THE EFFECTS OF CURRICULA, AND INSTRUCTIONAL PRACTICE ON STUDENT ACHIEVEMENT IN THE AREA OF MATHEMATICS

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ABSTRACT

This paper examines the effects of curricula, and instructional practice on student achievement in the area of mathematics. An examination of the history of this question reveals two distinct yet opposing philosophies behind the goals for teaching and learning mathematics. Currently this division is seen between those who would have goals for mathematics instruction and learning emphasize memorization and computational accuracy; while others advocate for conceptual understanding with procedural fluency, while fostering in children a healthy disposition toward the discipline. A critical review of the literature shows 3 distinct patterns. First, in comparison studies of the learning outcomes from each of these instructional patterns, students in the conceptual understanding groups did no worse than traditional students on measures of procedural knowledge, while the traditional groups did significantly worse on measures of conceptual knowledge. Second, mathematics classrooms that build students conceptual understanding have been found to share key characteristics, but more importantly is the qualitative differences in instructional practices within these similar classroom environments that work to maintain conceptual development in the learning. Finally, studies that measured the differences in instructional practices and their effects on students disposition toward the discipline conclusively determined that when teaching for conceptual understanding and procedural fluency, students develop more healthy dispositions for learning mathematics, while teaching for memorization and computational accuracy show negative patterns of students motivation toward the discipline. Implications for classroom practice are provided.

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CHAPTER ONE: INTRODUCTION

Introduction

What are the effects of curricula and instructional practices on student achievement in the area of mathematics? At one level this question speaks of goals of instruction and learning outcomes. At another level it sparks fierce debate within our country, and more and more, this question has become one of central import for the lives and futures of countless American children. All young Americans must learn to think mathematically, and they must think mathematically to learn (NRC, 2001). How best can we educate all of our children to successfully meet the increasing demands of our society? The significance of this question has motivated me to make this investigation into the nature of successful teaching and learning of Mathematics.

This paper will examine the effects of curricula and instructional practice on student achievement in the area of mathematics. In chapter one, I will put forth my rationale for this investigation, as well as provide important definitions of terms used throughout the discussion. In this chapter, I will also provide a description of the limitations of this investigation. In Chapter two, I trace the two predominant conceptions of successful mathematics learning and teaching throughout American history. In Chapter three, I provide a critical review of the current research literature surrounding this issue, and in chapter four I will summarize key findings found in this research and discuss applications and implications for future research.

Rationale

Throughout our history, schools have always attempted to ensure that students develop the mathematical knowledge needed to function in society at that time. This has placed an enormous responsibility on schools to determine what the mathematics curriculum should be and how instruction should be carried out. These decisions have proven to be difficult and have been the subject of much discussion over the decades. Should teachers assign a "long division" worksheet, or have students discuss the validity of different solution strategies to word problems? Is it better for children to memorize multiplication tables or investigate what multiplication is, and when to use it? Although such questions have been debated in our country as far back as colonial times, their implications are becoming more and more critical for future generations of American children.

Consider for a moment the increasing importance of mathematical knowledge for participation within our society and economy. Advances in science and technology are rarely made without the tools of the mathematics discipline. Just pick up a Newsweek or your local newspaper, and chances are you will find an article that requires you to use statistics or probability in order to understand both the content and validity of what is being reported. Need I mention Computers or information technology? Unless a computer is programmed, it is just a box made of metal, glass, and silicon. Programming is a mathematical language that is needed to specify what is to be done, how and when, and to verify that the programs and algorithms are doing their jobs (ASCL, 2006). Mathematics continues with increasing measure to be an essential part of computer based technologies, and more and more those technologies are being integrated into every level of our nations economy. Virtually every job in today's society requires mathematics and more important, mathematical thinking and reasoning (Van De Walle, 2004). What curricula and instructional practices will be sufficient to prepare students to navigate successfully within such a mathematical landscape? To answer this question, one must first answer another question. What are the key features of what it means to know and be able to do mathematics?

