EFFECTIVE TEACHING STRATEGIES FOR ALLEVIATING MATH ANXIETY AND INCREASING SELF-EFFICACY IN SECONDARY STUDENTS

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ABSTRACT

Math anxiety and low self-efficacy create stumbling blocks in math education. Teachers must learn how to effectively alleviate these problems using the most current research and best practices. In this paper, current research is reviewed and synthesized. It is found that math anxiety can be treated with direct interventions such as relaxation therapy, or indirectly, with teaching style and cooperative learning. It is suggested that future research focus on how math anxiety relates to achievement, and the possible benefits of relational instruction in secondary students specifically.

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CHAPTER 1: INTRODUCTION

Introduction

Mathematics anxiety can be defined as feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary and academic situations. This anxiety can come in many forms: worry, fear, high negative emotions, self-deprecatory thoughts, sweaty palms, or a racing heart. This paper focuses on the multi-dimensions of math anxiety, how this anxiety manifests itself in the classroom, and discusses the pedagogical approaches that researchers contend may alleviate math anxiety in students.

Rationale

Mathematics anxiety has a negative relationship with mathematics performance and achievement (Green, 1990; Hembree, 1990; Mevarech, Silber & Fine, 1991; Norwood, 1994; Wigfield & Meece, 1988), though it has also been found that a degree of cognitive anxiety (worry or concern) may motivate student to try harder. It is when this worry or concern becomes too strong that it may interfere with performance (Ho, Senturk, Lam, Zimmer, Hong, & Okamoto, 2000; Wigfield & Meece, 1988). The assumption that all students who perform poorly in math classes are incapable of understanding math needs to be challenged; there may be something else at play. Math anxiety needs to be given its due attention and strategies for overcoming math anxiety need to be taught to students.

Math anxiety is a problem in classrooms all across America and in many countries around the world (Ho, et al., 2000), threatening both achievement and participation. A major negative consequence of mathematics anxiety is mathematics avoidance (Hembree, 1990). Students with math anxiety take fewer elective math classes and avoid college majors and career paths that depend heavily on quantitative skills or mathematics (Hembree, 1990). This avoidance of math leads to a limiting of career choices, eroding our country's resource base in science and technology (Hembree, 1990). Modern American students, upon graduation, will be entering a global job market where competition is fierce and proficiency in the use of technology and mathematics is a necessity. "While US achievement has risen across our nation, we still lag behind our international competitors. It is important that we, as a nation, take steps to improve mathematics education for grades kindergarten through 12" (Thorpe, 1999).

There are currently math requirements currently in place in Washington State for all high schools, and soon the state of Washington will be increasing that requirement. Starting with the graduating class of 2013, all high school students will have to pass Algebra II to graduate and pass all sections of the WASL (Washington State Board of Education, 2008). Standardized tests like the WASL are becoming the measures for student success, teacher merit and school failure. These scores will have far reaching effects; on budgets, school closures and potentially on teacher pay if the proposals are approved. This is a lot of pressure for students and teachers to be under. When students are under pressure, their stress levels rise and therefore they feel more anxiety, and this could have a negative effect on their math test scores (Wigfield & Meece, 1988). Math anxiety is negatively correlated with math achievement, and if this issue is not dealt with, it could have a terrible effect in many areas of our education system.

In my practice as a math tutor, I have seen many students who do not necessarily lack the requisite skills or content knowledge to succeed in mathematics, but rather their high anxiety and low self-efficacy lead to doubt their ability to complete the work. I have handed out many tissues to students when their tears are threatening to smear the writing on their math paper; sometimes my job as a math teacher and tutor feels like part counselor, part instructor. I hear peoples' spontaneous confessions of math anxiety when I tell them I am entering the math teaching profession, the most common response being "oh, you are going to be a math teacher? I was horrible at math. Just thinking about it makes me nervous." This negative response is far more common than a positive response.

I have also seen the debilitating effects of math anxiety in my own life. I personally grappled with the cognitive (worry) aspect of math anxiety and with abstraction anxiety starting in my middle school years, lasting in to my sophomore year of college. Because of the anxiety I experienced in late middle school and the failing grade I received because of it, I was tracked into remedial math classes in 9^{th} grade. I stayed in this track all throughout high school, coasting through with not much effort and no passion for the material. I gave up on math.

In college, it took me until I reached my junior year to take an algebra class (on a whim), and was surprised to find that the material was exciting and engaging. My professor presented the material in an exciting and engaging way that made me want to learn more. I was inspired to take more math classes and raced through algebra, pre-calculus, discrete math, logic, calculus and real analysis until I graduated with a BA and an emphasis in mathematics. I am an example of what can happen when a student is afflicted with math anxiety that is not recognized and not treated, but pushes past it with the help of a highly capable and passionate math teacher.

Description of Controversies

Researchers do not agree on the best way to treat math anxiety and increase math self-efficacy. Some researchers feel that teachers' beliefs about mathematics have a powerful impact on the practice of teaching (Uusimaki & Nason, 2004; Charalambos, Philippou & Kyriakides, 2002; Ernest, 2000) and that teachers with math anxiety can lead to students with math anxiety and low math achievement; "Teachers appear to have the greatest influence in either direction over students' attitudes (in math)" (Murr, 2001). Other researchers have reported that these math anxious teachers respond by avoiding math all together. During classroom observations, Trice & Ogden (1987) found that those with higher anxiety planned fewer math lessons and taught non-math content more often. These researchers warn about the negative effects that the math anxiety of a teacher can have on student's math skill development, especially at the elementary level Hembree (1990) found that The highest levels of mathematics anxiety occurred for students preparing to teach in elementary school. In summary, one intervention recommended by researchers is to treat the math anxious teacher, especially those at the elementary level, to prevent students from developing math anxiety at all.

Another angle that has been researched is the effect of teaching style and pedagogy on math anxiety in students. Regardless of whether a teacher has math anxiety, they may have an effect on the math anxiety level of their students because of the way they teach math. Researchers have studied constructivist and behaviorist teaching methods, the use of problem solving, teacher comments, and the math anxiety level of students before and after the use of these teaching practices.

Other interventions are focused on math anxiety in the students themselves. This also poses a problem for treatment, for math anxiety is multi-dimensional. Studies have uncovered multiple unique aspects of math anxiety and have shown that the approach to treating math anxiety needs to be different depending on what aspect of math anxiety a student is facing. I will describe further various approaches researchers have used to treat math anxiety and increase math self-efficacy in students in the pursuit of an effective treatment for math anxiety.

Definition of Terms

Cognitive anxiety is described as self-deprecatory thoughts about one's performance. A student who experiences cognitive math anxiety may have feelings of low self-efficacy, worry, poor concentration, indecision, a sense of confusion or defeatist self talk. Affective anxiety is described as the feeling of nervousness, tension and unpleasant physiological reactions to testing situations. A student with affective or somatic anxiety may experience the sweats, an adrenaline surge, a need to urinate, increased blood pressure, increased heart rate, sweaty palms or dry mouth (Ho, et al., 2000). These two aspects are empirically distinct, though they are correlated (Wigfield & Meece, 1988).

A student with abstraction anxiety may feel fine in math class until they become anxious and panicked when presented with mathematical information in abstract form. This could include the use of variables, the construction of a conceptual proof, the use of set notation (Furguson, 1986). At this point, the students with abstraction anxiety may experience symptoms of cognitive or affective anxiety, or both.

Math self-efficacy, though not an aspect of math anxiety, is related. Self-efficacy refers to the belief that one is capable of succeeding at a task. This is related to self-esteem, but does not refer to how one feels about oneself as a person. Math self-efficacy is included in the math anxiety field of research because it refers to whether a student feels they can do math. If a student is not confident in their abilities, this can eventually manifest itself as math anxiety.

Different studies use different tests to measure math anxiety. Many use the Math Anxiety Rating Scale (MARS) in its entirety, others use only select pieces of it, others use a mix of the MARS and their own questions, and there are others still who use a different test altogether. When an instrument is introduced and its nature is not clear from the title, it will be described in concise detail.

Statement of Limits

How does gender affect math anxiety? How can an intervention in early elementary school stop math anxiety from manifesting in students? What preventative measures can be taken so that students never develop low math self-efficacy? These questions are important to the educational community and to the study of math anxiety. If students could be prevented from ever developing math anxiety, there would be no need for treatments when they reach the high school level.

The fact is that many students who reach my high school math classroom will have already had many experiences with math, both good and bad. They may have already developed anxiety and low self-efficacy and need treatment to succeed. I am interested in what I can do for these older students who have come up against these math barriers and are looking to get past them to math success. For this reason, I am only interested in strategies that I can use to alleviate math anxiety and increase self-efficacy in my high school math classroom.

Summary

Mathematics anxiety affects students across all abilities and levels. The national spotlight is on math education in our country, and a great deal is riding on the math achievement of our students, as measured by standardized tests. Giving our students more homework, raising the stakes, threatening school closures, and adopting new and better curriculum is not going to necessarily fix the problem of math achievement in our country if at the root of the problem is math anxiety. Low math achievement is potentially a symptom of a deeper problem, and math anxiety needs to be recognized and treated as a valid problem in our math students.

In the next chapter, the history of the study of math anxiety will be discussed; from its proposed roots in the education movements dating back to the early 1900's to present day curriculum and standardized testing practices.

CHAPTER 2: HISTORICAL BACKGROUND

Introduction

To look at the history of math anxiety in the U.S, one needs to analyze the history of math curriculum and education theory in the U.S during the past 100 years. From the early twentieth century onward, we have moved through the Progressive Movement of the 1920s, Activity Movement of the 1930s, Life Adjustment in the 1940s, New Math in the 1950s and 1960s, and then the Progressive Movement made a comeback in the 1970s, but with a new name: the Open Education Movement (Klein, 2003). With each new movement came a new focus in education and math curriculum was revamped countless times as a result. This historical exploration will make the connection between the inconsistencies in our nations' math curriculum, the shift from applied to abstract mathematics and the emergence of math anxiety as an issue in our schools.

The Progressivist Movement

William Heard Kilpatrick, influential twentieth century education professor at Teachers College at Columbia University, and a protégé of John Dewey, proposed that the study of algebra and geometry in high school be discontinued except as an intellectual luxury. According to Kilpatrick, mathematics is harmful rather than helpful to the kind of thinking necessary for ordinary living. In an address before the student body at the University of Florida, Kilpatrick lectured, "We have in the past taught algebra and geometry to too many, not too few" (Royer, 2003, p. 179). This is all very puzzling, as Kilpatrick excelled at math from an early age and was a mathematics professor at both Mercer University and the University of Tennessee.

Perhaps Kilpatrick's attack on the teaching of mathematics was due to his belief that only subjects that have a direct practical value to students should be taught in schools. He felt that students should only study mathematics if it had a direct relevancy to their lives, or if students independently wanted to learn math. Kilpatrick's views were supported by others in his professional community. According to David Snedden, the founder of educational sociology, and a prominent professor at Teachers College at the time, "Algebra...is a nonfunctional and nearly valueless subject for 90 percent of all boys and 99 percent of all girls--and no changes in method or content will change that" (Osborne & Crosswhite, 1970, p.126). This view of mathematics education was spreading through the progressive movement, and caused a great backlash from those who felt mathematics education was important.

In 1920, the National Council of Teachers of Mathematics (NCTM) was founded, created in part in response to the actions of those like Kilpatrick. The mission of this Council was to keep the values and interests of mathematics before the educational world and the first NCTM President, C. M. Austin, urged for curriculum reforms and adjustments come from the teachers of mathematics rather than from the educational reformers. This council grew in membership and power, greatly influencing the direction of math education in the U.S., and is still in existence today. Although this council worked to keep the best interest of the mathematics world at heart, the focus on math education continued to dwindle over the subsequent years.

Progressive education continued to be popular through the 1920s and 1930s, and spread through elementary and secondary classrooms. The focus was on active learning and the breaking down of such rigid walls between the subject areas. Many educators believed that math above the arithmetic level was not as important a subject as other content areas. Only math that had a practical and direct real world application for students was seen as important in education, and as a result, there was a general de-emphasis on anything above arithmetic in U.S. schools.

Flash forward to the 1940s, and we see public outcry at the state of mathematics education and the lack of workers able to do basic calculations. Focus was therefore put on purely practical problems such as consumer buying, insurance, taxation, and home budgeting, but not on algebra, geometry, or trigonometry. This “Life Adjustment Movement” was closely tied to the progressivist movement in that it stressed the importance of teaching skills that could be applied to real life and that were interesting to students. Public criticism was based on the lack of attention on basic skills and academic content.

The New Math Movement

New Math reform in the US came primarily from the cold war completion and the growing public appreciation of the importance of mathematics, science and technology. “By the end of the decade, the appearance of radar, cryptography, navigation, atomic energy, and other technological wonderments changed the economy and underscored the importance of mathematics in the modern world. This in turn caused a recognition of the importance of mathematics education in the schools” (Royer, 2003, p. 182). Before this point, the focus in mathematics education was on basic arithmetic that could be used in the home, in commerce, and in everyday life. Advanced math even into algebra was not stressed as important, and so many students were not prepared for careers that involved math more advanced than arithmetic.

The U.S.S.R launching of Sputnik, the first space satellite, in the fall of 1957 was seen by the American press as a major humiliation, and it called attention to the low quality of math and science instruction in the public schools. In response, Congress passed the 1958 National Defense Education Act to increase the number of science, math, and foreign language majors, and to contribute to school construction. As a result, math curriculum in the U.S. changed dramatically.

The New Math movement of the late 1950s brought the introduction of calculus courses at the high school level, when in 1955 only 24.8% of high school students were enrolled in Geometry, and only 2.6% were enrolled in Trigonometry (Royer, 2003). Royer (2003) wrote, “Some of the New Math curricula were excessively formal, with little attention to basic skills or to applications of mathematics. Programs that included treatments of number bases other than base ten, as well as relatively heavy emphases on set theory, or more exotic topics, tended to confuse and alienate even the most sympathetic parents of school children” (p. 184).

There were instances in which abstractness for its own sake was overemphasized to the point of absurdity. Many teachers were not well equipped to deal with the demanding content of the New Math curricula. These teachers were a product of the education system of their time, which did not emphasize mathematics to the extent that the new math movement was. This dramatic turn in math curriculum moved the focus from applied mathematics [primarily arithmetic] to abstract mathematics with no time in between to properly adjust. As a result public criticisms increased.

Students and parents had for decades experienced math education with a focus on arithmetic, and then within a span of a few years there was a dramatic re-focusing of math education on to more advanced math. The link between math anxiety in students and the anxiety levels of their teachers has been shown by researchers (Rule and Harrell, 2006). These teachers who were thrust into a new direction, having to teach math above the arithmetic level were not prepared and their anxiety levels may have been apparent in their teaching. Their students were ill-prepared by their previous schooling to tackle higher math, and their parents, a product of the previous decades' math-light education system, were ill-prepared to provide math support at home. Teachers were not prepared. Students were even less prepared. It is no surprise that here, in 1954, is where an awareness of math anxiety entered the scene.

Mathematics Anxiety is Recognized

The emergence of the study of mathematics anxiety started with the observations of mathematics teachers in the early 1950s. Elementary school teacher M.F. Gough published "Mathemaphobia: causes and treatments" in 1954 after seeing her students struggle with math. In 1957, Dreger and Aiken published the article "The identification of number anxiety in a college population" in the Journal of Educational Psychology, thereby introducing 'Mathematics Anxiety' as a new term for students' attitudinal difficulties with mathematics. They defined it as "the presence of a syndrome of emotional reactions to arithmetic and mathematics" (p. 344). These studies were coming out of what was seen in classrooms across the country. Teachers were seeing math anxiety at the high school level and in higher education and this got the attention of researchers in education.

When math anxiety first entered the research scene in the 1950s, researchers were interested in defining what math anxiety was: how did it relate to general anxiety, how did it relate to test anxiety, what did it look like in the classroom, what did it look like in a student, and in a teacher. Then the focus shifted to what the complexities of math anxiety were. Researchers broke it down into the cognitive and affective factors, and named abstraction anxiety as a distinct type of math anxiety. At this point researchers started becoming curious about how math anxiety could be alleviated, and so focused on testing various treatments on various populations.

That is where we are now, entering the fourth period in the study of mathematics anxiety: investigation of effective treatments. Researchers are just now starting to test treatments in the classroom setting at all levels of education. The work from the previous three periods are giving direction to these studies, as depending on what the researcher see as the cause of math anxiety, they may choose a drastically different approach to treating it.

Math Self-efficacy

Physical Alderian Psychiatrist, Rudolf Dreikurs, asserted that many people suffer from an 'assumed disability' in mathematical literacy. Shannon & Allen (1998) wrote that most children with difficulties in mathematics are found to also doubt their problem-solving ability. This doubting of one's own ability is related to self-efficacy, the "sense that one is competent and effective. Distinguished from self-esteem, a sense of one's self-worth. A bombardier might feel high self-efficacy and low self-esteem" (Meyers, 1993, p. 101). Students with low math self-efficacy may be see themselves as incompetent and ineffective in math tasks and thinking mathematically.

In the past 10 years, some researchers have taken a turn from seeing math anxiety as the problem, to seeing it as a symptom of a deeper problem: low math self-efficacy.

Shannon & Allen (1998) presented this equation for describing mathematical performance: *Performance = (Potential * Preparation) – Self-Interference.* They interpreted this by stating that traditionally, the focus of the problem of innumeracy has been directed to the first two terms, but evidence has been accumulating which suggests that the latter term is the true cause for disappointing performance in mathematics. It is not enough to use best practices in our math classrooms, or provide resources and scholarships to students with great potential; if they are held back by their worries, fears, anxieties and low self-efficacy, their performance will suffer. This changes the plan of action for treating math anxiety and shifts the focus of educators to the question of how we can improve students' view of themselves as learners and as mathematicians.

Summary

Knowing where the math anxiety a student is experiencing comes from is essential in any treatment that is going to be effective. This is why the walk through the history of math curriculum in the United States is relevant to investigating the roots of the question "what are effective strategies for alleviating math anxiety in the public school classroom?" There was a long road taken to get where we are today with math education in this country.

What has happened along the way in the past 100 years continues to have an impact on the way math is taught in this country, and math anxiety is one of the problems that have risen because of this tumultuous history. Students have not had a consistent path to follow through their math classes from elementary school through high school or college. Curriculum changes drastically every textbook purchasing cycle, and schools are not consistent in their approach to teaching math across states, districts, or even classrooms within schools.

Researchers are still trying to make the connection between various factors in math education and the emergence of math anxiety in students. They have yet to find the solution to the problem of math anxiety. Yet teachers continue to notice and take on math anxiety in their classrooms, using the research as a compass to guide their efforts.

In Chapter 3, the most current research about math anxiety and effective strategies for treating anxiety in the classroom will be outlined and discussed. Possible treatments include relaxation therapy, instrumental and relational instruction teaching methods, group work, student lead classes, the use of technology as treatment and other methods. Chapter 4 summarizes and concludes on effective treatments given the research in Chapter 3.

CHAPTER 3: CRITICAL REVIEW OF THE LITERATURE

Introduction

Researchers do not agree on the most effective way to treat math anxiety. Some focus on treating the student directly for their anxiety issues, breaking up math anxiety into the affective, cognitive, abstract dimensions. Given the clinical status of mathematics, anxiety as a type of specific phobia, it is not surprising that many researchers approach the treatment of math anxiety through affective (emotional) focused methods. Cognitive anxiety, on the other hand, refers to the worry component and some researchers focus on treating this nervousness and dread in order to alleviate math anxiety. Math self-efficacy research stems from the cognitive component of math anxiety and will also be outlined here.

Another way that treatment for math anxiety in students is approached is through treating the math anxiety of teachers. Some researchers (Cruikshank & Sheffield, 1992) have found that teachers with math anxiety or a negative view of mathematics can negatively impact the mathematical development of students, and in some cases can contribute to the development of math anxiety in their students. Many of the studies involving teacher anxiety focus on pre-service teachers currently enrolled in university courses, and have positive effects on their math anxiety and self-efficacy. The treatments used to alleviate the math anxiety of these teachers can also be potentially used with students experiencing math anxiety.

Others hypothesize that changing the teaching methods of instructors will help alleviate math anxiety in their students, so they focus their research on changing the way teachers teach. In summary, this chapter will be divided into seven sections. The first will describe the research on the dimensions of mathematics anxiety and the implications for treating math anxiety. Teaching methods will be the next section, focusing on how teachers can create an environment that will aid in alleviating math anxiety. Technology as a tool for math anxiety treatment, self-efficacy interventions, and journal writing will then be explored. Research on manipulatives as a tool in math methods courses for teachers will be described, followed by the last section, mood altering treatments.

Math Anxiety Defined

Wigfield & Meece (1988) studied math anxiety in elementary and secondary school students to assess math anxiety as part of a comprehensive longitudinal investigation of children's beliefs, attitudes, and values concerning mathematics. This study was observational and comparative, comparing gender and grade levels.

