, 1998) and focusing questions (Herbal-Eisenmann & Breyfogles, 2005), help promote higher level thinking. Even with this knowledge, I did not ask higher level questions more frequently than lower level questions. Only 27% of the questions I asked were higher level thinking questions. I would like to start asking higher level thinking questions more frequently. This might also get students to start using these types of questions with their peers as well. Stein, et al (1996) found that through pressing questions for justification, a message of the importance of justification was sent to the students. In the future I should ask more high level questions to send a message of the importance of these types of questions to my students.

Hufferd-Ackels, Fuson, & Sherin (2004) describes questioning as one part of a math talk community. I believe my class is currently at level two for questioning. I would like to continue to work my way towards level three. Questioning and explaining mathematical thinking (Justifying) was only one of their components needed for a math-talk community. To further develop my classroom norms, I would like to also start incorporating the other two components described in Hufferd-Ackels, Fuson, & Sherin's study: Source of mathematical ideas and responsibility for learning. I will work with my class around all four of the components Hufferd-Ackels, Fuson, & Sherin describes in their study around math-talk communities. This will hopefully help improve the current classroom norm of questioning and develop more classroom norms that will support my class.

Suggestions For Future Research

The first finding explored the two types of justification, justifying an answer and justifying a strategy. Future research projects could investigate how these two types of justifications are related. Should one type of justification be taught first or should they be taught together? Are both types of justification equally important or is one more valuable when it comes to student understanding? Further study into these two types of justification may help educators better understand how to help their students provide each type of justification and potentially increase student understanding of mathematics.

One of the findings in this study was the development of questioning as a classroom norm. Hufferd-Ackels, Fuson, & Sherin (2004) described the benefits of a math talk community. Students started questioning each other instead of walking each other through the steps. A future research study could look into how teaching students to work in groups might affect their ability to work together to come up with quality justifications. If students are taught how to help their group mates by asking questions would it led to students providing more mathematical justifications?

Another area I would suggest for future research is to examine the way in which students provide written justification and how this affects their understanding and academic achievement. Having students justify their work helps students understand the math better (Yackel & Cobb, 1996). If students understand the components of a valid justification and were required to write down their justifications, would their achievement and understanding increase? If so, would it increase by a significant amount? What teaching practices would the research implement to get students to provide accurate and complete written mathematical justifications? A study in this area would help educators understanding the best teaching practices to help students provide written justifications and how written justifications affect student understanding and academic achievement.

Finally, I would suggest research be done around the area of how questioning can promote other higher-level thinking skills such as generalizations. Generalizations are also part of Adaptive Reasoning (Kilpatrick, 2001). Justifications are why a procedure will work for a specific problem or why a student's answer makes sense. A generalization is why a strategy or procedure will always work. When students make generalizations they develop a deep understanding of certain strategies.

Closing Comments

The purpose of this study was to investigate the relationship between my questioning practices and students' mathematical justifications. Through this action research project, I was hoping to better understand what types of questions would help support my students' development of their justifications. Through my analysis I have further developed my understanding of the differences between the two types of justifications and which questions best support students' development of their justifications. I've always known that it is important to have students talk about the math concepts they are working with, I am pleased that the emergence of a sociomathematical norm of questioning was an outcome of my study. The level of math-talk and group cooperation during this study seemed far superior to any previous class I have taught in the past. I look forward to further developing my teaching practice around justification, math-talk, and generalizations.

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Appendix A

Justification Rubric

	Far to Go 1	Almost There 2	Proficient 3	Exceeding Expectations 4
Criteria	• Little or no mathematical basis • Can't explain why they did something. • Little or no evidence of understanding • Only provided an answer • Cannot communicate their understanding	• Some evidence of understanding why what they did works • Mathematical work supports reasoning • Evidence shown to support their answer/claim • Starts to communicate their understanding, but may be incomplete	• Shows evidence of an appropriate strategy • Justifies most steps to solve the problem, but may leave some things out that the reviewer would need to infer • Evidence shown clearly supports their answer/claim • Can clearly communicate why they did something and why it works with the mathematics, but the listener may need to infer into some details.	• Shows complete evidence of an appropriate strategy • Justifies all steps needed to solve the problem, with no details left out to infer • All evidence shown clearly supports their answer/claim • Can clearly communicate why they did something and why it works without the listener inferring anything.