Mathematics instruction can take on many forms. Each form of instructional practice has at its center specific goals for student learning. Currently there is a battle between two widely divergent philosophies in teaching mathematics. On the one side, there is a community of mathematics researchers, teachers and teacher educators, being led by the National Council of Teacher of Mathematics (NCTM) who believe passionately, that from the earliest grades and throughout their school experiences, children must be made to feel the importance of personal success in solving problems, figuring things out, and making sense of mathematical ideas for themselves (NCTM, 2000). With a similar vision, the National Research Council (NRC) used the term mathematical proficiency as their model of successful mathematics learning. Mathematical proficiency is a composite of five intertwined characteristics of mathematical knowledge and ability which consists of: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and a productive disposition toward mathematics (National Research Council, 2001). Both the NCTM and NRC assert that successful mathematics learning is Conceptual understanding guiding students to use procedures flexibly, accurately, efficiently, and appropriately (NCTM, 2000; NRC, 2001). This balanced view of procedural knowledge with conceptual understanding is directed at enabling students to comprehend for themselves the mathematical concepts underlying the procedures, seeing the relationships between various operations.

Procedural fluency and conceptual understanding are two aspects of both the NCTM, and NRC's vision of successful mathematics learning, and these mathematical educators and philosophers have argued that full understanding of mathematics consists of more than knowledge of mathematical concepts, principles, and their structures (Kitcher, 1984; Lakatos, 1976; Schoenfeld, 1992). Complete understanding, they argue, includes the capacity to engage in the process of mathematical thinking. That means students must learn to engage in the process of doing mathematics, or not only obtaining specific pieces of knowledge but also a degree of metacognitive knowledge about how to organize mathematical activities (NCTM, 1989). Successful mathematics learning is accomplished when students have a capacity for adoptive reasoning, which is the ability to think logically, communicating clear explanations, and justifications for their own mathematical ideas, as well as evaluate criteria for mathematical arguments generated by others (National Research Council, 2001). This capacity for high level thinking is applied with the student's capacity for strategic competence, which is the ability to formulate, represent, and solve mathematical problems. The final aspect of these reformers’ vision is the disposition students have toward the discipline. When a student is mathematically proficient, they have a habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with the belief that through diligence and hard work they can be successful at learning mathematics. Though this vision is comprehensive and has been well received among a broad range of sectors within American society, this view has not gone without criticism and challenge.

On the other side of this issue stands a group of mathematicians, researchers, philosophers, psychologists, politicians and parents who share a different conception for the goals of mathematics education and would like to see American public school mathematics taught as it has been for many decades, with a strict emphasis on procedural knowledge, and rote learning. This group consists of many loosely organized advocacy groups whose primary means of interaction is through internet sites. These consist of two large national groups, Honest Open Logical Decisions on Mathematics Education Reform (NYCHOLD), and Mathematically Correct. Along with these two national groups are scores of state and district level groups with similar aims. Interestingly these groups do less promoting of their conception of mathematics instruction, and instead seem to focus on criticizing the instructional goals of groups like NCTM. These groups criticize the learning goals of groups like the NCTM and NRC because they believe that the learning goals forwarded by reformers neglect "the systematic mastery of the fundamentals and lack the mathematical depth and rigor that promotes greater achievement in mathematics" (Mathematically

Correct, 2006). The major instructional technique involved in their perspective of mathematics teaching and learning is done by repetition, based on the premise that, you need to know skills before you apply them to problems, and that by becoming procedurally fluent first, you can pull ideas together to attach more complex ideas later. Essentially they posit that one will be able to recall the meaning of the material the more they repeat it (Sovchik, 1996). Like the NCTM and NRC, these groups consider learning mathematics with conceptual understanding a goal of curriculum and instruction, yet their main instructional and curricular technique focuses on memorizing the material so that it can be recalled by the learner exactly the way it was read or heard. It is through this rote style of mathematics instruction that these groups propose students arrive at the inner complexities and relationships of mathematical ideas.