The subjects were 564 sixth through twelfth grade, predominately white, middle class students (298 males, 266 females). The classes that participated were chosen randomly by the researchers from a pool of the classrooms whose teachers volunteered to be a part of the study. All students in each classroom participated.

The researchers administered the Student Attitude Questionnaire (SAQ) and a scaled down version of the Mathematics anxiety Questionnaire (MAQ). The items on the MAQ focused on negative affective reactions to doing math activities in school and on students' concerns about their performance in mathematics.

It was found that math anxiety is highest in ninth graders (M = 5.46), intermediate in 7^{th}, 8^{th}, 10^{th}, 11^{th} and 12^{th} graders, lowest in sixth graders (M = 4.63). They also found that a degree of worry or concern (cognitive anxiety) may be needed to motivate students to try harder; without that, students may see no reason to try. However, if this worry or concern becomes too strong and is focused on possible poor performance, it may interfere with performance. This conclusion is from the fact that there was a positive correlation between cognitive anxiety and math achievement values (.12*, .36**, .11*), while there was a negative correlation between affective anxiety and math achievement values (-.35**, -.18**, -.18**). Also, there was a negative correlation between math performance (grades) and affective anxiety (-.22** for year one, and -.26** for year two) (*$p < .05$) (**$p < .01$). It was also concluded that math anxiety should be conceptually distinguished from perceptions of math ability (math self-efficacy).

The researchers concluded that techniques to build anxious students' confidence in their math ability might not be enough to alleviate the strong negative affective reactions to math that they experience. Math anxiety students also may need trainings to reduce their fear and dread of math. As has been found in the test anxiety area, intervention efforts focusing on both the cognitive and affective components of math anxiety may prove to be the most effective way to reduce its debilitating effects. It was also recommended that programs should be implemented during the elementary school years, before children's' anxiety about math becomes strongly established.

Based on the researchers' findings, cognitive math anxiety is not entirely negative and could have its place in helping motivate students to work hard in math class. It is when the worry gets so extreme that it interferes with student performance. For this reason, the affective dimension of math anxiety should be more strongly focused on in treatments, for this type of math anxiety has only negative effects on student performance.

Ho et al. (2000) conducted an observational study on the affective and cognitive dimensions of math anxiety. It was a cross- national study that included the US, Taiwan and China. They defined math anxiety as having two unique components: affective test anxiety is the emotional component of anxiety, while cognitive anxiety is the worry component of anxiety. General mathematics anxiety is the convergence of the two.

The measures of math anxiety and mathematics achievement were group administered by researchers and a teacher in each nation. Standardized instructions were given and each test was translated for use in China and Taiwan. The tests given were: the Math Anxiety Questionnaire (MAQ) which tested students for cognitive and affective math anxiety, and a mathematics achievement test developed by the researchers for this study.

The subjects were 671 sixth grade students: 211 from China (92 girls, 119 boys), 214 from Taiwan (106 girls, 108 boys), and 246 from the United States (111 girls, 135 boys). The students were selected based on where they lived: rural or urban. The urban sites were Beijing, Taipei, and Claremont, California. The rural sites were Men Tou Gou district in China, Miao- Li and Yang Ming Shan in Taiwan and Cuyama and Santa Ynez, California.

They found that distinct affective and cognitive factors of math anxiety could be identified. Also, affective and cognitive factors of math anxiety differ in their associations with math achievement. In regard to gender difference, Taiwanese girls showed higher mean levels of both affective and cognitive dimensions of math anxiety ($M=4.49$, $SD=1.76$ for affective, and $M=5.58$, $SD=1.25$ for cognitive) as compared with Taiwanese boys ($M=3.88$, $SD=1.68$ for affective, and $M=4.77$, $SD=1.41$ for cognitive).

U.S. girls showed higher mean levels of cognitive dimensions of math anxiety (M=4.57, SD=1.42) than U.S. boys (M=4.16, SD=1.34). Chinese boys and girls were about even ($p < .05$ for all results).

Finally, they found that the effect of math anxiety on achievement differed depending on whether the subject was male or female, or from China, Taiwan or the U.S. For example, for Taiwanese students, the cognitive worry factor may serve as a motivator; for those students who were found to have cognitive math anxiety, their math achievement was higher by 25%. ($p < .05$).

The cognitive and affective split that they are working on is supported by other studies that they reference. This split is useful to the study of math anxiety, because it is useful to determine how to address math anxiety in students, but one cannot do that if math anxiety is not defined clearly. If it is true that some students have cognitive but not affective anxiety, it is going to affect how those students are treated for math anxiety.

The researchers said in their discussion that their results "prove" that affective and cognitive elements of math anxiety are distinct and should be treated differently. This study, paired with the other one they cite, do not alone prove this distinction. More research would need to be done on the distinct elements of math anxiety.

Because gender is not included in the scope of this paper, those results will not be considered. Though they are included here in order to provide readers with a full view of the study. This study is included because in order to talk about the affective and cognitive dimensions of math anxiety in my treatment methods, it must be established that there is such a split in mathematics anxiety. This study, in showing that some have affective but not cognitive anxiety, and visa versa, supports the distinction made in previous studies about the cognitive and affective split in math/test anxiety.

Furguson (1986) studied abstraction anxiety as a factor in mathematics anxiety. This was an observational study meant to explore whether there is a component of mathematics anxiety distinct from mathematics test anxiety and numerical anxiety. Abstraction anxiety is defined by the researcher as "a factor of math anxiety that reflects a qualitative difference from the type of anxiety illustrated by the items that loaded heavily on Numerical Anxiety. Students often express this difference with a statement like "I understand 2 and 3, but I don't understand x and y" (Furguson, 1986, p.146).

The subjects were 365 community college students at a college with a large Hispanic population. During the last 10 minutes of a community college math class, the researchers administered a scaled-down 20-question version of the MARS (the researchers named their version Phobos, which means fear and is a satellite of the planet Mars) with an additional 10 questions having to do with abstraction anxiety. The students responded to the statements on the test on a five level likert scale. A response of one meant that the statement did not frighten the student, a five meant that they were very frightened by the statement.

The results of the study were that all of the items labeled as pertaining to abstraction anxiety had loadings of at least .5 on Factor 1. Furguson concluded that there was strong support for the hypothesis that abstraction anxiety is an important and distinct factor to mathematics anxiety. It should be said that the interpretation of abstraction anxiety is not clear in this study, in part because the subjects in the study ranged from students in elementary algebra to students in calculus. Identification of another factor of math anxiety allows an additional refinement of treatment for math anxiety.

What strengthened this study was that the items from the Phobos inventory were clearly laid out so that the reader of the study can know exactly what was being observed. Also, this study could be easily replicated, thereby verifying the results.

Although there is no treatment in this study, it applies to the research question of this paper because the study of math anxiety is so new to the research world (past 30 years), clear definitions are in order. To understand and treat math anxiety in classrooms necessitates a strong definition of it. This paper is strong in that the sample size is so large and the population so academically diverse. The test given was simple and short, and the results were clear and conclusive. Given these strengths and weaknesses, abstraction anxiety is a valid part of mathematics anxiety, separate from general math anxiety, test anxiety and numerical anxiety.

Teaching methods

Teachers have a great deal of control over the classroom community and the environment created in the classroom. They have the last word on classroom rules, protocol, grades, punishments and rewards. Patrick, Turner, Meyer and Midgley (2003) were interested in how these teacher practices contributed to student perceptions and avoidance of mathematics. The researchers were specifically interested in the psychological environments established by the teachers in the first days of school and the effects of those environments on students. This study was qualitative; the treatment groups were not pre-determined and the researchers did not interfere with the teachers' practice. The researchers acted as quiet, unobtrusive observers at the back of the class.

The subjects were eight sixth-grade teachers and their students from seven K-6 elementary schools in an ethnically diverse school district in the Midwest. One hundred and seventy six students were administered surveys during the course of the study. Out of a pool of 20 classroom teachers that agreed to participate in the study, eight teachers were randomly chosen to be full participants. All classrooms followed the Connected Mathematics curriculum and survey data was collected during the same units of instruction. All classrooms had similar structures; teachers used heterogeneous whole class instruction with some small group work.

Qualitative data was collected through observations done on the first and second days of the school year. The students were also administered multiple math-specific survey measures in their regular classes at the end of both fall and spring semesters. One of these measures was on student perception of their classroom environment where they reported the extent to which their teacher was supportive, promoted mutual respect among classmates, and promoted mastery and performance goals in math class. Another measure was of their use of avoidance strategies in math class, including their self-handicapping, avoiding seeking help, disruptive behavior, and cheating.

The qualitative data collected through classroom observations were separated in to two types: student discourse and teacher practices. The data was coded so that the researchers could systematically focus on different aspects of the classrooms. Themes and patterns were found within and across the classrooms. Teacher talk and messages were coded using two categories: motivational and organizational discourse. Then they were put into one of two categories: supportive and non-supportive.

From this data, three different classroom environments were established from the beginning of the year: supportive, ambiguous, and non-supportive. Although this study was observational, it could be argued that the treatments were these three types of classroom environments. The researchers observed the effects of these environments on the students' math attitudes and compared the results as if the treatments had been pre-determined and controlled.

The first treatment was a supportive classroom environment. This treatment group had teachers who were respectful, humorous, and were enthusiastic about learning. They stated that they expected all students would learn, and their practices were based on respect. Three teachers fit in this category. These teachers spoke about aspects of the math curriculum they were excited about and portrayed learning math as enjoyable, valuable and worthwhile. They expressed high expectations for their students' success, and confidence that they could teach them. They actively minimized anxiety or discomfort students were feeling and conveyed that they would be helpful in any way the students needed. Relationships with students were actively worked on and the teachers connected their caring with their students' needs. They directed warm positive and personal comments to the students. There was also a strong sense of developmental appropriateness; they expected adolescent responsibility, yet still recognized the child in them. A strong atmosphere of community was created in these classrooms, in which respect among students was paramount. Students were encouraged to help each other and work collaboratively. Finally, classroom rules and management structures emphasized fairness. Clear examples of appropriate behavior were outlined, along with examples of behavior that was not acceptable. Student responsibility was stressed and there was the expectation that teacher power would not have to be wielded for order to be maintained.

The ambiguous classroom environment was sometimes learning-oriented and academically supportive, but teachers underestimated or overestimated students' development and failed to connect with their students in a personal way. Also, classroom procedures and management were inconsistent. Three teachers were in this category. These teachers sometimes contradicted themselves and were not consistent with the messages they sent to their students on discipline, expectations and personal connections. The students were told to behave positively and to get along with each other, but the teacher did not insist on holding students accountable for their actions. Classroom management plans were vague and inconsistent. There was a lot of scolding of students observed, and few supportive motivational statements were made.

Finally, unsupportive classrooms did not appear supportive of students intellectually or socio-emotionally. The teacher used extrinsic motivation, and they expressed that they expected students to find the work difficult and try to cheat. Authoritarian management was used and the teacher assumed students would get in trouble. Two teachers (Clark and Parsons) were in this group. The teachers implied often that students would not enjoy the class work, and statements often seemed intended to around fear and anxiety in the students. Threats were made that students may not survive the year. One teacher used the word "survive" twelve times in a ten-minute period when talking about the upcoming school year. These teachers made themselves the focus of the classroom, spending a great deal of time building an audience rather than a community. Much emphasis was put on the power balance in the classroom, and it was stressed to the students that the teacher was in control of their academic fate, such as removal from the school or not moving on to the next grade. The teachers made comments that appeared to model the very opposite of respectful, supportive behavior, outright making fun of students to other students. Classroom management plans emphasized a great deal of teacher control and the focus was on following classroom procedure. A "penalty card" system was used by both teachers, where a sequence of cards was turned over with each succeeding infraction. The percentages for supportive motivational discourse were 11.2% and 6.1% for the two teachers. The teachers blamed the students for not recalling information or implied that students were going to cheat or not try. One of the teachers used the exclamation "DUHHHHH!" often, and in a manner that appeared to indicate that students should have known what to do or what an answer was. These teachers were often sarcastic and mocking of their students.

After analyzing student surveys at the end of the fall and at the end of spring, it was found that students in supportive classrooms engaged in significantly less ($p < .05$) self-handicapping and disruptive behaviors than did students in ambiguous or non-supportive classrooms, who did not differ from each other on this measure. Also, students in supportive classrooms reported avoiding seeking help less than students in non-supportive classrooms. Students in ambiguous classrooms reported significantly more cheating in the fall then those in supportive classrooms, though students in non-supportive classrooms did not differ from those in supportive classrooms on this measure. By the spring measure, students in the non-supportive classrooms reported significantly more cheating ($M= 1.31$ in Fall, $M=1.90$ in Spring; $p<.01$) than those in supportive classrooms ($M=1.27$ in Fall, $M=1.44$ in Spring; $p<.01$). Those in ambiguous classrooms did not differ in the spring ($M=1.65$ in Fall, $M=1.66$ in Spring; $p<.01$).

The researchers only observed the first two days of class in fall quarter, and then made hypotheses about student behavior and math anxiety/avoidance that would be seen at the end of fall and spring. It was assumed that the teachers continued behaving in the way that was observed in those first few days, and the data supports this assumption. Reading the descriptions of the environments created, one can get a real sense for the way that students must feel.

This study is strong in that the researchers provided great detail on the teachers' classroom environments and painted a clear picture of the treatment groups. The qualitative data was collected and analyzed by a team of researchers, and they crosschecked their results and triangulated when possible. The sample size was large enough to validate the results, and the researchers had at least two teachers representing each grouping (supportive, non-supportive and ambiguous) so as to give a well-rounded picture of the grouping types. The students did not know what grouping the researchers deemed them to be in, and therefore could not have been affected by the knowledge that an objective outside party had judged their teacher to be one of the three types. The students could have discussed their teacher with other students in other classes, and therefore knew what they others were experiencing, but this happens in normal, non-research settings also.

Another researcher who was interested in teaching methods was Norwood (1994) who studied the effects of instructional approach on mathematics anxiety and achievement. This study was different than Patrick, Turner, Meyer and Midgely's (2003), in that Norwood set up the research groups instead of just observing natural classroom instruction. Norwood was trying to determine if instrumental instruction (rote memorization of formula and rules where mechanical computation was stressed) was more or less effective than relational instruction (focus on development of concepts and their relationships to the fundamental principles of mathematics. Rules and formulas were de-emphasized) in reducing math anxiety and increasing achievement in college students.

The subjects were 123 students enrolled in six different math 116 courses, over a 14-week semester in an urban community college in the Northwest. Of the 123 students, 62 were randomly placed in the instrumental groups, and 61 were randomly placed in the relational groups. They were not made aware of the fact that they were being studied. The six sections of the developmental arithmetic courses were taught by three mathematics instructors, one of whom was the researcher. Each instructor taught two sections: an instrumental section and a relational section. All three used the same textbook, lesson plans, handouts, quizzes, tests, and final examination. All materials were written by the researcher in cooperation with the other instructors. The researcher also observed the other instructors on a weekly basis to monitor the instructional interventions.

The variables for this study were pre test mathematics anxiety scores, pre test mathematics achievement scores, post-test mathematics anxiety scores, and post-test mathematics achievement scores. These data were collected at various times throughout the study, and the tests were the same for both groups.

Both treatment groups met for the same amount of time in class, but as the relational approach took longer to complete, the instrumental approach group received worksheets with extra problems to complete in class to equalize the time spent in class. The worksheets covered the material that had been taught during the lesson.

The results of this study were that the Instrumental group (group I) experienced a 5.03 decrease in the mean mathematics anxiety score (M=32.03 pre, M=37.06 adjusted post), while the Relational group (group R) experienced a 1.55 increase in mathematics anxiety (M=36.03 pre, M=34.48 adjusted post) , as measured by the Mathematics Anxiety Scale (MAS). It should be noted that the instrumental group started the quarter at a lower level of measured math anxiety than the relational group. A high score indicated less anxiety.

In the discussion section, the researcher concluded that (a) the more structured, instrumental approach tends to reduce mathematics anxiety when compared to the less structures, relational approach; (b) either of the two instructional approaches can be successfully employed to improve mathematics achievement of remedial college students; and (c) there is a slightly negative relationship between mathematics anxiety and mathematics achievement meaning that the lower the mathematics anxiety level, the higher the achievement level.

An issue that the researcher discussed is that there is the serious issue of how to make students more comfortable with instruction than the most current curricular recommendations support, namely relational instruction. The researcher argued that there was evidence that the source of the math anxiety problem at the higher grades was the rote memorization of facts (instrumental instruction) at the lower grades. This worked for students up to a point, where memorization no longer enables a student to get by.

The study was focused on a remedial arithmetic course, where rote knowledge of algorithms was sufficient, since the tests were computational. Norwood argued that this may be why there was no difference in the mathematics achievement post-test scores between the relation and instrumental groups. More research needs to be done in this area, using more advanced math courses where rote memorization will not suffice.

Another issue evidenced in this study was the threat to external validity, novelty and disruption. Several of the students in the relational group expressed discomfort with this new style of teaching and learning mathematics. Most were not familiar with studying the “why” of a math problem, but rather were practiced at focusing on the answer. They reported pressure and frustration in trying to use this new relational approach in learning mathematics. The mere fact that this relational approach was new to these students and the instrumental instruction was what the students were used to may have caused more anxiety in the relational students, or at least did not reduce anxiety levels. Norwood hypothesized in his conclusion that perhaps with more time and experience with relational instruction, students could find success in focusing on the “why” instead of the answer once the novelty of the method wore off.

One of the negative aspects of the study was that experimental treatment diffusion might have been an issue because these students all attended the same school. They may have heard something from the other group about the treatment and tried using it themselves. But they did not know they were in a study, so it may not have been an issue. Overall, having the subjects not be aware that they were in a study makes the study stronger.

Differential selection was also an issue, as group I was initially more anxious than group R by four points. Statistical regression was also an issue because the subjects were in this study because they earned a low score on their math entrance exam. Because they were already at the lower end of the score set, they were more likely to improve their score with any treatment.

Given the strengths of the study, students who have historically received low math test scores could benefit from Instrumental instruction and lower their math anxiety as a result.

Yusof and Tall (1999) studied the effects of a treatment for math anxiety and low self-efficacy that included problem solving and reflection on thinking activities during a university course. This study is very similar to Norwood's in that the treatment emphasized understanding over procedure and the instructor did not dictate how a problem should be done and did not provide a solution procedure for problems. This study also takes one step further and collected data on the instructors' expectations of student ability, whereas the previous study only briefly mentioned this variable.

Yusof & Tall hypothesized that problem solving would cause a positive change in students' attitudes, and that some of these changes would be reversed when the students returned to standard mathematics. The subjects were 44 students (24 males and 20 females) aged 18 to 21 studying Industrial Science and majoring in Mathematics and Computer Education. The course was taught by author Yusof and lasted 10 weeks, for a total of 30 hours. Class time was divided into two-hour group problem solving session and a one-hour meting in smaller groups once a week. The group problem solving was structured with the instructor pointing out a specific aspect of problem solving followed by students working on problems illustrating this aspect.

Thirty minutes in, the instructor reviewed the situation and made sure that everyone was focusing on the same problem and considering the ideas of their group mates. The instructor provided no clues as to how to solve the problem. Students were open to solve the problem with whatever method they wished and were encouraged to explore all aspects of mathematics while looking for a valid process. For the one-hour meeting time, students reflected on their mathematical experience and discussed their process for solving the problem. The instructor acted as facilitator, pushing the students to look for where things may have gone wrong in their process and reflecting back what he heard them say.

Data was collected in various ways during these class times. The researchers observed the class, administered an attitudinal questionnaire before and after the study as well as six months after the study's end. Staff was also given the same questionnaire to say what attitudes they preferred and expected from their students, and comments from students and staff were invited at any point. The questionnaire had two parts: attitudes to mathematics and attitudes to problem solving; and the responses were based on a 5-point likert scale.

From the classroom observations, the researchers noted that the students were initially very confused. They started the problem frantically, jumping right in to it then get stuck and ask, "What shall I do now? Is this the right way of doing it?" The students seemed to struggle with having no "correct" procedure given to them by the instructor for solving a given problem. It took four weeks before the researchers could observe that they students were coming to accept this new way of approaching a problem. At this point, they started making decisions and think for themselves. Some of them even started keeping track of their procedure as they made it up, creating their own running commentary of their methods.

The problems given to the students started out simple so as to build the students' confidence and not overwhelm them. The instructor did not work out any of the problems before hand and at times would work out problems in front of the class to show the students that even mathematicians did not come up with neat and perfect solutions at first. This was to model the type of risk taking and conjecture the instructor wanted the students to use and show the students that it was okay to make mistakes and be wrong sometimes. The students eventually found joy in problem solving in this way and expressed that they for the first time were seeing how creativity plays into math. They also found increased intrinsic motivation; no longer feeling like they were just doing the work to please their instructor and get a good grade.