Both of these opposing viewpoints claim their goals for student learning will benefit children with the abilities they need to successfully participate in our society. Is it possible to know the key features of instruction and curriculum that will give students the ability to do mathematics? Through this investigation I will determine what the current literature has found about the effects of instructional practices and curricula on student learning outcomes, in terms of empirical evidence based on research that meets standards of relevance, soundness, and generalizability.

Definition of Terms

Here I would like to define key terms as they will be used throughout this paper. The term "curriculum" refers to instructional material and content, such as textbooks, lesson plans, and units of study. "Instructional Practices" are classroom based processes that encompass all patterns of discourse and learning that teachers and children experience in class. Examples are lecturing, problem posing, question asking, commenting, discussions in pairs, small groups or whole class settings; argumentation, evaluation, and use of representations like manipulative, models, and pictures. Terms that refer to instructional practices that stem from the reform movement and the NCTM standards documents will be used interchangeably. The terms "reform-oriented", "problem-based", "inquiry/argument", "standards-based" and "discourse-oriented" will all refer to the same general form of instructional practice. Studies examined throughout Chapter Three will employ a variety of these terms, and the reader should note that they all have very similar meanings. To contrast, "traditional", "computational-centered", "fact-centered", "procedurally-oriented", "skill-based", "traditional-textbook," instruction refers to that which specifically emphasizes rote learning of algorithms and other mathematical procedures, with minimal focus on the learning of underlying concepts. Finally, the term "mathematic proficiency" will be the domain of successful mathematics which entails five aspects: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition, as described by the NCTM standards documents, and the NRC.

Limitations

The issue of teacher training and teacher knowledge can not be separated from the effects of curriculum, instructional practices on student learning outcomes. Yet for the purpose of a more in-depth analysis of the effects of instructional practices and curricular qualities upon student achievement, this paper does not directly seek to determine what role teacher knowledge and training play in that process.

Statement of Purpose

Mathematical knowledge serves an important function in our society. Without the ability to think mathematically, students will not be able to attain fully the opportunities our society will present them. The discussion surrounding how best to educate children in mathematics has been contentious; both sides of the debate make claims that their will serve our children the best. For the sake of my teaching career, and for those children who are relying upon me to have a comprehensive understanding of nature of mathematics learning and teaching, I seek to know what is the effects of curricula and instructional practice on student achievement in the area of mathematics.

CHAPTER TWO: HISTORICAL BACKGROUND

Introduction to Historical Background

As chapter 1 explained, our country is currently divided by two widely divergent views of the goals of American mathematics education. This chapter will examine the historical background of this division. I will examine this trend as it relates to American mathematics education and the curricular and instructional practices that have been used throughout our country's history. I also address the development of the modern mathematics education research community and its impact on modern views of mathematics education.

Colonial America and Mathematics as a Core Topic

Just as our culture, economy, and social norms have changed from the time of colonial America, so has mathematics education. Schooling during colonial times largely did not include mathematics as a topic. Those who received an education in mathematics did so via apprenticeships, academies, or from private tutors with use of commercial textbooks (Cohen, 2003). Arithmetic was the primary topic used by most people and was seen as a vocational subject. The arithmetical operations of adding, subtracting, dividing and multiplying were necessary as a vehicle for commerce; with the primary purpose of helping sellers, buyers, and investors calculate quantities and prices (Stanic, 1986).

Beginning in the middle of the 1700's colleges required arithmetic for admission and gradually mathematics made its way into primary and high school curriculum (Cohen, 2003). Following the American revolution, the new government put in place a decimal system of money. Suddenly markets and daily commerce went from pounds and shillings, to dollars, dimes, half-dimes, and pennies. With this change ordinary citizens needed to know more than just the basics of arithmetic to function in day to day life. Along with this measure of practicality, the new republican principles of the nation, which gave value to a common education for its citizenry, influenced mathematics rise in importance as a school subject. It was widely thought that voters ought to have minds trained to reason, and increasingly arithmetic was held up as a mental discipline that could increase a person's rationality (Stanic, 1986). Based on the faculty psychology of Christian Wolff, this belief held that the brain was a muscle; and like any of our muscles, the brain needed to be trained and empowered through the exercise of arithmetic. For these reasons mathematics slowly increased in influence and found a place in the school curriculum as a core subject.

Qualitatively Different Methods of Instruction

Mathematics educators of early nineteenth century America were joined in debate about how mathematics should be taught, in a similar fashion that they are today. Arithmetic curriculum at that time was primarily done through textbooks, yet instructional practices took the form of three different methods. They were the rule method, the inductive method, and analytic method (Michalowicz & Howard, 2003). It is striking how these three methods are comparable to the major divisions of instructional practices today. Indeed, the last two methods were in contrast to the first and complementary to one another; they could be thought of as the ancestors of the modern reform movement in mathematics education, while the first method is in every way similar to instructional practices forwarded by opponents of the reform movement. These three methods guided the way in which content was delivered to students in the nineteenth century.

The most commonly used texts from early colonial times in North America until about the 1820's were based on the rule method, and were little more than a collection of rules, written in sentences for the purpose of memorization (Michalowicz & Howard, 2003). Within the rule method, the four operations of arithmetic were introduced to the reader as basic rules. The reader was presented definitions, rules, and tables to be memorized. The rules were then practiced in exercises. The proponents of the rule method argued that arithmetic should be explained clearly and concisely to give children skills for the computational complexity of weights, measures, and monetary systems (Cohen, 2003).

The textbook industry exploded in the years leading up to 1820, with many new books entering the market. Despite this growth, many people had a general dissatisfaction with the rule bound textbooks, which caused many textbook publishers to search for new methods of delivering arithmetic to children (Michalowicz & Howard, 2003). Beginning in 1821, Warren Colburn published a series of textbooks that presented problems in addition, subtraction, multiplication, and division with whole numbers and fractions, and without giving any formulaic rules. Colburn wanted children to learn arithmetic as a mental exercise, done in the mind without pencil and paper, before they learned abstract symbols of numbers and operations (Cohen, 2003). He insisted that students discover the basic rules of arithmetic themselves, by working carefully chosen examples.

> Without telling him what to do. He will discover what is to be done, and invent a way to do it. Let him perform several in his own way, and then suggest some method a little different from his, and nearer the common method. If he readily comprehends it, he will be pleased with it, and adopt it. If he does not, his mind is not

> yet prepared for it, and should be allowed to continue his own way longer and then it should be suggested again. (Michalowicz & Howard, 2003).

Colburn's textbooks are an example of the inductive method of instruction and are based on the theories of the nineteenth-century Swiss educator Johann Heinrich Pestalozzi, and involved posing carefully chosen questions to students. Pestalozzi, who was influenced by Jean-Jacques Rousseau, believed that by using the senses as the basic tools of education, mathematics teaching could help children abandon rote and mechanical approaches to problem solving and help them discover the basic principles involved. Throughout the 1820's Colburn's texts became very popular. Colburn's work was praised for being both practical in that it encouraged fluency with daily marketplace calculations and theoretical in that it also developed inductive reasoning (Cohen, 2003).

The third method of instruction being utilized during this time period was the analytic method, a companion to the inductive method. Textbooks that used the analytic method presented an operation with a detailed explanation of a particular way to think through the solution of the problem (Michalowicz & Howard, 2003). The illustration of the operation and the analysis was followed by examples for practice. It was used to show children how to think through a problem using logic and reason rather than using rules. The textbooks recommended that teachers encourage students to ask questions, and cautioned them to be careful that the processes of analytical thinking do not degenerate into the mere repetition of repeating a formula.