It was found that on twelve out of the seventeen items on the questionnaire, teachers expected students to have attitudes which were opposite of what they desired. All but one of the twelve differences is statistically significance ($p < .05$). For example, the lecturers expected typical students to think mathematics was very abstract, that they would not understand quickly, they would consider that mathematics does not make sense, and they would not relate mathematical ideas. In every case, the lecturers preferred that the students have the opposite attitude. One has to ask what effect that expectation has on how the lecturer teaches the course, and therefore how well the students perform. But that is for another study!

Another result of note is that teachers also reported that they expect from their students emotions such as anxiety and fear of the unexpected. The researchers commented:

> The expectation of emotions such as anxiety and fear of the unexpected intimates an alternative anti-goal of avoiding failure rather than a goal of achieving success. This, coupled with the expectation that they will give up when faced with difficulty, is consistent with the idea that students are expected to settle for the lesser goal of being able to carry out procedures to solve routine problems rather than attempting to build a broader conceptual understanding (Yusof & Tall, 1999, p. 74).

This analysis of the teachers' mindset is spot on. This paragraph describes the reason teachers should address math anxiety in their classrooms and analyze their own view of math anxiety and student achievement.

The results on student anxiety and self-efficacy show that the attitudinal changes were positive for all but pleasure, which was almost at a maximum at pre-test and post-test. Three items were found to change significantly ($p < .05$) post-study in the desired direction: ability to relate ideas and confidence both increased and anxiety diminished.

Six months later, after having returned to mathematical lectures, the positive effects of the study were near reversed. The least effect was on anxiety at this post-study measure, with the total number of students who felt anxious increasing from 6 to 9, and those not anxious decreasing from 39 to 36. (p values are all $< .05$). The attitudes of the students were back where they were at pre-test levels at this sixth month post-study time. The only three problem-solving attributes that retained their improvement were confidence and unwillingness to give up which remained high, and fear of the unexpected remained low.

The student and instructor open-ended comments collected showed similar results to the numbers described above. Students responded that they were more confident and found math more enjoyable after the treatment. “Mathematics has always given me a lot of problems because I don’t have the ability for memorization….Now that I know about mathematical thinking, my interest and desire to learn maths have increased” (Yusof & Tall, 1999, p. 78), wrote a 4th year computer education student. After returning to lecture classes after 6 months, students expressed their frustration, “there is little discussion and it provides no encouragement to do maths. The content is emphasized over everything else. We are crammed full of lots of bland mathematical abstract theory” (Yusof & Tall, 1999, p. 78), stated a 3rd year industrial science student majoring in mathematics. There were some students who expressed reservations about using problem solving, “the main disadvantage is time. It would take several hours maybe days to understand each new concept…..sometimes we are too bogged down in the technical details and we end up purely taking down the notes without even considering”, but this student goes further to say, “but I think with further support from good teaching as well as tailoring the courses to suit the needs of the students the situation can be improved” (Yusof & Tall, 1999, p. 79).

The course lecturers stated that they genuinely desired to change the system, but they were not sure how. “…I am not sure of these (processes). I have not thought about them and I don’t know how to go about (teaching) them….we developed certain abilities to look at problems but we are not sure how those abilities came to be with you” (Yusof

& Tall, 1999, p. 79) says one. “The system has been proven a failure. It has not been successful in producing good mathematicians, or engineers that can use mathematics effectively. They only know how to use procedures or computer packages without really understanding why they use them….it’s all down to the system. We are not training students to discover patterns, or how to prove a statement is true, for example. What we teach them is mainly how to use the procedures” (Yusof & Tall, 1999, p. 80) says another. These teachers seem very aware of what they see the problems to be yet don’t have the tools to fix them. They continued to teach in a way that they deemed to be insufficient and misguided because they don’t know any other way.

Paired with the other studies in this paper on cooperative learning and problem solving, a strong argument for a treatment of this nature is forming. The strength in this study is found in the fact that pre-treatment traits were changed for the better directly following treatment, then the levels were returned almost entirely to their pre-treatment levels six months after treatment was ended. What would have made it even stronger would have been to reintroduce the treatment at this point to see if the levels returned to the desired levels. This may have shown causality.

Alsup (2004) examined the effectiveness of constructivist instruction in reducing the math anxiety of pre-service elementary teachers. Alsup also studied the effect on the teachers’ efficacy beliefs and perception of autonomy or empowerment. Although this study focused on math anxiety in teachers, the results can be extended to students at the high school level. The pre-service teachers were students in this study, their instructor was treating them and other studies described in this paper support the conclusion that constructivist instruction can alleviate math anxiety.

The participants were students in the fall semester of 2001 in Math Concepts I and Math Concepts II. These courses were required for pre-service elementary teachers at a small, 4000 student, rural liberal arts university in the Midwest. The same instructor taught both courses. One of the math I sections and the only math II section were experimental courses and the other section of math I served as the control group. The courses were randomly assigned to the control and experimental group. At the conclusion of the study, there were a total of 44 students in the experimental group and 17 students in the control group. The curriculum for math I included problem solving, sets, algorithms for basic operations, number theory, integers, fractions, decimals and percents. Math II covered elementary statistics, probability and geometry. Both courses were one semester long.

There were multiple instruments used in this study. A 25-item abbreviated version of MARS was administered as a pre-test on the first day of class for both groups, alone with the 21-item Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) to measure confidence to teach math and strength of the belief that effective teaching influences student learning, and finally, a simple rating scale was used to measure the participants' sense of empowerment (autonomy) over instruction, curriculum, and evaluation in the course and previous math courses. The next-to-last day of class, the students were given the same three instruments. Only those students who completed all three instruments both times were included in the study. Students were also eliminated from the study if they dropped the course or were simultaneously in math I and math II.

The treatment, constructivist instruction, emphasized active learning and student involvement, modeled after pedagogy employed by progressive, constructivist educators.

The format of the experimental class was that each period began with a short lecture (10-20 minutes) about key concepts. On occasion there was no introductory lecture. Then the students worked in groups of three or four to solve problems they had chosen or created during the previous class. They would then prepare a presentation of the solution and methods to show the class. The problems they could not solve to the satisfaction of all the group members were added to a "work list." When the work list was completed, the professor asked if any of the students wished to present their solution. Whenever a solution was presented, the group presenting had to explain their method and convince the entire class of its reasonableness. If another group disagreed with the solution or the process, they could challenge the presenting group by requesting further elaboration or could offer to present their own solution.

During this entire presentation and work process, the professor was a facilitator of thinking and discussion; he did not offer solutions to the problems. His role was to clarify a point of mathematics, suggest an idea to investigate, focus the discussion, or summarize his interpretation of the groups' solution. If the students could not come to a consensus about the solution to a problem, it became an "open problem," as in the community of mathematicians, had no generally agreed upon solution.

Once during the semester each group taught one section of the textbook to the class. They presented the essential concepts and procedures, assigned homework, and reviewed the assigned homework. The group could teach using whatever method they liked, whether it be through games, lecture, class discussion or hands-on activities. The course was designed to be student-centered and the professor made every effort to emphasize active learning, communication, reasoning, and the development of deep conceptual understanding of mathematics through a problem-solving curriculum.

The control group was taught the same material, but through a traditional lecture-recitation format of instruction. The class period started with 10 minutes for students to work together looking over their homework. Then the professor worked out requested problems on the board and answered content questions. The professor focused on using his own methods of solving problems and did not explore the validity of students' own creative solutions. After the review of homework, the professor lectured on new content and assigned homework for the next class period. Manipulatives were used and hands-on activities were done during the control group class, but the professor always prescribed how they would be used and described in detail how the hands-on activities would be completed.

In both the control and experimental groups, conceptual understanding was stressed in addition to the expectation that the students would have procedural competence. This was re-enforced by the tests, which required the answers to be accompanied with a short essay regarding the process they used to solve each problem. This was the same with both groups. The difference between the two was the method of instruction. The experimental class was student-centered, gave as much control to the students as possible, and promoted debate and creativity. The control class was teacher-centered; the students were not encouraged to think creatively or debate the teacher and they had no control or input in the class.

Another aspect of the study was that all participants were required to compose a math autobiography, keep a daily journal about their experiences in the class (with a focus on the classroom instruction), and to audio record a half-hour personal interview with the instructor at the end of the semester.

The pre-test, found that students in the control group had a slightly lower level of math anxiety as compared to the experimental group, although this result was not statistically significant. Students in both groups had a similar post-test level of perceived teaching efficacy, while students in the experimental group showed a significantly higher sense of autonomy than those in the control group ($p < .05$). The only significant difference was found in the Autonomy scale.

The math II students (from the experimental group) had the highest level of math anxiety (Adjusted M=58.2; $p < 0.05$) and the lowest level of teaching efficacy (Adjusted M=75.9, $p < .05$) of all three classes at the conclusion of the study. Also of note was that, in general, math anxiety declined dramatically for all participants in the study, when viewed collectively. Finally, the math II students, while showing a slight decrease in math anxiety (M=61.3 pre, M=58.4 post), also showed an increase in their math course anxiety (M=8.8 pre, M=9.1 post).

Alsup stated that while the results of the study do not support his conjecture, they are “encouraging, curious, and thought provoking”. The fact that the level of autonomy was increased for all students, but more so for the experimental group, was promising. He also posits that his teaching style and focus on concepts in both control group and experimental group classes were the reason that both groups showed a decrease in math anxiety and an increase in math self-efficacy. This hypothesis is not based in research of course, but it is in an interesting note that warrants further research.

Another point to make is that the math I course was, for most students, a review course. They were mostly familiar with and comfortable with the material and so they were able to enjoy the group work and presentation aspect of the treatment. Those in the math II group were not as familiar with the material, had little exposure with statistics and expressed intense loathing for geometry. They struggled with key concepts, were less secure (in comparison with math I students) about teaching the class, and were not as comfortable participating in group discussion or with putting their work on the board. In their journals, the math II students wrote they disliked being "guinea pigs" and complained that it would be easier to understand the content if the professor would teach the class instead of having the students at the board. These factors may have greatly affected the anxiety level of the group and the results of the study.

The strengths of this study were that there was a true control group, which experienced everything as the experimental did, save for the treatment. The instructor was the same for all of the classes, and the instructor was also the researcher. The instruments have been tested in prior studies and have been shown to be effective. The treatment was well described and, for the most part, could be replicated.

The fact that students dropped out of the study (the researcher did not state how many) and the fact that there were only 17 students in the control group, compared to 44 in the experimental group, weakened this study. Also, the fact that a different level of math was being taught in half of the experimental group complicated the results and obviously was an unintended complicating factor. If only math I classes were used, the study would have been stronger.

Although not directly connected to teaching method, the use of cooperative group work is a choice a teacher actively makes and cooperative group work is an integral part of constructivist teaching. Townsend, Moore, Tuck & Wilton (1998) studied whether utilizing co-cooperative learning strategies in a psychology class with a statistics component would change mathematics self-concept and mathematics anxiety. It was expected that self-concept would improve and anxiety would decrease after the course, especially for those students who had less mathematics experience when they started the study.

The subjects were 153 British University students (137 females, 16 males) in a second- year course in educational psychology. They were included in the study if they were present at both the first and last lab session of the course, and consented. Based on their math scores on the measures and on their self-reported academic history pertaining to math, the subjects were divided into 2 groups: an advanced math group (39%) and a non-advanced group (61%). There was no control group for this study. This was a correlational study.

The educational psychology course consisted of 25 one-hour lectures and 20 one-hour laboratory classes taught weekly during the academic year. Four academic tutors ran one laboratory section each and were trained in how to administer the treatment and run the class. They were directed to provide opportunities for students to work together in collaborative groups and to focus on conceptual understanding. On average, 60% of the class hour was spent in small cooperative learning groups, with the rest spent in large-group discussion or direct instruction. The class is described in further detail in the study so as to see what type of treatment was being administered.

As measures, the researchers gave the mathematics self-concept scale (a 27-item self report scale based on a five-point Likert scale), the mathematics anxiety scale test (10-item self-report scale based on a five-point Likert scale), and the students were invited to make written open-ended comments about how confident they were in doing statistics at the beginning of the course and at the end.

It was found that the more math a student had taken previously, the more confident they were in their abilities before the treatment ($M=96.76$, $SD=16.17$), compared to students without advanced math experience ($M=77.71$, $SD=18.28$). The more advanced students also had lower math anxiety levels ($M=26.41$, $SD=8.07$) when entering the study than the less advanced students ($M=35.47$, $SD=7.86$). The perception of math anxiety did not decrease over the course for either group (low anxiety or high anxiety). It should be noted that although the instruments did not show any decrease in the math anxiety level of the students, in the written evaluations of the course, many students wrote that they had gained a great deal of confidence in using statistics. It was also found that self concept in mathematics at the end of the course was significantly related to test performance in the course ($r = 0.22$, $p < .01$), and to overall performance in the course ($r = 0.20$, $p < .05$). It should be noted that these were weak correlations.

Some strengths were that no one dropped out of the study, and the number of participants was high enough to get a valid result. The study was conducted over an entire semester, with a great amount of qualitative data collected. The data was also collected at various points throughout the semester using various methods.

Weaknesses were that there was no control group for this study, but there were two subgroups the students were put in to: advanced and non-advanced. Both groups were given the same treatment and were in the same environment. Also, there were multiple people giving the treatment, and although they were all trained in the methods, they could not have all given the treatment in the exact way it was intended. The tutors were only given a two-hour training in the methods of the study.

Another point of concern is that the post-test was given before the final project was turned in, so the researchers admitted that the students' anxiety levels may have been partly due to the looming deadline. This was not good timing for giving a test on anxiety. The type of results they were looking for may not have been possible over such a short period of time. The researchers admit this in the discussion. Such a deep-rooted problem cannot be fixed with a constructivist class and cooperative group work over only one semester.

Townsend and Wilton (2003) fashioned a follow-up study to their cooperative learning study they completed in 1998. The researchers explained that they believed the previous study did not take in to account all of the factors that were at play and a follow up was needed. They stated that their current hypothesis was that a response-shift bias might have been operating in the previous study. A response-shift bias is where a pre-test measures math anxiety in a set of subjects, then the treatment is administered. The treatment then alters the mindset of the subjects and when the post-test is administered the subjects now see the questions in a different way. Their post-test scores may show no change in their math anxiety, when in fact there was a change. The researchers likened this to a group of untrained counselors rating their counseling skills as 'competent' on a pre-test early on in their training, but during the course they discover that their initial skills were actually less than competent when judged against the standards of the training.

Using these new standards, they rated their counseling skills as ‘competent’ on the post-test. These two assessments were based on different understandings of dimension of competence, so were not measuring the same thing at all.

The solution the researchers came to was to recreate the study, but with a retrospective pretest (a ‘then’ test) obtained in addition to the usual pretest. At the conclusion of the study, the subjects would also be asked to complete both a post-test and (of their perceptions now) and a retrospective pretest (of their perceptions then, before the intervention). They argued that this format would be able to measure the response shift through seeing the change in score between the pre-test and the retrospective pre-test, thereby making it possible to factor that shift in to the results of the study.

In summary, the purpose of this study was to examine the effects of a cooperative learning intervention on mathematics self-concept and anxiety in undergraduate students, using the retrospective ‘then-test’ to detect the possible mediating influence of response-shift bias. The participants in this study were 141 students (117 females, 24 males) enrolled in a second-year course in educational psychology at the University of Auckland. The students were put in to three groups: the Retrospective group (n = 60), the Pre-Post group (n = 27), and the Pretest-Only group (n = 48). The assigning of students to different groups was random.

There were three instruments used in this study. The 27-item Mathematics self-concept scale measured attitudes, beliefs and feelings about one’s ability to learn mathematics. The 10-item Mathematics Anxiety Scale measured anxiety related to doing mathematics. And finally, during the post-test session, open-ended comments were collected. These comments were written responses to five questions asking how well they had coped with the statistics, the amount learned, the key elements that had helped or hindered their learning, whether the research exercises should be continued in future classes, and whether their attitude toward mathematics had changed. The responses were coded depending on if they were negative or positive answers and data was compiled.

The undergraduate psychology course had a laboratory component that lasted for two hours each week for the 12 weeks and dealt with basic research design and statistics for the social sciences. The class was taught through three situated-learning exercises in which students participated as researchers and/or subjects and then wrote individual reports for assessment. Eight laboratory groups of 20 students each were taught by five experienced tutors trained in the methods of the treatment. They were trained to engage students in whole-class discussions and to focus their teaching in collaborative small-group work based on the principles of cooperative learning. They focused on positive interdependence, face-to-face promotive interaction, and individual accountability. They used these groups to explore different problem-solving approaches and to focus on conceptual understanding of the material. The tutors worked to create an open and non-threatening classroom environment.

The tutors also promoted a positive attitude to learning, promoting the fact that the students that they had skills to cope with problems and that they could achieve success in the class. No attempts were made by the tutors or instructors to overtly assist the students in the management of their anxiety, for the researchers wanted the students to work on their self-efficacy and be in control of their own development. The researchers went further to explain all of the details of the treatment, but it can be summed up by saying that everything about how the tutors and instructors structured the class was explained in the treatment description and they followed the treatment to the best of their ability. Every aspect of the treatment was intended to create an environment where cooperative group work was the focus.

The self-concept and anxiety tests were administered at the first and last laboratory sessions of the course. There was a strong and significant correlation between the two measures on the pre-test ($r = -.87$, $p < .001$) and post-test ($r = -.75$, $p < .001$), meaning that the lower a student's self concept, the higher their anxiety.

Students were randomly assigned to either the Retrospective group or the Pre-Post group. Those who were present on the first day of class but not the last day were later assigned to a pretest-only group. Students in the Pre-Post group completed the two instruments at the beginning and end of the course. Those in the Retrospective group did the same but were then given the instruments again and were asked to retrospectively judge their self-concept and anxiety at the beginning of the course. This was to measure the response-shift bias. Finally, students were asked about their prior math education to determine the highest level of math they had studied. Based on these responses, students were classified as low, medium or high. This was because it was shown that math background was significantly ($p < .001$) correlated with mathematics self-concept ($r = .42$) and anxiety ($r = .40$).

The univariate effects revealed that mathematics self-concept was higher at post-test ($M = 90.30$) than at pre-test ($M = 84.14$) ($p < .001$), while mathematics anxiety was lower at post-test ($M = 28.88$) than at pre-test ($M = 31.27$) ($p < .05$). Overall, more positive attitudes were present at the end of the study than measured on the pre-test.

When the researchers looked for evidence of response-shift bias, they found some interesting results. Students with high mathematics background had less initial anxiety and appeared to retrospectively judge it as being even lower, whereas students with low or medium mathematics background had more initial anxiety and retrospectively judged it as being even greater. But overall, the students' responses on the pre-test and their retrospective responses were similar enough to say that there was not a significant response-shift bias at play.

The open-ended responses from the students were also analyzed. Of the 82 students who submitted comments, 38 responded positively, 35 students reported no change in their attitude (either they were positive coming in and were positive coming out, or they were negative about math before and after the study), and a further 16 students could not be classified as either positive or negative. Two students reported that they experienced a negative change in their attitude about math as a result of the treatment. Finally, seven comments could not be classified as to whether there was a change or the direction of the change. In summary, 46% of the participants responded that they benefited from the treatment.

Some strengths of this study were that it was a continuation of prior research by the same researchers. They entered the study with a clear idea of what the variables were and tried to control for the variables they had previously overlooked. There is a clear description of the treatment, so it could be replicated fairly well.

There are weaknesses that affect the validity of the results of this study. The researchers chose not to have a control group for this study or the one they did in 1998. The treatment cannot therefore be isolated from the environment and the instructor; one cannot know the effects of the treatment alone without a control group for comparison. The researchers also point out that the changes in math anxiety and self-efficacy, though significant, were only modest. Also, the subjects they chose were not being treated for a pre-diagnosed math anxiety problem. The treatment may have had different effects if it had been performed on either very high-level math anxiety students or very low math anxiety students.

It seems curious that a study conducted by the researchers seven years earlier did not have the same results, though the treatment was very nearly the same. The researchers did improve on their design, so perhaps these improvements led to a more effective treatment or more carefully taken measurements. They waited until the course was over before administering the post-test and they controlled for response-shift bias. They did not extend the length of the course, which was a point of critique with the last study.

Even though there are weaknesses, this study is an improvement over the 1998 study, and showed better results. With these two studies side by side, it can be said that cooperative learning may have a positive effect on math anxiety and math self efficacy.

The next piece of research regarding cooperative learning and self-efficacy is a qualitative case study performed by Forster (2000) based on her observations of a Year II Geometry and Trigonometry class where students were studying vectors. The subject was a high school student named Katie, and the analysis centered around two classroom episodes, both in the context of whole-class discussion. The focus of the research was on Katie's feelings of competence or incompetence in each episode and her change in self-concept as shown in comments made by Katie, her classmates and her teacher.

The first episode Forster described was an exchange between Katie, her best friend Jenny and Mr. C. Katie and Jenny were working with their graphing calculators, calculating the magnitude of various vectors, when they worked out a short cut method for quickly calculating the values, and they shared this method with their teacher. In the exchange, Jenny was obviously the one that initially came up with the method, but it was only with Katie's help that she perfected it and made it work. When Mr. C. approached them about their process, Jenny explained, "we made an aplet," and "we just did ABS, then I and J." When Mr. C. tells told them that he thought their idea was clever, Katie jumped in for the first time and said, "It's Jenny's idea." Later, when Jenny shared the idea with the class, all of a sudden she was referring to "my aplet" (Forster, 2000, p. 230) She changed from expressing shared ownership to claiming sole ownership. The teacher followed suit and starting referring to 'Jenny's idea.'

Forster wrote that assessed ability is not an attribute of a person, but rather, it is socially constructed. Katie's did not form her self-concept alone. The change of ownership of the aplet idea on Jenny's part, and then the change in Mr. C's words confirmed to Katie that she was not a valid part of the process. Yet, when Jenny went to present "her idea" to the class, Katie did lay some claim on the idea by saying "Yea, go Jen" to encourage Jenny to go explain the aplet. Forster wrote that this was making public that Katie had inside knowledge of the aplet and therefore had some sort of claim on the idea.

Mr. C used a great deal of whole class discussion and small group work, as evidenced by the transcriptions provided and Forster's observations. This exchange begs the question "what responsibilities do students have to each other when working together?” In this last exchange, it is evident that Katie and Jenny’s collaborative group work helped them to come to a solution to a problem that they may not have solved alone. As a team, they were stronger than had they been working alone. Yet, Katie’s self-efficacy is evidently low and therefore Jenny earned the credit and the confidence boost from the work they did together. The treatment recommended by Forster would be for Mr. C to use hermeneutic listening, which involves listening to the text of students’ conversations so that, for example, subtleties like the use of “we” might be heard. If this was done, Katie’s self-efficacy could improve when she saw that she had been a useful and contributing part of the creative math process.

The next episode Forster described was when the students were, as a class, working out a problem involving vectors describing wind and plane velocity. The students had worked in small groups on the problem, and then they brought their work into the discussion. Jenny volunteered an answer that she and Katie had worked out, then when Mr. C. challenged that answer, Katie said “oh, I think I have done something really wrong.” Before this moment, Katie had not been involved in the whole class discussion. She took blame for there being an issue with their solution, instead of sharing the blame with Jenny or not saying anything at all. Then when the teacher pressed her further, she made a statement that Forster described as “declaring error and thereby getting off the hook.” This was normal behavior for Katie.

Katie also often voiced what Forster described as “public expression of incompetence or error” in whole-class discussion. She spoke of her incompetence or non-understanding in public on five occasions over the month of observations. The statements that she made were, “I have done something really wrong,” and “I am probably wrong anyway." She also publicly stated that she was having trouble twice and she admitted she had made a mistake once. Mr. C. also responded at times to students' answers in a way that implied incompetence or failed to confirm competence. Katie received a "not quite" as a response to her answer to Mr. C., three times her answers were almost dismissed, and one question she asked was put aside and Mr. C. did not return to it. Another time, she gave an explanation instead of an answer and the ownership of the response was shared amongst the collective.

During the course of the class and month long observation, Katie started to exhibit a change in her self-efficacy. The source of the change was not clear, but there are factors that may have played a part. She achieved a mark of 71% on her test on vectors, putting her at seventh out of 18 in the class. The class average was 62%. Katie also started receiving more positive remarks from her peers and Mr. C. When Katie had the opportunity to demonstrate the solution to a problem at the board, Jenny said, "Yea, go Katie" when she completed the problem, thus receiving public praise for her actions. On various occasions, Katie's answers were confirmed with "Yea," "That's right," "Okay," "Exactly," and by Mr. C. repeating her answer. Other times, Katie's answers were confirmed by implication by Mr. C. He also acknowledged her work publicly by saying, "Okay. That was a good thing Katie was thinking about," and "Katie and I and a number of others have discovered in our conversations that…."

On occasion, Katie challenged Mr. C. on his methods or on a definition, and received backing from her peers. She was not always correct, but, being supported by her peers, she stood behind her response; something she had not done before. Katie continued to improve in the class, as well as with her self-efficacy. She achieved a score of 82% on her mid-year evaluation, putting her third in the class. The mean score was 66%.

Forster suggested that perhaps her newfound confidence came from Katie being asked to explain to the class her innovate graphing calculator solution to a test problem, or from her friend Jenny saying that "Katie thought she couldn't do it, but now she knows she can." Given that Katie and Jenny are close, and in the past Katie bowed to Jenny, giving her the credit, hearing that Jenny believes she could do it helped Katie believe in herself too. Being asked by her teacher to show her calculator process to the class showed that Mr. C. was assigning her competence, and Katie could have internalized this and boosted her self-efficacy.

A point of weakness in the study is that Forster looked back through her observation notes and played back her audio recordings to gather her data, and could not check in with students about their responses; she could not triangulate the data. She did have the teacher look over the data, and he found the text acceptable.

Technology

This next study uses Computer-assisted instruction (CAI) to adapt instruction to individual differences. It tailors problems according to students' responses and supplies immediate feedback for each response. There are two types of CAI, individualistic and cooperative. I-CAI has been shown to help raise examination scores of college student and high school students, and has had a small but significant effect on students' academic self-concept and math anxiety. Mevarech, Silber and Fine (1991) fashioned a study to see what effect having students work in small groups with CAI would have on math anxiety and self-concept. They hypothesized that having children work together would lead to less errors when entering data, give students the opportunity to collaborate to solve a difficult problem they would not have otherwise been able to do alone, give students an opportunity to be a more capable peer, and would lead to higher cognitive processes being used.

The researchers were not sure if having the students work in pairs would lead to some students decreasing their self efficacy and increasing their math anxiety due to the fact that others would be able to see them make errors. Especially for those students who had high levels of math anxiety, the researchers were unsure how they would respond to this factor.

The participants were 149 sixth grade students (78 boys, 71 girls) in an Israeli school in Tel-Aviv. The students were of low socioeconomic status and most of them had no experiences with computers prior to the start of the study. The study was done over the span on one semester, lasting five months.

The software used was Computer-Based Practice in Arithmetic (TOAM) system for drill-and-practice in mathematics. This system was used by students in many countries around the world and was developed in Palo Alto, California. It covers the common elementary school mathematics system, including the four basic operations with whole numbers, fractions, and decimals, as well as word problems, weights and measures.

The first 10 sessions were devoted to diagnostic purposes. After the 10th session, each student was drilled and practiced problems according to his or her tested level of mathematical achievement. The computer tracks the students progress and tailors the questions to the student's ability level, tallying their score at the end. The scores use a system that outputs a two-digit number. The tens digit represented the equivalent year and the ones digit represented the equivalent month level within that year. For example, a score of 63 indicated that a student was performing at a sixth grade level in the fourth month of school.

Subjects were put into three ability groups of equal size: low, medium and high according to their TOAM pretest diagnostic scores. TOAM scores were obtained three times during the study: prior to the study beginning, immediately after the study ended and at the end of the school year (two months after the study ended). All scores were obtained when students were working individually at the computer.

The Arithmetic Achievement Test (AAT) was administered prior to the beginning of the study to determine the students' level of mathematical achievement. This test was a thirty two-item paper and pencil examination that included computational and word problems about the four basic operations with positive integers and fractions.

Mathematics anxiety and Mathematics self-concept were measured with a thirty-item questionnaire. Eighteen of the items measured mathematics self-concept and twelve items measured mathematics anxiety. The test was scored using a 3-item likert scale where the child was to respond how much he or she identified with a hypothetical child in various math situations. Higher scores corresponded with higher levels of math self concept and math anxiety. Data were collected three times during the course of the study: before the study (pretest), immediately after the end of the study (immediate posttest) and at the end of the school year, two months after the end of the study (delayed posttest).

Prior to the start of the study, each student received two separate weekly 20-minute sessions at the computer. The students were randomly assigned to one of two treatment conditions: using CAI in pairs ($n = 90$) or individually ($n = 59$). Those in the cooperative learning condition (C-CAI) were instructed to take turns at the keyboard, help each other, discuss the difficult problems, and decide together on the correct solution. The individual students (I-CAI) were instructed to work alone at their computer. Both groups used the same program. The same tests were given to all students in the same manner, individually. During the two-month period between the post-test and the follow up test, the students continued to use the system, though all used it individually.

The results of the study were that the main effects for treatment and ability were significant ($p < .05$), but the treatment to ability interactions were not significant ($p < .001$). Students in the cooperative group performed better (M=51.54 pre, M=59.14 delayed post) on the mathematics achievement test than those in the individualistic setting (M=50.69 pre, M=56.63 delayed post).

The treatment x ability correlation on math anxiety was statistically significant ($p < .02$) as well as the ability main effect ($p < .01$). Low achievers had higher math anxiety (M=1.58 pre in C-CAI, M=1.67 in I-CAI) than medium (M=1.41 pre in C-CAI, M=1.57 in I-CAI) and high achievers (M=1.32 pre in C-CAI, M=1.27 in I-CAI) . Low achievers also reduced their levels of math anxiety to a greater extent in C-CAI (by .12) than in I-CAI (by .01) setting, but high achievers did not change their level of math anxiety in either condition (by .0 in C-CAI, by .03 in I-CAI). The treatment did not significantly affect math self-concept in either group.

The researchers discussed how previous studies showed that in individualistic and competitive settings, social comparison is likely to occur, thereby lowering the self-concept and raising the math anxiety of low achievers. It seems to have the opposite effect on high achievers, providing them with feelings of confidence and superiority. They argued that when students are put in groups, this effect gets neutralized. In cooperative situations, students are less likely to make social comparisons

One of the weaknesses of this study was that students were first introduced to the use of computers just shortly before the start of the study. Given that the study was computer-based, it would have made more sense to use students who were comfortable with the use of computers. The novelty of these computers added another variable to the study. Granted, all students were having this same novel experience, so in the end it may not have had a negative effect on the study.

The study was strengthened by the fact that that researchers returned two months after the completion of the treatment to test for the long-term effects. This study is very useful given that it could be easily used in a classroom setting, the treatment was simple and the results were conclusive. Cooperative group work used with computer based math software can help alleviate math anxiety in school age students, though it may not have an effect on their math self-concept.

Gundy, Liu, Morton and Kline (2006) studied the effects of web-based instruction on math anxiety, the sense of mastery, and global self-esteem in undergraduate statistics students studying at a predominantly white University in the northeast United States. The 175 students enrolled in the fall 2002 and spring 2003 statistics class received extra credit from their instructor for participation in the study.

The research question, more specifically, was whether uploading and downloading files from Blackboard and online interaction through discussion forums would affect the participants' math anxiety, perceived mastery and self-esteem. Pre-test and post-test surveys were administered at the beginning and the end of the course instruction periods. Researcher Gundy was the instructor for the course.

Instruction in the use of the computers, software (PowerPoint and StataQuest), and web-based materials was taught to the students during at least four class meetings in the computer lab. Class 1 and 2 met twice a week for a total of two hours and forty minutes in fall quarter. Class 3 and 4 met for the same amount of time, but in spring quarter. A number of participants dropped from the each class and there were some differences between those who stayed and those who did not. This will be discussed further in the critique.

There were three treatment conditions. Condition A required students to use Blackboard and the assignments and data were only available via the web. In addition, class email and an online discussion board was made available to students via Blackboard, but the students were not required to use these technologies. Finally, the students were required to upload Stat\Quest data and log files via Blackboard's digital drop box. Condition B students did not have access to Blackboard. The web based resources used were general information (syllabi, assignments and data) posted on a one-page class website. This web-based information was also available in paper form. Students could use email for communication if they wished, and students were required to turn in StataQuest data and log files on disks. Finally, condition C was required to use the same resources as condition A, but assignments were also available on paper. These students were also required to use the online discussion board in Blackboard at least four times during the semester. The discussion board in this condition group served as an alternative source of information and/or tutoring unavailable to students in the other conditions.

Math anxiety was measured using a ten-item modified version of one of nine Mathematics Attitudes Scales (MAS) at pre-test and post-test times. The modifications were to add questions specifically about statistics, using a 4-point likert scale. For measuring sense of mastery, a seven-item scale was used at pre-test and post-test. This measured the subjects' feeling of a sense of personal control over life circumstances. Self-esteem was measured with a six-item survey. Other data was collected on the subjects, including their self-reported expected course grade, whether this class filled a major requirement, whether they were on-campus residents, their gender and their age.

It was found that at both pre-test and post-test, math anxiety levels were lowest among students who expected better course grades (M=23.04 pre, M=19.69 post) and were taking the class to fill a major requirement (M=25.25 pre, M=22.77 post). It was also found that at post-test, older students had greater self-esteem levels (M=22.46 post for 21+ years) than younger students. All post-test math anxiety scores were significantly lower ($p = .000$) than pre-test scores (the least change was .19, the greatest was 3.35). It also appeared that the use of web-based instructional materials eased math anxiety more so in the students using Blackboard courses (M=24.95 pre, M=21.95 post). The online course components did not noticeably affect the students' level of mastery or self-esteem overall, though condition C students did see a significant rise in their self-esteem levels at post-test (M=21.21 pre, 21.89 post; $p = .028$).

The attrition rate for each of the treatment groups was 41.4%, 19.5%, 26% and 28.5% for Class 1, 2, 3 and 4 respectively. Some of the students that dropped out had a higher level of computer anxiety, which was measured using a modified version of the Computer Anxiety Rating Scale (CARS). This could very well affect the results of the study, for the intervention was computer based. Those subjects with computer anxiety would have showed a higher level of anxiety overall, thereby changing the results for the worse. The assignment of students to class conditions was non-random, which weakened the study. Also, the sample size was small (n = 175), increasing the type II error decision, as pointed out by the researchers. Finally, there was no control group.

The treatment software is commonly used and therefore the study could be easily replicated. Also, this treatment could be easily adapted for use in a high school classroom. Giving students access to class assignments, providing a message board for students to communicate with each other and the teacher, and tapping in to the computer skills and interest of adolescents could be very beneficial, not only with math anxiety but in many other ways.

Yushau (2006) studied the effects of blended e-learning on mathematics and computer anxiety in a random sample of 70 students at King Fahd University in Saudi Arabia. Blended e-learning is a method of using technology in the classroom mixed with traditional instructor-led training. The technology aspect includes synchronous online conferencing or training and self-paced study.

The study was done over one semester of a pre-calculus class. The students in the study were in their first year of university, and therefore they were required to take one year of intensive English training (university classes are taught in English, while pre-university classes are in Arabic) and two pre-calculus algebra courses. Due to the fact that the students were undergoing intense academic, social and emotional adjustment, the

blended e-learning treatment was given to them in its simplest form. The students had lecture three times a week in a traditional manner, and a weekly computer lab session. They also had online resources and Internet available to them for use in the class.
As part of the technology component, the students were taught to use MATLAB (a language for technical computing that integrates computation, visualization, and programming in an environment where problems and solutions are expressed in familiar mathematical notation) for use in solving mathematical problems. They were directed to the course materials on the university's website, given math problems to solve and submit online, administered tests that provided immediate feedback, provided solutions on WebCT (Blackboard), and provided access to online discussion forums, email and other components of Blackboard.

The Mathematics Attitude Scale measured students' attitudes towards mathematics. The Computer Attitude Scale (CAS) was used to measure students' attitudes towards computers. A score of 50 or above and 100 or above showed that a student had a positive attitude toward math and computers, respectively. The MAS showed on the pre-test that students had a positive attitude toward mathematics ($M = 55.22$), while their post-test showed non-significant ($p < .08$) decrease in their positive attitude toward math ($M = 51.51$). Similar results were found for overall computer attitudes, with a mean of 111.9 at pre-test and 106 at post test (non-significant at $p < .109$). It is worth noting that computer anxiety was significantly decreased ($M=28.64$ pre, $M=26.19$ post; $p < .023$).

One of the major weaknesses of this study was that there were 50 students who took the pretest and 65 students who took the post-test. First of all, why did those 15 students not take the pre-test, and what happened to the other five of the 70 students? Also, there was no control group for this study. The researcher could have used one of the other required math courses at the university as a control group, but they did not.

Self-Efficacy

Hodges (2005) put together a pre-test and post-test control group design quantitative study to examine whether weekly efficacy enhancing email notes increased self efficacy for learning math. The control group received neutral email notes at the same frequency as the treatment group in order to mimic the treatment conditions.

The email notes the students in the treatment group received were meant to enhance efficacy and regulate, like “Excellent! You correctly answered every question on quiz 2 in Math 1015. Aim to do the same next week” and “Your math 1015 quiz score improved this week. Nice job! Be sure to budget enough time for Math 1015 so that you can do that well; or even better; next week.” The control group received emails that simply gave information and were neutral, like “Math Emporium tip: to access the Math Emporium, students must present a valid Hokie Passport to the check-in staff. Make sure to bring your Hokie Passport with you on each visit to the Math Emporium.”

Math self-efficacy was measured via a web survey and math achievement, which was measured using the highest score on the Math 1015 class test. Three surveys were used: self-efficacy for learning math asynchronously, a demographics survey, and an exit survey.

A $100 cash prize was offered for participants to sign up for the study, and 196 students (143 female, 53 male) signed up. They were randomly assigned [using computer randomizer] to the control group or the experimental group. Out of the pool of 196 students, five dropped out during the course of the study for unknown reasons, and 66 did not complete all of the instruments, leaving 125 in the study. In the end, there were 57 participants in the control group, 68 in the experimental group. The average age of the subjects was 18.21 years.

One of the findings of the study was that there was no significant difference in self-efficacy scores between the control (M=114.26 pre, 119.11 post; $p < .05$) and experimental groups (M=113.00 pre, M=119.62 post; $p < .05$). There was a weak positive relationship between post treatment self-efficacy for learning math asynchronously survey and math achievement. The r^2 value of .093 indicated that self-efficacy to learn math asynchronously explained 9% of the variance in math achievement.

The results showed that the method used to increase math self-efficacy in students was not effective. Therefore, the improvement in math self-efficacy does not necessarily come from students receiving positive messages, but perhaps from a more proactive approach. Also, this may show that extrinsic motivation does not effectively increase math self-efficacy. This hands-off, automated and canned feedback email treatment was not effective in the college classroom setting.

Luzzo, Hasper, Albert, Bibby and Martinelli (1999) implemented a combined treatment program, described as performance accomplishment and vicarious learning, in 94 students (55 females, 39 males) in a large public University in the South with the intention of increasing their math self-efficacy and interest in math. All students were in their first year of college and enrolled in an elective two-credit orientation course. They did not receive credit or grade incentives for participating in this study.

As a measure of math achievement, students provided their ACT mathematics score. The ACT is required of all incoming students to this university. Only those students with an average or above math score were allowed to participate in the study. Math self-efficacy was measured with a math/science course self-efficacy scale, self-efficacy for technical/scientific fields-educational requirements, and a math/science occupational self-efficacy scale. The subjects also completed a Courses and Major study to indicate the course the planned to take during the next quarter, the major they had selected, and their current career aspiration. During the initial phases of the study, the students were given a demographic questionnaire.

Students were randomly assigned to one of four experimental conditions: no treatment (n = 24), vicarious-learning only (n = 22), performance-accomplishment-only (n = 22), or the vicarious learning and performance accomplishment combined (n = 26). All participants completed the same measures they had completed prior the study immediately following treatment delivery and again four weeks later.

The vicarious learning treatment group viewed a 15-minute video presentation in which two graduates described how they were undeclared majors early in college but that they chose math and science related fields after having several successful experiences in math. They also described how they were successful in these fields following graduation. The interviewees did not try to directly persuade the audience to take math; they just described their positive experiences.

Immediately following the video, participants were asked to respond to the question, “To what degree would you rate the career satisfaction of the [name of university] graduates who were included in the video?” The responses were rated on a

10-point likert scale from “not satisfied at all” (1) to “very satisfied” (10). Of the participants who viewed the video, 58% responded with a 10, with a mean rating of 9.35. This showed that the participants believed the people in the video to be satisfied with their math/science related careers.

The performance accomplishment treatment group was informed that the number series task they were given was a test of their mathematical abilities and that they needed to successfully solve at least six out of the 12 number series in order to pass the test. The students had 10 minutes to complete the task. The questions required the subjects to determine the formula describing a sequence of numbers and give the next number in the sequence; for example “1, 2, 4, 8, 16” could be described as “double the previous number, and the next number is 32.” Of the 12 questions, six were rated as “relatively easy,” three were rated “moderately difficult,” and three were rated “somewhat more difficult.” Immediately following the completion of the task, scores were tallied and students were told their score. All of the participants received a passing score on the task, as predicted.

Immediately after receiving their score on the math task, participants were asked to answer the following question to ensure that they considered their performance on the task to be a successful one: “regardless of whether you passed or failed the math problem-solving task according to your criteria, how successful do you rate your performance on this task?” The responses were rated on a 10-point likert scale from “not successful at all” (1) to “very successful” (10). Responses ranged from a six to a 10, with a mean of 8.22. This result showed that all of the participants indicated that they perceived their performance to be relatively successful.

Those in the vicarious learning treatments/ performance accomplishment combined treatment group completed both treatments and both follow-up questions. The control group received an orientation to the university's career center.

The results of the study showed the superiority of the performance accomplishment treatment over the vicarious learning treatment. Although the participants thought that those in the video were satisfied with their math/science related career choices, viewing their video interviews did not produce any significant changes in math/science efficacy (M=5.70 pre, M=5.71 delayed post). The only statistically significant effect of the treatments was from the performance accomplishment condition (M=5.60 pre, M=6.21 delayed post).

Only those with average or above average scores were included in the study, because the researchers did not want to skew the results with students having low self-efficacy because of a lack of math experience, rather than for other reasons. This exclusion of a portion of the population for the study may hurt the generalizability of the results. Also, the treatment was very short and different results may be been found if the treatment had been more extensive. This point is weakened though by the fact that the self-efficacy results from the performance accomplishment group were maintained as measured on the 4-week post study test. The vicarious treatment, on the other hand, could have been strengthened with the use of a longer video or with other interventions used along with the video.

The results were strengthened by the fact that the performance accomplishment treatment was simple and that the treatment was the only difference between the control and vicarious groups. Although the short length of the treatments is indeed a weakness, in some ways it helps the validity of the results. There was less of a chance that the participants would experience drastically different unintended variables during a 15-minute treatment and therefore it can be known that the results are based on the treatment alone.

Biesinger, Crippen and Muis (2008) were commissioned by a large urban southwestern school district to study the effect the block-scheduling model on student self-efficacy, attitude and instructional practices within the context of mathematics. For this study, block scheduling is an alternative scheduling format for high school students, where they enroll in eight courses and each course meets every other day for 85 minutes.

Three treatment schools new to block scheduling were invited to participate in the study, with 10^{th} graders from these schools used as subjects. Most of the students had experienced a traditional schedule in grade 9, and would be experiencing block scheduling for the first time in grade 10 during the study. One traditional schedule school was used for comparison. To maximize validity, similar courses were selected for study in each school. A total of 242 students from the four schools participated in the study. Of these 242, 118 were in the treatment group, 124 were in the comparison group.

Participants were administered a revised version of the Fennema-Sherman Attitude Scale (1976), a pre-post test to gauge changes in attitudes towards mathematics. This test focused on five distinct attitude factors about mathematics: success, effectance motivation (the motivational preferences of an individual), usefulness, teacher, and confidence. A 12-item mathematics self-efficacy survey was constructed by the authors and administered at pre and post-test. A 12-item self-efficacy instrument was also administered for students to self-assess their confidence.

Four students were randomly selected per course level at each school (12 total from each school) to participate in student focus group interviews. The lead researcher led these 60-minute groups interviews and recorded and transcribed them for thematic analysis. The responses were coded in to different categories: class activities, connections with teachers, attitude toward block schedule, balance of schedule, student attitude toward learning mathematics in block scheduling, and learning. Classroom observations were performed on a random sample of nine of the 22 classrooms. Five were observed twice, two were observed once, and the remaining two were observed once.

The comparison group showed a statistically significant higher mathematics attitude pre-test score (M=244.08) than the treatment group (M=236.14; $p = .006$). Student attitudes toward mathematics declined significantly for the comparison group (M=240.64; $p = .026$), but remained stable for the treatment group (M=234.27). This showed that block scheduling did not better students attitudes, but that traditional scheduling hurt student attitudes in math.

The comparison group showed significant decreases in Effectance Motivation (M=44.66 pre, M=43.72 post; $p < .05$), and usefulness constructs (M=51.76 pre, 50.43 post; $p < .05$). The treatment group showed no changes in any of the five constructs: success ($\Delta = -.34$), effectance motivation ($\Delta = -.10$), usefulness ($\Delta = -.19$), teacher ($\Delta = -.60$), and confidence ($\Delta = -.33$). Both groups experienced significant increases in mathematics self-efficacy (treatment group, $t = 7.192$, $p < .001$; comparison group, $t = 2.548$, $p = .012$) but changes were significantly greater for the treatment group ($F(1,240)=2.91$, $p = .009$).

From the observations, it was noted that there were no significant changes in teaching style over the six months of exposure to the alternative block scheduling model, and in 90% of the teacher interviews, teachers indicated that the observed classes were typical of their teaching routine and normal behavior for their students. The classes observed were very similar in their teaching methods, use of technology, number of distinct instructional activities and use of lecture and small groups. The majority of instruction was teacher directed. This shows that the main difference between the observed classrooms in the treatment and comparison groups was block scheduling.

The data showed that implementation of block scheduling resulted in significant changes in student attitude toward mathematics, though the effect size was small. Of greater interest is the fact that the comparison group experienced a significant decrease in math attitude. This showed that although the effect of block scheduling was not dramatically positive, at least the effect was not negative as in the traditional group.

Hannel (1990) observed and treated an eight and a half year old girl, Sophie, who was highly intelligent, yet was significantly underachieving in formal mathematics. Based on the Wechsler Intelligence Scale for Children-Revised (WISC-R), discounting the math score, Sophie placed in the 99.9th percentile. Her math score placed her at the bottom of the average range. Hannel looked at the potential factors that may be causing her low math achievement and briefly discussed the interventions that led to her success in math.

Sophie's teachers had observed that she seemed confused and uncertain when doing what were objectively easy problems that could be completed by her classmates with ease. Her approach to math was uncharacteristic when compared to how she approached her other subjects. With math, she was slow to start the problems, not enthusiastic or willing to perform the problems and was dependant of others to help her. With new math subjects, she acted anxious and flustered. With other subjects, she was enthusiastic and self-directed. What was very curious to her teachers was that she was able to quickly and effortlessly complete complex math problems when they were embedded in what seemed like a non –mathematical task.

During the study, Sophie showed a high level of interest in her own achievement and frequently asked if she was right or wrong in her responses. She was very focused on how well she was doing and wanted to succeed in her tasks. She made comments during her work that showed she had a very subjective way of judging "hard" and "easy" questions. When told by the researcher that she was doing well during her work on a problem, she commented that she was only being asked easy questions. When told that in fact, no they were not easy and that a lot of older kids would not even being able to do the problems, she responded that the older kids must be stupid then because the questions were easy. When given questions she perceived to be difficult, she would say "I can't do this….it's hard…you'd have to be clever to do this one."

Hannel summed up her perception of Sophie's beliefs in two ways: 1) "hard" work was the work that she could not do, "easy" work was the work that she could do. 2) clever people can do "hard" work, "stupid" children can do "easy" work. This way of thinking about herself put Sophie in a position where it was not possible for her to be clever or be able to do hard math because she was, by her own definition, stupid. This was quite a conundrum.

Another complexity come to the surface when a conversation was had between Sophie and the researcher about the difficulty with the earlier, easier questions on the WISC (R), but her ease in answering the later, more difficult questions. What came out of the conversation was that Sophie could not find the "trick" in the earlier questions, and therefore assumed that they were difficult questions and she could not do them. She felt that the trick was so well hidden and complex that she was not able to find it. For example, she could not answer how an apple and a banana were similar, because her initial thought "that they were both fruit" seemed too simple. She felt that there had to be a trick, so her mind ran through all sorts of complex connections between apples and bananas (for example, that they were both "fresh fruit", but that could not be it because she knew that bananas had further to travel to get to her and therefore apples were more fresh) and then she gave up and moved on when she could not figure it out. With more difficult questions, she was able to see the "trick" and felt confident that she had the right answer once she had uncovered that "trick." For example, when comparing a telephone and a radio, she said that the obvious answer was that they were both plastic, but that was too easy. The best answer was that they were both used for communication, and that was the "trick," so she was confident in her answer.

Sophie had set up a world in which she was, by her own definition, stupid and where "easy" questions were difficult and "hard" questions were easy. Also, Sophie's high intelligence gave her superior metacognitive abilities and she could therefore see that questions have many levels and possible interpretations. Her search for complexity came from this understanding, and in that way her intelligence worked against her.

Many math tasks that Sophie was presented with were reliant on her choosing the one right answer, while in other subject areas, the questions were open-ended and her skills at finding creative and complex relationships worked to her benefit. The simpler the math task, the more complex Sophie perceived it to be.

Sophie's math curriculum was based on graduated levels of work. As a student worked through the levels of modules, the work became objectively more difficult, and Sophie was aware of this. This became a block that she could not work past, for she believed that she could not do difficult work, therefore she had a high expectation that she could not do the work in the next module level and so could not move up. When given encouragement to try, she was held back by her conviction that she could not find the right answer until she could uncover the trick. If she did not find the trick, (or there was no trick to be found) she could not do the problem.

Sophie had no expectation of future success, gave herself no credit for math she could complete because she deemed it to be easy, and overcomplicated every problem she was presented. She was so achievement oriented that she became anxious when faced with perceived future difficulty, and she only saw problems as either "easy" or "hard." She did not see any problem as "moderately difficult," which is where the researcher stated was the level at which achievement-oriented students are generally seen to thrive.

This case study is fascinating. Here is an intelligent, capable and motivated student who cannot excel in math because of a complex she has created that has roots in her own intelligence. Her math anxiety is cognitive and self-efficacy based in that she worries about her own performance, does not believe she is capable of succeeding and uses self-deprecating self-talk. She cannot move past this place of frustration and confusion because she is not able to see the problems objectively and is so caught up in her own view of herself.

The treatment used by the researcher was to discuss with Sophie what it was she was doing and how she was seeing the math work. She was administered an intensive treatment of supervised math practice with overt attempts to modify her misleading cognitions regarding task difficulty relationship to her ability and her expectation regarding results. After this treatment, she moved up several modules in mathematics and came to a point where she was ahead of all but two of her classmates in math achievement. She became more involved and self-initiating in her math work and was very interested in complexities in math and problem solving rather than mechanical computation.

This qualitative case study was not very strong in that the measures were not explicitly described; the treatment was vague and read more like an after-thought. There was no pre-test or post-test, but the module level did act as a measure of the change in her achievement level. A math anxiety test administered before and after treatment would strengthen the study. The diagnosis was made by the researchers own observations and success of treatment was also determined by the observations of the researcher.

The description of the subject and her complex math difficulties are useful to include in a study of math anxiety. Most studies involving math anxiety are quantitative and focus on impersonal measures and overall change in mean anxiety scores. This case study provided an in-depth look at a child suffering from a version of math anxiety and could be used to better understand the very personal and unique reasons a student suffers from math anxiety and low math achievement. This study can be used to help build perception of math anxiety in students and to show that there is not just one type of student that suffers from math anxiety. Although this does not prove that there is not a correlation between math anxiety and intelligence, it is evidence that intelligent students can struggle with math too.

Journal writing

Salinas (2004) used reflective notebooks in an attempt to change the perceptions of learning and mathematics anxiety in 24 pre-service elementary school teachers (four males, 20 females) enrolled in Math 201, Structure of the Number System.

The course was intended for pre-service elementary school teachers, though one participant identified himself as a future high school math teacher. The content covered was set theory, whole number and rational number operations, number theory, and proportional reasoning. The format of the class was that there was in introduction of a problem, followed by a lecture and "discovery of knowledge." Students then participated in group-work to find a solution to the problem. The problems were mathematically rich exercises that required upwards of seven days come to a solution. The problems generated discussion as students shared their methods and debated with their classmates. The students learned to work collaboratively and didn't rely on the instructor for their solutions. The textbook used in the course was Mathematics for Elementary Teachers.

The treatment was for the students to keep a notebook for the class. This notebook contained their class notes and assignments, thoughts, questions, and assigned writings. The students were instructed to write in the notebooks after each class; they had the freedom to write in it in any way they liked and be as creative and colorful as they liked. They could comment on what they liked, as long as the writing was on some aspect of the class. The length of the writing was not mandated and they were not marked or graded on the notebooks. Notebooks were collected every week to two weeks and to be read and commented on by the instructor. The instructor wrote comments, responded, and provided encouragement. The notebook became a conversation between the instructor and each individual student.

At the end of the course, students were graded on these notebooks. They received two grades, one for the notebook itself and another for the ten excerpts they chose that they felt demonstrated his or her growth during the semester. These grades made up ten percent of the students' final average.

Data was collected in a variety of ways. The notebooks themselves were a source of data, specifically for the content and frequency of writings. The instructor herself maintained a notebook in which she noted events from class that related to the students' notebooks. Surveys were administered at the end of the semester to ask about their thoughts on the use of the notebooks, and the researcher interviewed a random sampling of the students. The interviews were done at the end of the semester and consisted of questions relating to students' prior mathematics experiences, thoughts on writing in the classroom and observations form their own journals.

Three themes were found by the researchers: students developed new understandings of the meanings of mathematics, students gained awareness of their place in the learning community, and students began to evaluate themselves- their motivation, feelings and learning. Early reflections and students' autobiographies tended to be negative and showed fear and dread of mathematics. Many students also expressed that they considered math to be a non-critical thinking activity. They felt that learning math was a passive experience where students learned through plugging in numbers. Some of them even said that math was just a mimicking experience where they were supposed to merely copy what the instructor did. Others expressed that they felt that when doing math there was just one way to complete a problem and one answer that was ‘right’ above all other answers.

Interestingly, some students expressed that they “learned” their negative attitudes toward mathematics. One student wrote,

> I would sit and shake so bad. I remember our first test. It was over place value. They had just changed and gotten these new textbooks, and they were just so different. I got a U! I was like, “Oh, no! I got an F!” I would just cry, and I couldn’t eat my lunch, I was so upset (Salinas, 2004, p. 319).

This experience is representative of many of the students’ math autobiographies. Others said that they had teachers that they felt were bad and subsequently they grew to hate mathematics. There were a few students who wrote about positive experiences they had in math. These students, as the students with negative experiences, also felt that learning math was a passive experience and that there was only one right answer.

In their notebooks, some students wrote about a change in their thinking about mathematics. One student wrote, “It is not just to know what steps to do, but how and why we use the steps”. Another wrote, “I see and enjoy the ‘whys’ of mathematics. I like to see how things work”. And yet another wrote “I have a new outlook of math and how to apply it” (Salinas, 2004, p. 320). Most students wrote that they found there to be more than one way to do a problem, and even more than one possible answer. For some of the students, they were not even aware of the change in their thinking until the instructor reflected back the students' ideas to the class.

Another aspect of the class that students wrote about in their journals was group work. Some students wrote that it was not until they worked on problems in a group that they started increasing their math self-efficacy and decreased their math anxiety. One student wrote that they were nervous in the math class at first because they assumed that everyone else was smarter than they were and knew more math than them. Then during group work, "it took only a few minutes and I realized that we were all on the same level and this problem was going to be challenging for everyone" (Salinas, 2004, p. 322). The students developed a sense of unity with their group members and successes and failures became shared. They were less nervous because they felt supported by those around them.

Students also commented on how they felt about having the connection with the instructor that they journal provided. "Being able to turn in my work and questions and then receiving responses from the teacher helps me understand better." Another student wrote, "[writing in the journal] is a great method to let the teacher know where the student is at and why they are there" (Salinas, 2004, p. 323). Salinas commented that the journals became an indispensible part of the class, an informal assessment of student thinking and development.

Finally, the reflection piece of the journal writing process was found to be a useful sort of self-evaluation. One student wrote,

> Fractions have always scared me. I had a hard time motivating myself to even attempt this homework. But, somehow reflecting in my notebook

> gave me a way to let out what I felt…. This was the point in the class when I realized how important reflecting in my notebook was. From this part on reflection became a serious part of the class. It helped me understand math and the way I feel about math (Salinas, 2004, p. 324).

Other students shared similar experiences where they read over their journal entries and noticed how drastically their thoughts and feelings about math had changed. They make comment that show the development of their math self efficacy, like "if I stick with any problem, I can reach an answer" (Salinas, 2004, p. 325). These comments differed dramatically from those students made in their notebooks early in the semester.

The students had to come to a comfortable place with journaling in math a class before the treatment could be effective, as seen in one students' writings late in the semester, "at first I thought [writing in a journal] was crazy. I thought it would be a waste of time…. It's not been that way at all… I can write down when I was frustrated. I can write down when I resolved that frustration" (Salinas, 2004, p. 325). Many students wrote about frustration. It was a common theme throughout the study. They were frustrated about their anxiety, their past experiences, and with their feelings toward challenging problems. One student wrote that writing down his frustrations in his journal got them out of his system so he could concentrate better.

It was apparent from their writings that students began realizing that their own knowledge had been hampered by their attitudes. One student wrote, "I did not give up when I got frustrated. I did not stop if I was unsure of the method. I did not leave everything alone hoping it would come to me on the day of the test. This would not have been possible if I had not changed my attitudes about math over the course of this semester" (Salinas, 2004, p. 325).

A major strength of this qualitative study was that the student voices were included a great deal in the conclusion and results. The researcher did not just sum up what the students said and try to fit students in to groups based on their responses. The student voices are a clear and strong showing of the results of this intervention. It is obvious that a great deal of the students benefitted from the intervention, based on what they wrote in those journals.

On the other hand, there is no way for the reader to know how many of the students responded positively to the treatment. The researcher does not say how many students say they benefitted vs. how many did not or in any way rate the positive and negative entries. For all the reader knows, the researcher could have chosen a few journals that supported the hypothesis and just not included the others.

Although not a feature of qualitative research, a control group would have shown whether a group of students taking the same class with the same instructor would have reported increased self-efficacy and reduced math anxiety based on the teaching of this instructor. Without a control group, it is hard to say if the treatment was the reason for the change. But the students did write that they felt the journals themselves were the reason for their change in attitude.

Students reportedly left the class feeling better about math, less anxious and more comfortable getting past frustration to success. Based on the strengths and weakness, standing alone this study does not prove the effectiveness of journaling in reducing math anxiety.

Other researchers who studied the effects of journaling on math anxiety were Sgoutas-Emch and Johnson (1998), who examined whether journal writing was an effective method of reducing anxiety. They put together a classic pre-test/ post-test control group design, with the treatment being the use of an academic journal. The control group took the same class, but with no journaling. The same instructor taught both courses. The subjects were 44 undergraduate students (11 males, 33 females) enrolled in statistics courses at UC San Diego. The mean age of the students was 20. Students received extra credit for participating.

Prior to start of study, a basic math test, Spielberger trait anxiety scale, test anxiety inventory, and an achievement anxiety test were administered. The tests given on the first and last day of study to examine effects were a statistics anxiety inventory and statistics attitude scale. On the day of each exam, the Spielberger state anxiety questionnaire, perceived performance and difficulty scales, and journal efficacy scale were also administered. Finally, saliva was taken before and after each exam for cortisol analysis.

The instructions for using the journals were for the students to write down feelings, frustrations, and thoughts about the course. Also, a brief summary of the lecture was required to be written in the journal at the end of each lesson. They were allowed ten minutes at the end of each class to write, and the journals were collected three times throughout the quarter.

There was no difference in any of the baselines between the two groups, other than total test anxiety (.28 for trait anxiety, .0 for debilitating anxiety, .46 for facilitating anxiety, 6.27 for statistics anxiety, .92 for statistics attitude, 5.18 for test anxiety total, 2.08 for emotional scale, and 1.62 for worry scale). Neither group had high levels of anxiety or a negative attitude about the course before the study. Their findings were that neither group showed any significant changes in either anxiety or attitude. Although not statistically significant, the journal group showed a decrease in anxiety by the end of the semester from a score of 103.33 to 96.5. The control group showed an increase (though not significant) in anxiety post-study. Cortisol changes showed a decrease in anxiety from pre-test to post-test in the treatment group (decrease of 0.56).

Given the strengths of the study, journaling might have a positive effect on student anxiety in a math course, though the change was not statistically significant. More research needed to be done, although this study was a follow up to other studies that showed journaling helps math students with their achievement and anxiety. The problems with the study led to the results not being that significant. One thing that may have affected the results of the study was that they used a group that did not have math anxiety to start with. If they had used subjects that were more on the extreme end of math anxiety; they might have had more positive results. Treating people for math anxiety that do not have math anxiety does not seem to make a great deal of sense.

Finally, the journaling was very “light”, not very structured, and student responded that they did not think the journals would help. They went in to the study believing that journaling would have no affect on their academics or self-efficacy. This may have had an effect on the results of the study.

Manipulatives as Tools for Conceptual Understanding

Vinson (2001) studied the use of manipulatives as an effective method for alleviating math anxiety in teachers. The researcher placed heavy emphasis on concrete learning of mathematical content by use of manipulatives to help the teachers have a better understanding of the mathematical concepts and to give the teachers a teaching tool that differed from the modeling of a procedure at the board that is used heavily in math classrooms. Vinson argued that if the teachers focused on teaching for understanding rather than procedure, anxiety would be reduced for the teacher and their students.

The subjects for this study were 87 pre-service teachers enrolled in a large University. The study was spread out over four quarters, with different n values for each quarter (Fall: 24, Winter: 17, Spring: 23 and Summer: 23). The subjects chose to enroll in the mathematics methods course and there was no control group. The researcher was also the professor of the class.

The treatment was a hands-on approach to teaching mathematics with manipulatives in the methods and materials course for pre-service teachers. The research method was to give a MARS pre-test for math anxiety, then the treatment was given, then the posttest (same MARS test) was given at the end of the study. There were also qualitative observations performed in different forms: informal observations of the pre-service teachers in their methods classes and informal interviews, although this portion of the study was described in vague terms. The treatment's results were measured by the difference between the score on the MARS before treatment and after treatment.

The findings of this study were different for each quarter the study was done. Fall Quarter evidenced no *significant* change in anxiety (pre-test score: 186.79, post-test score: 171.87, gain: -14.92, $p < .05$). An interesting note to make here is that the researcher theorized that because this was the first time he had taught a math methods course, and that because this was his first quarter to teach at this particular college, his math anxiety level as professor was possibly higher and they probably exhibited more stress. Therefore, his own anxiety contributed to the anxiety levels of his students. All other quarters evidenced highly significant differences in math anxiety levels as shown by the gain between the pre and post-tests. Winter gain: -48.06, Spring gain: -27.26, Summer gain: -30.22.

One result of this study was that some students showed an *increase* in anxiety after treatment. Interviews suggested that this was due to the fact that the use of manipulatives was new for these teachers and they were struggling to relearn mathematics as they were learning to use manipulatives. It can be hypothesized that, as with the greater overall reduction in anxiety seen after fall quarter when the professor was more comfortable himself, the teachers may come to be more comfortable with the use of manipulatives and may come to see them as a helpful tool in their teaching practice. The level of anxiety seen in these teachers may reduce when they are more comfortable with using manipulatives.

The presence of a control group would have strengthened this study, such as providing a measure for the affect of the researcher's nervousness or inexperience. Also, multiple- treatment interference may have had a negative impact on the validity of these results, for as there was there was only one treatment, it could be argued that the primary treatment was teaching how to use manipulatives was boosted by the secondary treatment of simply being in a math methods class focusing on math anxiety.

Gresham (2007) conducted a study of mathematics anxiety in pre-service teachers to see if there were changes in their levels of math anxiety after taking a methods course using Bruner's framework of developing conceptual knowledge before procedural knowledge and using manipulatives to make concepts more concrete and meaningful. The research was conducted during different classes in fall and spring semesters over four years. Two hundred and forty six pre-service teachers (237 females, nine males) from a large southeastern university participated in the study while enrolled in a math methods course meant for educators. All of the students were training to become elementary teachers. Their participation did not affect their grade in the course.

The Math Anxiety Rating Scale (MARS) was administered on the first day of class for the semester. Bruner's model of instruction, which would be used as part of the treatment, was also introduced at this time. The course consisted of discussions, journal writing, teacher directed large and small group activities, literature based mathematical activities, student groups presentations involving hands-on manipulatives, implementation of hands-on approaches to teaching mathematic content, and a 12-week practicum in a K-6 classroom. The teacher taught four or more lessons which used manipulatives and integration of literature during this practicum. The teachers wrote detailed lesson plans for these lessons and kept a journal of their thoughts and processes. The MARS post-test was administered during the last week of the semester.

As qualitative measures, Gresham performed informal observations during class time, interviewed each participant using a pre-written questionnaire, had informal discussions and informal interviews. These interviews and discussions were recorded for data collection and detailed field notes were taken. Gresham was the instructor for the class.

According to the data, the greatest difference in change scores from pre-test to post-test existed between fall of 2003 (-18.13) and fall of 2005 (-49.40). A negative difference in score meant that the participant's anxiety decreased. This means that the class from fall 2003 showed less of a reduction of math anxiety than those in fall of 2005. The researcher hypothesized that this was because fall 2003 was the first semester to teach at that particular college and the first time she had taught such a large class.

Overall, it was found that pre-service teachers' mathematics anxiety was reduced ($p < .001$). The qualitative results showed that 213 students stated that the methodology of the course and the use of concrete manipulatives in their teaching helped reduce anxiety. Students also said that the personality of the professor, the environment created in the classroom, and the use of journal writing helped to reduce their math anxiety. The use of journal writing was said to have helped with both teaching students and with the course itself. They commented that when taught in a concrete and practical format, they could understand math concepts that had previously eluded them.

Six students showed an increase or no change in math anxiety. Many of them stated that the use of manipulatives was new, unfamiliar and intimidating to them and that this novelty caused them anxiety. Some also stated that they preferred doing the work in their head and working with others to using manipulatives. They therefore struggled not only with the math in the course, but the use of the manipulatives as well. Some said that the integration of literature in to their lessons was difficult and anxiety causing. Others stated that this was their first time teaching math to students and that alone was anxiety causing.

This study could have benefitted from a control group to compare results. Also, the vast majority of the subjects were female, affecting the generalizability of the study. The treatment was so complex and multi-faceted that it is very difficult to tell what part of it caused the decrease in math anxiety.

Data was collected over a two-year period where the instructor remained the same and the researcher was the instructor. Also, the sample size was large. Both of these factors strengthen this study and help to validate the results. There was only one measure used for math anxiety, the MARS, which showed test-retest reliability from .78 to .85 and has been shown to be a valid test ($p < .001$).

Taken in parts, this study could give insight into treating math anxiety in other populations, specifically high school students. The use of manipulatives and journaling has decreased math anxiety in other studies and therefore the evidence is even stronger that this aspect of the treatment could be extracted and used in high school classrooms.

Rule and Harrell (2006) used images drawn by pre-service teachers describing their emotions regarding math to determine how a methods course utilizing manipulatives affected their math anxiety levels. As a pre and post test, the subjects were asked to visualize images that represented associated feelings and experiences related to teaching and learning mathematics. Then they were then given a large white piece of paper and crayons and asked to choose three experiences or emotions from their past math experiences and draw the images that symbolized these mathematical experiences. These three images were drawn on the left side of the paper and each one was titled. After 10 minutes of drawing, they were asked to make emotional associations with each of the images and they were directed to list the associated thoughts and emotions on the right side of the paper. Finally, on the backside of the paper they wrote interpretations of what the images meant to them in relation to teaching and learning mathematics.

The subjects were fifty-two junior level pre-service elementary teachers (47 females, five males) in two sections of a math methods course taught in the same manner by the same instructor. The study took place at a medium sized, 4-year college in central New York State. The subjects were assured that their grades in the course would not be affected by their performance on the image-drawing tasks.

The mathematics methods course included daily use of a variety of manipulatives for concrete representation of concepts, cooperative group work, an inquiry or problem solving approach to tasks with the instructor acting as facilitator, and an emphasis on mathematical discourse with questions, conjectures, discussion, and justification of thinking. The students also wrote on their reasoning and thinking, mapped their ideas, integrated math and language arts through a math picture book, and used technology through electronic presentations. The course addressed national and state standards for teaching K-6 mathematics and mathematics content.

When the pre-test images were analyzed, 63.2% of them were determined to have a negative tone. The negative images included poor grades, sad or crying faces, embarrassment at the chalkboard, clocks indicating time pressure, frustration with equations, angry or judging teacher, and images of confusion. Many of the images were deemed to be from the elementary school years, given the mention of times tables and the relative size of teachers and students. These images were associated with words of inadequacy, incompetence, anger, loathing, fear, anxiety, frustration, discouragement, failure, embarrassment, humiliation, shame, isolation, confusion, pressure and trauma. Of all the statements made on the pre test, 60.4% were negative.

The self-interpretations of the drawings supported and explained the negativity. Several students also expressed desire to develop a better attitude and skill set for mathematics, and acknowledged that it would take time and effort. Two students said that the help of a good teacher would improve their math skills and feelings and others said they had mixed feelings about math.

Of the images drawn, 36.8% were positive and showed good grades, fun games, happy faces, use of manipulatives, awards, success with equations, good and affirming teachers, light bulbs, and real world connections. Positive emotions connected to the images were excitement, happiness, joy, ease of understanding, pride, accomplishment, relief, motivation to learn, desire to help others, valuing of mathematics and sense of connection. In their self-interpretations, some students noted their positive attitudes and success in mathematics and their excited anticipation of teaching mathematics.

The post-test showed fewer negative images; only 27.9% of the images were negative, compared to 63.2% on the pre-test. The post-test images showed happy faces, lesson plans, and group work. There were still some negative images drawn of crying, sad faces, trouble with numbers, poor grades, losing at games and clocks indicating time pressures. Almost all of the images referred to past negative experiences, not occurring during the math methods course.

A two-proportion z-test showed that the proportion of positive images on the post-test increased significantly. The emotions students associated with their images also decreased in intensity. The words “shame” and “humiliation” were not expressed on the pretest. Their disposition was described as “annoyance,” “dislike,” or “irritation,” rather than anger and fury. Another two-proportion z-test showed that the proportion of positive emotions stated on the post-test also increased significantly (36.8% pre, 72.1% post).

Two negative images were depicted on the post-test that referred to the methods course. Students drew several confused faces to show difficulty understanding the manipulatives presented during the course: an abacus used for subtraction, wooden people for long division, protractor to measure angles, and Pentomino pieces used for symmetry exercises. The students expressed frustration with the manipulatives, given that they were new to them and they had not learned math that way. Another negative image seen on the post-test that referred to time during the course was a dollar bill representing money spent on purchasing materials for hands-on lessons.

The post-test images and associated descriptions showed that, overall, students were positively influenced by the math methods course and had less negative feelings about math after the treatment. Before the treatment, many of the students showed signs of having math complexes. Rule and Harrell believed that the conscious reflections on attitude altered these negative complexes, tapping in to the unconscious mind of the subjects to bring their unconscious negative math thoughts to the surface.

Rule and Harrell’s study has weaknesses. There is no control group to compare the results. The researchers determined whether images were either negative or positive, meaning that there may have been bias at play in these determinations. The fact that the students themselves provided interpretations of their own images did provide triangulation, so they did help counter this point somewhat. The use of drawings to measure math anxiety, though backed by research presented by the authors in their introduction, is not a clear measure. If the researchers had also used a math anxiety questionnaire or test, they could have further backed their math anxiety measures better. Also, if a subject did not consider himself a strong artist or was anxious about the drawing component of this study, that may have affected how well his drawing represented his true emotions and thoughts.

Altering Mood

Bryan & Bryan (1991) studied the effects of positive mood and math performance through a post-test only control group design. They used a two mood (positive, control) by two group (risk, non-risk) Chi Square design. The dependant variables were feelings of self-efficacy, performance on the math problems, and self-reports of mood states.

The subjects in study 1 were 32 black and Hispanic girls and boys, third to fifth grades in three inner city Chicago public schools. Students were randomly assigned to treatment groups controlling for sex, race, and level of test anxiety (high vs. low). Risk was the independent variable. There were seven high-risk students and nine non-risk students in the positive mood condition and six high-risk and 10 non-risk students in the neutral mood condition. They were of low socio-economic status, and were nominated for the study by their teacher because they were having trouble with reading and/or math. The control sample was chosen from students not nominated who matched the children in the study on sex and race.

The subjects in study 2 were 18 middle class junior high and high school students (15 males, three females) who attended a private school for students with learning disabilities in Chicago. In study 1, children were tested one by one in a vacant classroom.

Those children in the positive treatment group were told to close their eyes and think of the happiest day of their lives. They though about it for 45 seconds, then talked about what they were thinking. Children in the positive mood condition and in the no treatment control group were told that they were going to compute math problems. Then they were shown two pages of 50 addition and subtraction problems for two seconds and asked to estimate how many they could complete accurately in five minutes. This was to measure their self-efficacy. Then they were given five minutes to work on the problems. Following this, they were asked to indicate on two 4-point likert scales how happy they were and indicate their willingness to repeat the exercise. These items served as a check on the experimental manipulations. Study 2 followed the same set up.

One of the findings for study 1 was that there was a significant effect of mood states on the number of math problems accurately completed ($F(1.30)=5.02$, $p<.04$), though not on the number of items attempted. The fact that no differences were found on the mood questionnaire was not surprising. Researchers found that children in negative or neutral states often report themselves as happy (Cole, 1983). It was also found that there was a significant effect of mood states on the number of math problems accurately completed ($F(1.31)=4.86$, $p < .04$), though not on the number of problems attempted. There was no significant difference between the risk and non-risk groups.

Study 2 found that students in the positive affect condition estimated they could do more problems ($M= 33.7$) than students in the no treatment condition ($M = 25.37$) ($p < .03$). Similar to study 1, students in the positive mood condition computed more problems accurately ($M = 31.9$) than students in the control condition ($M = 20.38$) ($p < .004$). The number of problems attempted was not significant ($p < .074$), though were close to being statistically significant, as students in the positive affect condition attempted more problems (M = 33.7) than students in the control condition (M = 25.37).

The fact that they repeated the study to make sure that the results would be replicated with a different group of children makes the study very strong. This was a one-time test, so there was no way for the children to tell each other about the treatment and then use it themselves. Also, they did not know if they were in the control group or the experimental group, and they did not have time to find out, so they could not know if they were being treated or not.

In regard to weaknesses, the study was conducted with children who were diagnosed with a learning disability or were recognized as being low-performing. This negatively impacts the generalizability of the results. Also, the researcher stated that for the students responding whether or not they felt happy "conceivably could be the effect of the interpersonal consequences of dealing with an experimenter interested in the child's happiness". This would have affected whether the researcher put the child in the group with positive affect or not. Also, the kids may have been excited about thinking about their happy thought, and the fact that it was a new thing for them might have made their score better.

Despite the weaknesses mentioned, this study seems very strong, with not too many factors or complicating matters. Being happy made students perform better on math tests and have better self-efficacy. Although, the results of the first study did not show that the younger kids had better self-efficacy because of a happy mood, but the older kids did. Students in good moods perform better. In terms of how this fits in with other self-efficacy articles, this link between mood and self efficacy is new, but the idea that self efficacy helps math and test anxiety is useful and is a theme.

Haynes (2003) wrote her dissertation on the effect of background music on mathematics test anxiety and math achievement. The hypothesis was that playing background music for ten minutes prior to taking a test would reduce math anxiety, compared to students who studied in silence. The subjects were 160 students enrolled in four sections of Math 126, college algebra, at West Virginia University. The class met five days a week, and was designed for students who missed the cut-off score for College Algebra on the mathematics placement exam. The course was slower paced to help these students succeed.

The four sections were randomly assigned to either the control group (n = 150) or the experimental group (n = 170), with two sections per group. A 25-item abbreviated version of the MARS was administered to measure math anxiety. Mathematics achievement was measured using the first exam for the College Algebra course, and each class took the exact same exam.

The background music used was Music for the Mozart Effect, Volume I Strengthen the Mind. Haynes chose this piece based on research that suggested that listening to classical music by Mozart could improve concentration, memory, and spatial perception. At the time of the exam, students were given 10 minutes to study and instructed not to talk during this time. Background music was played for the duration of the study time for the experimental group, but not the control group. Both groups were then given the exam, and they both took the exam in silence. The MARS was administered the week prior to the exam, and directly after completing the exam.

There was not a statistically significant difference between pre-exam MARS scores of the control group (M=67.05) and experimental groups (M=66.26). There was a statistically significant difference found between the post-exam MARS scores of the two groups (Control, M=66.87; Experimental, M=62.07), no statistically significant difference between pre and post-exam MARS scores for the control group (M=67.05 pre, M=66.87 post), and there was a statistically significant different between the pre and post-exam scores for the experimental group (M=66.26 pre, 62.07 post).

No statistically significant difference in the College Algebra exam scores was found between the treatment and control groups.

There were no threats to validity regarding the subjects, the tests, the research design or the instruments used. This study was strong in its simplicity, and the results are accepted as valid.

Positive self-talk

Kamann and Wong (1993) taught students with learning disabilities (LD) to use self-instruction to control their distracting thoughts arising from math anxiety. They hypothesized that this intervention would enhance their performance in mathematics and decrease their mathematics anxiety.

The subjects were 20 students (11 males, nine females) enrolled in grades 4 through 7 at two Catholic Elementary schools in Vancouver, British Columbia. Ten of the subjects were children with learning disabilities (five from each school), the remaining were normally achieving. Those students with learning disabilities had been previously diagnosed and were of average to above average intelligence. The students with LD showed much frustration with and anxiety toward school, particularly in the area of mathematics. The normally achieving students were randomly selected (five from each school). They showed average academic performance as reported on permanent record cards.

The study took place in the Learning Assistance Centers located at each school and were rearranged and organized to provide the least amount of distractions as possible. Audio recorders were set up around the room to collect verbatim self-talk data during the pre and post-tests.

All of the subjects from both groups met prior to the intervention period for two days with the researcher (who was also the instructor) for baseline collection. Using a prepared written script, the instructor outlined how the subjects were to complete the mathematics task. He modeled the procedure by working through sample questions written on a chalkboard. During the task, three levels of self-talk were demonstrated that focused on affect-laden statements. These were intended to inhibit or enhance performance. The first type of statement was neutral and described an action, "I am going to have to carry this one." The second and third types of self-talk were task-approach statements: a positive statement, "I am doing great. Now I just need to move to this part," and a negative statement, "I can't do this. I am so stupid." The model questions were different enough from the task questions the subjects would be completing that it would not affect the students' working of their questions.

The students then engaged in a ten-minute discussion amongst themselves in which they shared the kinds of things they said to themselves while completing a math task. In order to recall their self-talk statements, students were instructed to close their eyes and play a movie of their past math experiences in their minds. Following the discussion, students were led to their individual desks and instructed on how to operate their personal audio recorders.

Then the students were given a math task to complete and instructed to verbalize everything that passed through their heads throughout the process of solving the problem. They were audio recorded during this task. In the second baseline collection, the students were given a verbal reminder of the instructions prior to completing the task. Each of the sessions lasted 45 minutes, 30 minutes of which were spent on the task, 15 minutes on directions for the completion of the task and for wrap-up. Two hundred individual sessions were recorded over the duration of the ten-week study, and all save one of these were transcribed verbatim.

Following the baselines, the normally achieving students were released and given instructions to return for a third and fourth session to be conducted in the final three weeks of school. Students in the LD group were given a schedule for the 6-week intervention period. They met once per week starting the first week of May and ending the second week of June.

Before the intervention period started, students in the intervention group were given a verbal and visual presentation on the role played by self-talk in maintaining poor performance. The purpose of this presentation was to make students aware of their self talk and its effects, teach them a strategy to help them cope with mathematics tasks, and to elaborate on the efficacy of the strategies provided in the upcoming study. The instructor then described the goals of the training to the students and then provided them an opportunity to ask questions and clarify the goals and procedures.

Cue cards were provided to the students that outlined the steps of the coping process and sample self-statements. After another modeling session, the students were fitted with their microphones and given a practice math task to complete. During their work time, the instructor did not assist the students in any way and they worked independently on the task. The only interference by the instructor during the work time was to remind the students to verbalize their thoughts. When a student had ceased to verbalize for a 3-minute interval, they were prompted with a neutral cue. The remaining sessions of the study were completed following the same procedure outlined above.

All math problems used in the study were taken from the fraction strand in *Investigating School Mathematics,* Levels 4 through 7. Forty-eight math problems were selected out of this text. Six operations with fractions were covered, including deriving equivalent fractions, converting fractions, addition and subtraction of unit fractions, addition and subtraction of mixed fractions, multiplication of unit fractions, and division of unit fractions.

For analysis purposes, self-talk was categorized in one of three ways: negative, positive and neutral, and divided into two levels: task specific and task approach. Task specific statements were relevant to the specific task at hand, while task approach statements focused on the learner's characteristic that facilitated or inhibited performance.

During the first baseline, 611 self-statements were generated by the students with LD. Of these statements, 17.6% were rated as positive self-statements that facilitated their task performance, and 53.4% were rated as negative. Normally achieving students generated 623 self-statements, 42.4% of which were positive, 19.4% of which were negative. Normally achieving students produced significantly more positive statements and significantly less negative statements than did subjects with LD ($p < .05$). During the second baseline, normally achieving students again generated significantly more positive statements and significantly less negative statements than did the students with LD ($p < .05$).

Pre and post-test data showed a complete reversal of the trend found in the baselines. The children with LD engaged in significantly more positive self-talk (17.6% pre, 45.7% post) and significantly less negative self-talk (54.1% pre, 22.8% post) as compared to change in the normally achieving students (Positive, 44% pre, 44.6% post; Negative, 20% pre, 23% post). Based on the post-test scores, no significant difference was found between the two groups on positive, negative or neutral self-talk.
The LD students very closely resemble the normally achieving students post-treatment.

Performance scores on mathematics tasks were collected for both groups. For the group with LD, the percentage-correct scores on the first pre-test ranged from 1 through 55 with a mean of 22.9%, while posttest I scores ranged from 19 through 80 with a mean of 57.9%. This is a statistically significant difference ($p < .05$). This improvement was sustained in the second pre-test/post-test comparisons, showing a statistically significant change in scores ($p < .05$). Correlation coefficients on the post-test data for the LD group indicated a strong, significant positive relationship between positive self-talk and performance ($r = .32$, $p < .05$). Essentially, this shows that the more positive self-talk a student uses, the better their mathematical performance will be.

The most exciting part of this study was the fact that student math scores improved by the end of the study, when the intervention did not include any mathematical instruction. The researchers successfully isolated the treatment variable to show that decreasing negative self-talk, and therefore increasing positive self-talk, alone could better math performance. This was a major strength of this study.

A weakness of this study was the small sample size (n = 20), which negatively impacts the generalizability of the study. Also, the population being treated was on the extreme end of the scale, as they had learning disabilities. This study showed that positive self-talk can help LD kids; it does not necessarily apply to normally achieving kids. It would be interesting to see a study with a lager subject population that was focused on normally achieving students. Also, the researchers pointed out that the intervention did not work as well on the younger LD students, specifically the fourth graders.

Wiley (2008) hypothesized in his thesis that progressive muscle relaxation (PMR) would reduce self-reported trait anxiety and enhance academic performance in children. The subjects were 28 5th grade students (14 males, 14 females) enrolled in a Southwestern Washington public elementary school. The students were administered the State Trait Anxiety Inventory for Children, which measures anxiety trait (A-trait) and anxiety state (A-state). The A-trait measures how a child feels at a particular moment; the student responds to 20 statements on a scale from hardly ever, sometimes, or often. The A-state measured perceived feelings of worry, tension, and apprehension that fluctuate over time; students responded to statements with either very calm, calm, or not calm. The students received a candy bar for completing the questionnaire.

The scores from these measures served to choose the participants for the study. Two were selected; one male and one female with the highest A-trait score for their gender group. They were randomly assigned to one of two treatment conditions, but were not informed of their assignment or score. These participants received school supply awards for their successful weekly participation.

The treatment was administered with pre-recorded audiotapes developed for the study. The tapes narrated progressive muscle relaxation treatment procedure with 33 steps. The narrator was a female and was instructed to strictly follow the proposed time sequences for flexing, relaxing and evaluating muscle groups. One of the audiotapes included a series of positive self-talk statements: “I believe in my ability,” “I can relax myself during tests,” and “I am successful at school.” Therefore, there were two intervention strategies: PMRT and PMRT with positive self-talk.

Both experimental groups were instructed to practice the entire segment of PMRT techniques at least four times per week at home. They were given a logbook to record their treatment times. Each participant was also instructed to use PMRT techniques if they felt tensions in the targeted muscle groups during classroom assignments, instruction or assignments. The positive self-talk participant was instructed to use the statements given during these times in class as well.

The researcher met with the students on a weekly basis to have the students demonstrate PMRT implementation to ensure proper execution of the treatment, and to hear the self-talk statements to ensure memorization. The students received a prize for the completed log they turned in each week.

Data from their classroom teacher was used to track achievement on weekly spelling and math classroom assessments. For the purposes of this paper, only the math data results will be discussed. Grades were also used for comparative purposes. Their classroom teacher was not involved in the study and was blind to STAIC results and participant conditions.

Jane was the participant in the PMRT and positive self-talk condition. Jane's baseline ability was assessed until week five, and then beginning with week six, she received the treatment. Her baseline math score was 88%. Her math performance showed a moderate increase to 94%. However, her performance rapidly increased just after implementation of treatment, then rapidly decreased for the last 2 weeks of the treatment, week 15 and 16. This lowered her average significantly to an average increase of 6%. Jane received treatment for a total of 10 weeks.

Jesus was in the PMRT-only condition. His baseline performance reached stability at week 3, so he received the PMRT treatment for 13 weeks. Jesus' average baseline was 100% before treatment, with no variability in performance until week 13, when it dropped to 95%, returned to 100% for weeks 14 and 15, and dropped to 84% for the last week of the study. He ended with an average math performance of 98% across intervention, a decrease of 2%.

There are many weaknesses in this study. First of all, there were only two participants, who were chosen because they tested the highest level of anxiety. Choosing participants from an extreme group impacts the generalizability of the results. One was chosen from the female testing group, one from the male. Why make this choice? Gender was not a variable being studied, so why choose one male and one female? The students chosen for the study were already performing well in mathematics, with one participant receiving an average math score of 100%. There is no room for improvement when treating a student with an average test score of 100%. Also, Wiley did not re-administer the anxiety measures to test for change in anxiety levels.

Another point of critique was that the researcher did not directly administer the treatment. The students were sent home with an audio taped treatment, to be completed on their own time and they self-reported completion. They had the added incentive of receiving a prize for completion, so they could have lied about listening to the tapes just to receive the prizes.

Also, the two subjects received treatment for a different period of time. Jane received ten weeks of treatment and Jesus received thirteen weeks. With this 3-week difference in treatment time, a comparison of the results is not as strong. Also, there was no control group. This study is so weak in its research design and method that the results cannot be taken as valid.

Tatum, Lundervold & Ament (2006) studied 20 undergraduate students at a small Midwestern university who self reported being test anxious in order to determine if Upright Behavior Relaxation Training (BRT) would be effective in alleviating the students' anxiety. Although this study focused on test anxiety, the measurement they used to assess test anxiety was an abbreviated math anxiety scale, so the results are applicable to the study of math anxiety. The subjects were aged 18-40, the majority were Caucasian females (66%), and the rest were African American females (33%). They were randomly assigned to either the treatment or control group.

The study was conducted over a one week period, during which the treatment group subjects were given instruction in 10 upright relaxation behaviors using behavioral skills training (e.g.- direct instruction, modeling, corrective feedback, manual guidance, shaping, and descriptive praise). There were two 30-minute sessions, given concurrently. The first session focused on acquisition of the upright relaxed behavior, and the second session was focused on proficiency of the training. These sessions were taught in groups with the researcher.

BRT is a behavior analytically based procedure used for teaching 10 overt relaxed behaviors, each with an operational definition: head, eyes, throat, shoulders, hands, body, feet, breathing, mouth, and quiet. The subject is instructed to, for example, “notice the sensations as you relax your hand in the curled, claw-like position in the arm of the chair.” The purpose of BRT is for subjects to observe anxiety and let it go through these relaxation exercises. This therapy has been used to manage tremor severity and anxiety of patients with essential tremor (ET) and Parkinson’s.

The control group received instructions to take the Subjective Unit of Discomfort (SUD) ratings before an exam or quiz while in the classroom. SUD measures the amount of anxiety a student is experiencing at the moment. The post assessments (SUD and Math Anxiety Scale) were given to control group and treatment group one week after treatment.

The results of the study were that the BRT group experienced a greater decrease in anxiety than the control group over the week of treatment they received. Based on the post-test, mean scores on the Abbreviated Test Anxiety Scale (ATAS) for the BRT group was significantly lower (m = 18) than for the control group (m = 22.4). Mean SUD rating for the BRT group was also significantly lower (m = 4.4) on the post-test compared to the control group (m = 6.7).

A point of critique for this study is that the researcher did not publish the pre-test scores for comparison, so the reader only has the results of the two groups to compare. The researcher did say that an independent t-test on BRT and control group pre-test mean ATAS scores was non significant ($p \geq .10$), which seems to mean that there was no difference on the pre test between the two groups, so a significant difference on the post test shows a result of the treatment.

Another point to note with this study is that the subjects were self-selected. They came to the researcher with self-reported test anxiety and were seeking treatment for this problem. They were an extreme group that may have, given novelty and disruption effects, seen improvement merely because they were being given a purported solution to their problem. It is not known whether any treatment that aimed to reduce anxiety would have done so.

Shannon & Allen (1998) studied the effectiveness of a Rational-Emotive & Cognitive-Behavior Therapy (REBT) training program in increasing the performance of high school students in mathematics. The subjects were 56 low-income black eleventh graders (18 males, 37 females) enrolled in Upward Bound at a mostly white Midwestern University of medium size. Upward Bound is a college preparation program intended for inner city students. The students were recommended for Upward Bound by their school counselors and had a C average or better. The students were randomly assigned to Perception Analysis Training or the control group.

The purpose of this study was to investigate the assertion that attitudes toward mathematics and beliefs about one's ability to do math (math self-efficacy) are strongly related to performance in the discipline. They were also looking to ascertain the influence of Perception Analysis Training (PAT) on mathematical achievement.

The control group met at the same times as the PET group but did not receive the treatment. They watched a series of films ("The Changing Middle East," "Dr. Martin Luther King, Jr.," and "Understanding Agression," with any remaining time spent discussing how to best prepare for the challenges of college.

The control group and PAT groups met for one hour, each week, for 8 weeks. There was a pre-test for both groups, the 13-item Dutton Attitude Scale, which measured the belief in the importance of math and their ability to cope with math. Motivation to do mathematical work was measured with the 12-item Fennema-Sherman Motivation in Mathematics Scale. Confidence in math was assessed using the National Science Foundation's Fennema-Sherman Mathematics Attitude Confidence in Learning Mathematics Scale. Math achievement scores were obtained from the California Achievement Test (CAT-M). Their high school grades (HSMG) and their Upward Bound math class grades also served as measures of math performance.

The researcher, Henry Shannon, under the supervision of the other researcher, Thomas Allen, an experienced Rational-Emotive Therapist, developed PAT. The training was designed to help minority high school students deal with the frustration and anxiety they reported experiencing with mathematics. The program developed the students' skills in managing that anxiety and negative emotions by replacing negative self-talk (cognitive aspects of math anxiety) with positive self-talk that encouraged them.

As part of the treatment, they are directed to record thoughts they had during math class and during homework. Then they described the facts of each situation they recorded and the behavior that ensued. They also participated in exercises meant to help them differentiate between objective descriptions of situations and their subjective responses to them. For example, they were asked to recall a recent experience with math in which they were "mildly uncomfortable" and describe the experience objectively and then subjectively, describe their self-talk and describe their behavior response. Then they were taught relaxation exercises they could use in a mathematics situation in which they were "very upset." Then they were divided in to groups of five for a discussion of the experiences they had described. For homework, they kept a journal in which they recorded at least one self-statement made about math for each class they attended. At the end of the eight weeks, the subjects were given math problems to solve under time pressures and asked to examine the nature of self-talk they employed during this effort.

It was found that pre-intervention high school grades (HSMG) and CAT-M math scores were positively correlated with attitude measures obtained through their pre-test ($r=.64$; $p < .01$). The Dutton "attitude toward math" scale had a correlation coefficient of .31 for the CAT-M and .30 for the HSMG ($p < .01$). Meaning that the better their attitude about math, the better their grades and test scores were.

The PAT groups' mean pretest CAT-M score was 49.55, and the posttest mean score was 57.69. The control groups' mean pretest CAT-M score was 47.70, and the posttest mean score was 51.07. This was an increase in score of 8.14 for the PAT group, and an increase in score of 3.37 for the ACG control group. The difference between the CAT post-test scores of the PAT group and those of the control group was statistically significant ($p < .02$), in favor of the PAT group.

PAT students' high school math grades increased by an average of almost one-half grade point (0.46), while controls' deteriorated by almost as much (-0.41) ($p < .02$). Also, mean PAT Upward Bound math grades increased by almost a full-grade point while the ACG average dropped by almost one-third of a point. The results for the attitude or self-efficacy measure were not as dramatic; the change was positive for the PAT group, but was not statistically significant.

This study had a very strong research design. Pretest/posttest control group design controls for history, maturation, testing, instrumentation, regression, selection and mortality. The researcher controlled for "placebo effects" by having the two groups meet during the same eight week period at the same location and by having them all from the same school. Also, both groups received the same pre and posttest, attended the same classes (grades in those classes were taken into account), and took the same CAT-M.

This study is strong also because they picked someone who would "mirror" the researcher from the PAT group to lead the control group: an educated African American male who did not know the students. Because they were controlling for history and maturation, there was no way the same instructor could lead both groups; they had to meet at the same time. Also, the researcher controlled for interaction of time of measurement and treatment effects, as the post-test was given directly after the study ended, and the CAT-M was given one week after the study was done. It is also ensured that the person administering gave the treatment exactly as it was prescribed, because he was the one who developed it.

There are weaknesses in this study that affect external and internal validity. For one, students in this study come from low-income families and attend an inner city school. This may be a threat to validity if these findings are applied to students not in this study group. The Hawthorne Effect may have also been in play here. All of the students in this study were recommended for this Upward Bound program by their counselor and are therefore getting special attention. But this effect may be "cancelled out" given the fact that all students, in either the treatment or control groups, were receiving the same special attention. Also, the students in the program were spending time together outside of the treatment time, therefore those in the control group could have heard of the treatment. Their reaction could have been to feel jealous that they were not receiving treatment and therefore would have performed worse on their assessments, or they could have tried the treatment themselves and improved their scores.

Given the strengths and weaknesses discussed, the results are valid. Students who have shown a negative attitude about math and have math self-efficacy issues can benefit from work on positive self-talk and practice with seeing their anxiety situations from objective vs. subjective perspectives. The treatment seems like something that could be modified for the classroom and used by teachers to get students to look at math in a more positive way.

Zettle (2003) studied math anxiety with the intention of comparing two treatments: Acceptance and Commitment Therapy (ACT) and Systematic Desensitization. Zettle stated that students seeking treatment for math anxiety who experience significant co morbid (pertaining to a disease or other pathological process that occurs simultaneously with another) levels of generalized or trait anxiety might be expected to benefit more from an intervention designed to reduce anxiety more generally than one that more narrowly targets math anxiety, such as systematic desensitization.

The subjects were 24 college students with self-reported math anxiety were recruited through campus flyers. The subjects were randomly assigned with a coin toss to two groups: ACT or systematic desensitization. They were each treated individually for 6 weeks. Students were not paid. In ACT the group were two males and two females, with a mean age of 30.2 In the SD group were two males and two females, with a mean age of 30.8

The purpose of ACT is to promote psychological acceptance and discourage avoidance by altering the normal social-verbal contexts that support dysfunctional behavior such as escape and avoidance of anxiety-eliciting behavior. ACT Treatment consisted of getting the subject to a state of acceptance of the anxiety and the math work, and to bring the subject's goals and intentions to the forefront of their minds to provide them with incentive to push past the anxiety. During the first two sessions, the researcher induced a state of "creative helplessness" in the subject. In the middle two sessions, the researcher introduced willingness and psychological acceptance as alternative ways of responding to math anxiety, and weakened factors that support experiential avoidance. During the last two sessions, the researcher clarified how successfully completing college algebra related to larger goals and values in the lives of the participants. They also outlined what behavioral processes the subjects would have to commit themselves to in order to realize these goals.

The researcher used Systematic Desensitization Treatment because of his classification of math anxiety as a phobia. The treatment consisted of a first session in which the researcher gave rationale for treatment and a demonstration of progressive muscle relaxation. The subject was given an audiotape of the exercises and practiced for homework. The second session was a continuation of the first. The third session was when the researcher assessed which situations sparked the math anxiety in the subject. During the fourth through the last session, the researcher presented each situation (imaginary) that caused anxiety and used relaxation techniques to reduce the anxiety felt.

Co morbid test and trait anxiety were all measured at pretreatment, post treatment and at a 2-month follow-up. Test of mathematical skill was also given at pre and posttest using a wide range achievement test. MARS was administered to test for math anxiety. Test anxiety inventory assessed test anxiety levels. Trait anxiety inventory assessed trait anxiety.

The findings were that only those who received systematic desensitization reported significant reductions in their level of trait anxiety as a result of the treatment ($t(11)=4.91$, $p < .001$). Reductions in trait anxiety for the ACT group fell short of the level required for statistical significance ($t(11)=2.10$, $p = .06$). Also, those who received ACT maintained their levels of test and trait anxiety post-treatment and showed even further reductions in math anxiety during the follow-up period ($t(8)=2.95$, $p = .02$). SD subjects reported further reductions in test anxiety ($t(8)=2.72$, $p = .03$), and continued maintenance of their post treatment levels of math and trait anxiety.

The researchers also used a system to determine if the subjects were either: recovered and improved (RI), recovered but not improved (RNI), improved but not recovered (INR), or neither (N). Based on their MARS scores directly following treatment, 50% of the ACT subjects were categorized as RI, 8.3% were RNI, 25% were INR and 16.6% were N. For the Systematic Desensitization subjects, 66.6% were RI, 0%

were RNI, 25% were INR, and 8.3% were N. Something to note though, is that neither treatment improved the math skills of the subjects as measured by the WRAT3.

There were some weaknesses in the structure of this study that should be noted. The PD treatment included “homework,” so there was more ‘treatment time’ for that group. The researcher admitted that this may be why the PD group showed a greater reduction in trait anxiety. Finally, the researcher stated that the non-improvement of the math score might have been due to the measurement used. The results were curious, given that other studies have shown that reduction in math anxiety should cause an increase in math achievement. Measurement may have been an issue, or perhaps the small sample size.

Summary

Researchers have found that teacher practices influence the math anxiety and self-efficacy of students in a significant way (Norwood, 1994; Patrick et al., 2003; Wigfield & Meece, 1988) . Students can benefit from cooperative group work (Mevarech, 1991; Patrick et al., 2003) and a focus on problem solving (Alsup, 2004; Norwood, 1995). Researchers disagree on whether the use of manipulatives can lesson math anxiety (Vinson, 2001), for some found an increase in the anxiety of those for whom manipulatives were a novelty (Gresham, 2007). Studies presented here have explored the use of technology in math curriculum as a way of lowering math anxiety and improving math achievement in students , but the technologies studied differed across research (Gundy, 2006; Mevarech, 1991; Yushau, 2006). Math anxiety levels have been shown to be significantly correlated with math achievement, but some have found a positive effect of cognitive anxiety on achievement (Ho et al., 2000).

Chapter four will summarize and synthesize the results of the studies from Chapter three. It will suggest implications for the treatment of math anxiety in the public general education classrooms, suggest further research in the field of math anxiety research, and draw conclusions on effective treatments for math anxiety based on the synthesis of the results in chapter three.

CHAPTER 4: CONCLUSION

Introduction

This literature review focused on effective teaching strategies for treating math anxiety and improving math self-efficacy in students. Chapter one defined math anxiety from multiple perspectives and justified the need for the study of this type of academic anxiety in classrooms. A negative relationship between math anxiety and performance was discussed through multiple researchers' work and the effect of this high anxiety and low performance was examined. The controversy discussed in chapter one was about effective treatments are for math anxiety; either teachers were the source of the problem and should be the focus of a solution, or teaching style and pedagogy were the solution, or finally that the students were the source and treatments should focus on generally reducing stress in students. Chapter two traced the history of math anxiety through pedagogical movements in education and attempted to link math anxiety to the view of math through those movements and the importance the schools placed on the study of mathematics. Math self-efficacy was introduced in chapter two as being linked to math anxiety. The recognition of math self-efficacy as the potential root of many cases of math anxiety was seen as a breakthrough in the study of math anxiety and may lead to new effective treatments. Chapter 3 reviewed the most current literature on math anxiety in the academic world. The research reviewed in Chapter 3 was organized into eight sections: Math Anxiety Defined, Teaching Methods, Technology, Self-Efficacy, Journal Writing, Manipulatives as Tools for Conceptual Understanding, Altering Mood, and Positive Self-talk. Each of the thirty studies were summarized, reviewed, and analyzed to examine how math anxiety could be effectively treated given the conclusions provided.

Chapter four concludes this paper. This chapter draws together the conclusions of the previous chapters, provides a summary of findings from chapter 3, implications for classroom practice, and suggestions for further research.

Summary of Findings

What are effective strategies for treating math anxiety? This was the guiding question for this review of professional literature. Much of the research focused on teacher practice and pedagogy as the solution to the math anxiety problem in students, rather than treating math anxious students directly. Teachers are not the source of the problem, per se, but rather that what teachers have control of in their classrooms is how they teach, not how students think. Teachers cannot make students lower their math anxiety; they can simply let the research influence teaching practice in a way that could help students access mathematical learning and achievement.

Patrick, Turner, Meyer and Midgley's (2003) study has great implications for how to create a classroom environment where students gain confidence in their math abilities, lower their math anxiety, increase their math self-efficacy and participate in a healthy learning community. Many of the characteristics of a supportive teacher are found in the treatment methods of other studies in this paper: cooperative learning, positive mood, and teacher expectation that students would succeed. The fact that these themes are arising and that each of these characteristics has been shown to decrease math anxiety strengthens the results of each of these studies.

Norwood (1994) found interesting and somewhat confusing results in his study on instrumental vs. relational teaching methods. She expected, and it seemed hoped, to find that relational teaching that focused on the development of concepts helped to reduce math anxiety in students because they would understand the math on a deeper level and not just memorize a collection of facts. What she found instead was the algebra students were more anxious after the treatment, partly because of the novelty of it. It can be assumed that the vast majority of students in public education receive a traditional education with the use of memorized multiplication tables, formulas, rules and procedures. To then be hit with a class that requires making connections, finding patterns, looking for big ideas and discovering essential principles could be stressful just because it is a new way of looking at math. This will be further discussed in the implications for further research section.

A similar issue was found in the Alsup's (2004) research: the results were curious and showed that the constructivist vs. traditional methods debate is indeed complex. It seems that when the environment and students are right for constructivist teaching methods, math anxiety of students can be decreased, self-efficacy can be improved and autonomy can be increased. Yet when the setting is not right, constructivist methods can have a negative effect on students' math anxiety and self-efficacy. Just taking Alsup's study at face value, constructivist methods could have a positive impact on math anxiety, autonomy and self-efficacy if used in the right environment and if the instructor is well educated about constructivism.

Yusof et al.'s (1999) study, on the other hand, showed that students were at first resistant to the use of problem solving and student-directed work because they were not used to it and did not trust the process. They wanted answers, they wanted direction, and they wanted the instructor to tell them what to do. Given time, these students were, for the most part, able to accept the treatment and benefit from it. The interviews showed that the students experienced a change in the way they saw the learning of mathematics. Some said that they had previously thought they were bad at math because they were bad at memorizing facts. Their anxiety decreased when they saw that math made sense, that it was not just a collection of facts to be committed to memory. This shift in thinking about math was what Alsup and Norwood were looking for but did not find.

Townsend et al.'s (1998, 2003) studies were similar to Alsup (2004), Norwood (1994) and Yusof et al.'s (1999) in that they studied cooperative learning with a problem solving and conceptual understanding focus. In their first study, their results did not show a positive change in math self-concept or math anxiety, but when they controlled for more variables, unseen at the time of the first study, they found success with their treatment. Their students increased their math self-concept and decreased their anxiety post-treatment. Students responded that being shown the process and logic behind the math they were doing helped them feel more confident and capable of doing math.

Forster 's (2000) case study uncovered the complexities that cooperative group work can cause. Katie was a capable and seemingly intelligent student who consistently gave credit to Jenny, her work partner. Group work helped Katie come to mathematical conclusions she may not have come to otherwise, but also robbed her of a sense of personal accomplishment. She needed others to tell her she was doing a good job and that she was capable of complex mathematical thinking. In a way, this was the treatment put in place by her teacher and her peers, without them even knowing it. Forster (2000) concluded, "the significance of the inquiry for teaching is that by uncovering the intricacies of classroom conversation, problematic aspects can be addressed." Mr. C publicly assigned Katie competence, and was careful with his wording, in that he used the word “we” when he was referring to work that was done as a collective. Although he was not the researcher in this study, and he was not intending to provide a treatment, Katie showed improvement in her self-efficacy over the course of the study. It is difficult to say what helped Katie improve, but there are factors that have been named that could have affected Katie’s development of math self-efficacy. When paired with Patrick, Turner, Meyer and Midgley’s (2003) study on establishing psychological environments, Forster’s (2000) study provides strong evidence that a supportive and encouraging classroom environment can help students develop a positive view of mathematics and increase their math self-efficacy.

Mevarech et al. (1991) found that cooperative group work with computers had a positive affect on the math anxiety of low achieving students. Also, that students who worked in cooperative groups performed higher than students working alone. These results fit in well with those of other researchers discussed here. Cooperative group work can provide students with a more capable peer and enhance higher cognitive processes. When working in groups, students are required to verbalize their thoughts, justify their answers, debate processes, and listen to the ideas of others. All of these things enhanced the learning process and when properly facilitated by a teacher, can give students confidence and momentum in problem solving. If not properly facilitated, students could experience what Katie did and actually lose their confidence to the collective, not seeing the merit in their contributions.

Gundy, Liu, Morton and Kline (2006) and Yushau (2006) both examined how technology can be used to lower math anxiety and increase math self-efficacy. Gundy et al. (2006) found that math anxiety was reduced in students post-treatment and further, that students using Blackboard had even more reduced levels of math anxiety. Yushau (2006) did not have as positive results in her study of web-based learning. The fact that this computer intervention had almost no effect on the students' attitudes and anxiety is contrary to the results of the Gundy, Liu, Morton and Kile (2006) study, which found that the use of Blackboard decreased student math anxiety. What was different about this intervention? The researchers did state in their critique that the students, being in a new environment and experiencing a lot of change, may have been more anxious overall. Some of the students were reportedly feeling overworked compared to those not in the study who were taking normal classes, though they admitted that they learned more because of the study. These issues would have had an impact on the results of the study, causing there to be no effect on their math anxiety. Due to the contradictory results of these studies, it is difficult to accept that the use of Blackboard and other web-based tools are effective at reducing math anxiety in students. More research needs to be done on this subject.

Hodges' (2005) study on motivational email's affect on math anxiety did not add much to the discussion. His canned automated emails to students were not effective. Perhaps motivational emails that are authentic and responsive to individuality in students would be effective. Technology has made its way in to classrooms in a big way, and students are comfortable with its use. Teachers and students could benefit from the communication opportunities technology could provide. This would be a useful direction to go in math anxiety research.

In terms of efficacy research findings, Luzzo et al. (1999) also did not have much to contribute with their findings, though it is worth mentioning that students who were provided an opportunity to succeed at a math task were more confident in their math abilities. Biesinger et al. (2008) found that block scheduling did not necessarily increase student self-concept in math, but that in comparison, students in traditional classrooms experienced a decrease in math self-concept. Sophie, the eight year old in Hannel's (1990) case study benefitted from overt attempts to help her re-shape her self-concept and re-define ability and intelligence. Once she succeeded in that, she was able to overcome her barriers and see herself in a new light, therefore increasing her self-efficacy.

Salinas (2004) and Sgoutas-Emch & Johnsons' (1998) studies found positive effects on student self-efficacy and anxiety, though it should be noted that Sgoutas-Emch & Johnsons' (1998) results were not statistically significant. Part of the issue with journaling in a math classroom is that students may be resistant to writing in math. Students have traditionally seen math as a procedure based discipline, devoid of creative opportunities. This is what these researchers were up against, trying to use journaling as a valid tool for reflection and self-efficacy improvement and butting up against student resistance because of preconceptions regarding math and writing. A more longitudinal study should be done to see the effects of journaling on self-efficacy and anxiety, giving enough time for students to get used to using journals and therefore eliminating that as a variable in the results.

The studies on manipulatives have similar findings to those on the use of journals; if students can get over the novelty of using manipulatives, they can experience a positive effect on their math anxiety and self-efficacy. The use of manipulatives is also similar to the use of constructivist teaching methods in that a conceptual understanding of the material is the goal. The researchers used manipulatives as a tool for representing mathematical ideas in a concrete way for student understanding.

Finally, the research on progressive muscle relaxation is not conclusive. Some studies found good results with the use of PMR, others' results were mixed. There is not enough evidence to take a great deal of the PMR, ACT and SD as fully effective treatments for math anxiety. Treating math anxiety in a student will not always lead to an immediate improvement in their math performance. Also, Systematic Desensitization may be able to have a significantly positive effect on the math anxiety of students. This conclusion is supported further by other research done that concluded that SD is a successful intervention for treatment of diverse forms of specific phobia (Task force on psychological procedures, 1995). In these studies, there was a focus on treating the anxiety and not the teaching or learning practices. The anxiety is seen as a phobia that has become so disconnected to the environment the student is in, that taking the student out of the environment to treat the anxiety as an isolated problem is seen as the most effective solution. In terms of usability in the classroom, one should rate PMR and relaxation exercises low on a list of interventions to utilize.

Classroom Implications

Teachers who created a supportive and encouraging classroom environment in the first days of school had the greatest amount of success in creating productive, non-math-avoidant, and high-achieving students. (Patrick et al., 2003) Furthermore, teachers who were positive and supportive most of the time but not consistent with this persona were not successful with their students. Teachers need to be consistent and gain the trust of their students early in the year, not follow the "Don't smile until Christmas" recommendation often given to novice teachers. There is a difference between being consistent with rules and boundaries with students and being tough, callous and unsupportive.

Students with math anxiety tend to become confused and overwhelmed with the constructivist, relational teaching style (Alsup, 2004) and prefer to be taught in a direct, concrete way (Norwood, 1994). With time however, students can come to appreciate a more relational and abstract teaching style that uncovers the “why” behind the mathematics they are being taught (Yusof et al., 1999). Although a relational approach to the teaching of mathematics will be more beneficial to students’ understanding of the material and ability to apply what they are learning to novel problems (Alsup, 2004; Norwood, 1994; Yusof et al., 1999), when working with students with high levels of math anxiety, a teacher must first be able to give the students a chance to succeed in solving math problems and show them that they are capable of completing the work (Luzzo et al., 1999). Using a concrete, procedural approach to teaching will provide students with that opportunity and then a teacher can transition to using a more relational and conceptual teaching method once students’ math anxieties have been quelled.

A way for teachers to make abstract math concepts more concrete is to use manipulatives in the classroom (Vinson, 2001). The move from the procedural to the conceptual can be supported with hands-on activities and physical representations that can be manipulated and explored by students. Although some students may experience an increase in anxiety at first due to the novelty of using manipulatives, the end result is that students can find relief from their anxiety when they reach a deeper understanding of the math they are learning (Gresham, 2007; Rule & Harrell, 2006; Vinson, 2001).

Students need to be given the opportunity to succeed in a math task if the desired outcome is to increase students' self-efficacy. If the students see how they can be successful, they can believe that they can be successful again in the future. (Luzzo et al., 1999) This intervention could be very useful in the classroom, perhaps in the form of gradual movement from easier material to more difficult material in math curriculum. Give them confidence early on so that they are willing to work toward success in later math, given that they believe that success is possible. Positive mood therapy can also help students to increase their math self-efficacy (Bryan & Bryan, 1991), so teachers should strive to create a positive and encouraging environment where students do not feel that they are under a negative pressure to succeed. Students who believe they are capable of succeeding in math but experience negative physical reactions to mathematics such as sweaty palms, a racing heart, or panic attacks may benefit from relaxation therapy targeted at the anxiety itself (Tatum et al., 2006, Zettle, 2003).

In Townsend et al.'s (1998) study of cooperative learning among students with math anxiety, students' math self-efficacy improved, but their math anxiety levels were not significantly reduced. Yet a follow up study (Townsend and Wilton, 2003) that repeated the same treatment while changing the research design found that students' math anxiety was decreased post-treatment. Forster (2000) found that positive student interaction could increase math self-efficacy. However, it seemed that a student could fail to attribute their task success to her own ability and may assign full credit to the group. On the other hand, some students are able to increase their math self-efficacy through working in small groups on math tasks because they can see others making mistakes. They are able to also see how others work through problems, comparing themselves with them and see that they are not the only ones who don't know how to solve a problem right away (Salinas, 2004). Students working in groups with computer-assisted instruction were able to increase their math self-efficacy while reducing their math anxiety more so than students working individually (Mevarech et al., 1991). Teachers should use cooperative group work in conjunction with clear and direct instruction that emphasizes procedure before transitioning to more abstract, relational understanding.

Technology is an integral part of modern education and could be used in ways that help to support learning and alleviate math anxiety. The use of BlackBoard, a web-based classroom environment, has been shown to decrease math anxiety in students through the use of an online community where students can communicate about the assignments and support each other in class work (Gundy et al., 2006). Other uses of technology have not had the same positive results, such as the use of motivational email (Hodges, 2005), so it is seen that the use of technology alone is not the solution.

Technology as a tool for communication among students and between the students and teacher can be beneficial and should be used in the classroom. Many schools currently employ the use of online grading systems, classroom websites, emailing as a communication tool and online interactive calendars. Taking the use of technology one step further and creating a classroom blog for student and parent use can increase the amount of communication between school and home, serve to remind students of the work they are doing during class, provide a message board for students to collaborate on assignments and provide students with a larger support system. This online community as a communication hub has the potential to alleviate math anxiety and increase math self-efficacy.

For students who have issues not necessarily with math, but rather with their self-concept, teaching them that they are capable of doing the work (Luzzo et al., 1999) and helping them to re-define intelligence can help them improve their math self-efficacy and increase their achievement (Hannel, 1990). Teachers need to know their students in order to teach them most effectively. They need to know how their students see themselves as learners. Teachers can then start from that point and help students redefine themselves as learners, and help students see themselves as mathematical thinkers who are capable of learning mathematics. Hannel (1990), for example, found that her case study student, Sophie, saw math tasks as either hard or easy and then approached each task accordingly, usually overcomplicating problems and then failing to solve them. The researcher, through creating a genuine relationship with Sophie, was able to see how her mind worked and then pinpoint the math anxiety and self-efficacy problem as not math-based but rather as the way Sophie problem-solved and viewed her own intelligence. In middle school and high school where teachers have upwards of 150 students in a day, the task of learning how each individual student learns is great indeed. The reality of the situation requires teachers to put forth their best effort to connect with their students but not overwhelm themselves and end up burned out. A fine balance needs to be found.

One tool teachers may utilize is a math journal. Journaling in the classroom can be a highly effective way for a teacher to learn what students are thinking and to see how they are problem solving (Gresham, 2007; Salinas, 2004). The studies reviewed in this paper were not conclusive on the benefits of journaling but the results were promising. Salinas (2004) found that journaling helped students to analyze their problem solving process, track their progress through the course, communicate with their professor, and relieve some of their anxiety about their abilities and about math in general. In essence, journaling made everything more transparent for both teacher and student (Gresham, 2007). In a middle or high school setting, journaling could be used as a communication tool as well as a tool for self-reflection, which would help teachers pinpoint the source of the math anxiety a child is experiencing and then bring in another method for alleviating that anxiety, such as using manipulatives, encouragement, group work, concrete instruction, or relaxation therapy. In summary, journaling alone may not treat math anxiety but when used as a diagnostic tool could be effective in pinpointing the appropriate treatment.

Based on Wigfield & Meece's (1988) findings, the cognitive aspect of math anxiety could motivate students to work hard in math class. It is when the worry becomes so extreme that it interferes with student performance that it becomes a problem. For this reason, the affective dimension of math anxiety should the main focus of treatments when the cognitive anxiety is at a healthy level in students. Teachers should monitor student comments pertaining to stress and worry but should not take action to treat minor anxiety as it may act as a motivating factor.

Suggestions for Further Research

Most of the studies outlined in this paper pertain to math anxiety in college populations, as that is where many of the researchers are employed and therefore that is the population they have access to. I would like to see more studies done in middle and high school classrooms or with afterschool enrichment programs in order to better understand how to treat the math anxiety present in students who are not yet college aged. If treatment could be administered before students made it to college, perhaps more students would be capable of being successful in math. Also, for secondary teachers to effectively use research findings to treat math anxiety in their classrooms, the studies should be performed on that age group for better generalizability.

Norwood's (1994) study was done over a short period of time, 14 weeks, and did not provide enough of an opportunity for the treatment to fully take effect. A study that takes this one step further and examines the long-term would be beneficial. Is it that instrumental instruction is a good early intervention, then relational instruction can be implemented so that students can understand the material better, thereby maintaining a low level of, or eliminating, math anxiety? Without the chance for students to get used to relational instruction, the conclusion that a relational approach is not effective for reducing math anxiety cannot be made. A relational teaching study should be performed over a span of a year in a secondary school setting using course materials also used by a teacher in a different nearby secondary school teaching an instrumental math class. This way, the two approaches could be compared using a similar population of students and the same materials. As a relational approach to teaching mathematics is currently viewed as best practice, it would be beneficial to the mathematical community to fully investigate the benefits and drawbacks of this teaching method as they pertain to math anxiety.

In Tatum et al.'s (2006) study, they were not interested in seeing the effect of anxiety reduction on mathematics achievement, only the reduction in general anxiety. It is not known, based on these results, whether this treatment would lead to a reduction in anxiety that would lead to the subjects being more capable of being successful in testing situations. Where these results may be useful in regard to the reduction of math anxiety would be in reducing anxiety in general in order for a student to work on their testing issues and be able to more clear headed in their approach to their math work. The treatment of math anxiety as a step towards the mathematical success of a student was the goal for some of the researchers, but for others, like Tatum et al., the reduction in anxiety was the sole purpose. More studies need to be done that focus on student achievement and success for math anxiety research to benefit students the most.

In Zettle's (2003) study, multiple treatment interference was at play. The researcher did not have a true control group to compare to, and may have intended the PD group to be the control group. If there had been a control group, the reader could know what the result of attention and inclusion in a study would have on a group of people not getting treatment would have. The way the study is structured makes it hard to tell which treatment was effective and whether just talking about their anxiety was helpful to the students. Did the treatment cause the improvement, or was it the one-on-one attention the students received?

Hodges' (2005) motivational email study had a noble goal, but was lacking in proper design and execution. His emails sent to students for encouragement were pre-written and were based on a point system. A student receiving a motivational email after a bad test grade may feel some sense of support coming from their professor after reading the text sent, but after the second email arrives, with potentially the same text after another failed test, it becomes obvious to the student that the support is not genuine. Perhaps if the researcher had written more genuine and less formulaic emails, students would have felt motivated and capable in their math class as a result. I would like to see the use of motivational emails researched further, as I believe that it would be beneficial for teachers to have the opportunity to help motivate students in a way that does not have to occur during class time when there are so many things to attend to and so many students to work with.

The Furguson (1986) study could be strengthened with a description of the racial makeup of the classes, gender distribution, a breakdown of how many students were in algebra or calculus, and breakdown of the scores according to these identifiers. Also, finding follow-up studies on abstraction anxiety that do indeed find that it is a distinct element of math anxiety would make these findings stronger. As it stands, there are not many other studies pertaining to abstraction anxiety.

Finally, Patrick et al.'s (2003) study on teachers establishing psychological environments during the first days of school and the importance of creating a positive and encouraging environment is a great asset to the body of research on math anxiety. While strong in its design and depth, the student surveys could have been described in greater detail so the reader could get a clearer picture of what was being measured. The students could also have been given a math anxiety or self-efficacy measure at pre and post-test to show changes in the students' math feelings. "Math avoidance" clearly is connected to potential math anxiety and low self-efficacy, though the researchers only skirt around that connection. The connection is loosely made in the researchers introduction, where they stated that "students use avoidance strategies…in an effort to protect themselves from anticipated failure and appearing unable; they therefore protect their self-esteem and save face by not trying or engaging in the task's demands." Students were protecting themselves from appearing unable, they were anxious about failing, and they were scared that they would be judged. Although not explicitly stated, these researchers studied the effects of teacher-created math anxiety and low self-efficacy. I would like to see further study of the effect of classroom environment on student achievement and math anxiety.

Conclusion

Chapter one outlined what math anxiety and math self-efficacy were and justified the study of them through personal accounts and professional research. The argument was made that students who perform poorly in math may have an undiagnosed anxiety issue that is keeping them from reaching their full potential. Chapter one discussed the consequences of math anxiety on a global scale in business as well as education. It outlined the math graduation policy in Washington State to argue that students need to be able to be successful in math to be successful in school. It described the controversies in the research world and set the limits for the scope of the paper.

Chapter two laid out the history of the study of math anxiety, arguing that the quick and drastic transitions between education movements had a negative affect on math curriculum. It outlined those movements, compared and contrasted them, and pointed out those movements that had the most negative impact on math education. Then it introduced math self-efficacy history and its link to math anxiety in the research world.

Chapter three reviewed research pertaining to the question: "What are effective strategies for alleviating math anxiety in the classroom", with a focus on what teachers can do in their own classrooms. The research in Chapter three was organized in to eight sections: Math Anxiety Defined, Teaching Methods, Technology, Self-Efficacy, Journal Writing, Manipulatives as Tools for Conceptual Understanding, Altering Mood, and Positive Self-talk. In the Math Anxiety Defined section, the research suggested that a healthy amount of the cognitive, or worry, aspect of math anxiety could be motivational and useful. The Teaching Methods section found that a positive, supportive, concrete and instrumental approach to teaching is best suited to students with high levels of math anxiety and that over time students will benefit from a more conceptual, relational approach. The Technology section found that technology used as a communication tool can help alleviate math anxiety. The research reviewed in the Self-Efficacy section concluded that if students are given an opportunity to succeed with a math task, they will increase their math self-efficacy and will believe that they can be successful with math in the future. The Journal Writing research found that journaling could serve as a communication tool between teachers and students, and that students could use the journals to track their progress and self-reflect, thereby making the process of learning math more transparent and therefore less anxiety-ridden. In the Manipulatives as Tools for Conceptual Understanding section, the research showed that while the use of manipulatives could relieve anxiety by helping students to better grasp the material, many students developed anxiety using the manipulatives because of their novelty. The Altering Mood section concluded that positive mood alone could reduce math anxiety in come cases. Finally, the Positive Self-talk section showed that, especially in learning-disabled populations, students who use positive self-talk and speak-aloud their process while they are working can reduce their math anxiety and improve their math scores dramatically.

In summary, math anxiety is a complex issue that can manifest itself in a wide variety of ways, and therefore teachers should not adopt just one method for treating it. The more methods a teacher is able to employ, the more likely that they will be successful with the highest percentage of students.

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