From 1820 to 1860, many schools embraced inductive and analytic arithmetic wholeheartedly. Yet these methods had critics and the scales of pedagogy used in schools tipped back and forth between inductive methods and the more rote learning methods.

The Growing Crisis: New Math & Back to the Basics

At the turn of the twentieth century the mathematics curriculum was being discussed with growing fervor. Advocates of Colburn's inductive method of instruction included prominent child psychologists G. Stanley Hall and William James who both thought that, not just mathematics, but all school instruction should be focused on child-centered methods (Sovchik, 1996). They stressed strategies that connected new learning to the child's previous knowledge and cultural ways of knowing. With their influence and others like them, the early twentieth century brought with it a gradual acceptance of the child as an important part of the learning process. Yet this progressive view did not move forward unchallenged. The prominent theorist Edward Thornedike called into question the theory of mental discipline which had justified the place of mathematics in the school curriculum up until then (Garret & Davis, 2003). Thornedike not only challenged Wolffe's psychology, but also the progressive pedagogy of the inductive method. Thorndike placed great emphasis on speed, accuracy, and memorization in arithmetic to ensure that basic facts would be recalled (Sovchik, 1996). As a result of Thornedike's influence, computational drill or the rule method regained prominence in school mathematics.

The term "New Math" is a part of our everyday lexicon and for many it is connotative of a failed reform or poor math skills, and still some associate those failures with specific instructional practices. Despite this image, "New Math" is not so much the label of a cohesive set of reform proposals and activities, as it is for the era during which a variety of reforms were undertaken (Stanic, 1986). As the National Advisory Committee on Mathematics Education (NACOME) noted in their 1975 report, the term new math, refers to "two decades (1955-1975) of developments that had a general thrust and direction but sprang from many roots, took many different and even opposing forms, evolved and changed with facets disappearing and new ones arising" (Kilpatrick, 1992). Many people see the launching of the Soviet Union's space shuttle Sputnik as the beginning of this period of sweeping math reforms in our public schools. With all the heightened attention mathematics education was given as a result of Sputnik, it is common to think of the changes during that time period as a unified, cohesive movement of change, yet in reality, the voices advocating change during this time period were not always unified, nor connected (Roberts & Walmsley, 2003). The launch of Sputnik did instigate a sudden increase of federal funds for curriculum development, through the United States National Defense Education Act of 1958. The media also created a frenzied mentality in our country with a flurry of articles about the failures of mathematics education in our public schools. Despite all the attention and the common misperception that the New Math was a unified movement, there were many different reform projects at this time (The School Mathematics Study Group, the University of Illinois Committee on School Mathematics, the Boston College Mathematics Institute, the University of Maryland Mathematics Project, and the Madison Project) and some of them had pre-Sputnik roots. Some of these projects focused on child-centered pedagogy, but some of them focused on other pedagogies like learning mathematics through reading.

Furthermore, a central emphasis on some of these changes during this period was on the subject content for the preparation of mathematical and scientific college preparatory students. In 1959 the Commission on

Mathematics of the College Entrance Examination Board noted a discrepancy between what students were learning and what professional mathematicians perceived mathematics was (Kilpatrick & Stanic, 1995). As a result, schools began using new topics such as logic, modern algebra, probability, and statistics so that school curricula more reflected new facets of pure and applied mathematics. Even the content of elementary textbooks became modified. These new texts were criticized as being more formal, more difficult to read, and more concerned with mathematical terms than children's developmental stages.

Despite the good intention of these various forces for change at this period of time, commentators have noted that in the history of education, it is not easy to find examples of rapid change in content or teaching of any school subject, much less mathematics; which at this time was especially resistant to change because mathematical achievement served as a critical gatekeeper to opportunities in higher education and none of the gateway tests reflected the changes that these reformers were proposing (Romberg, 1992). The result was that it became more difficult to gain access to higher education.

The need for change during this time period seemed obvious, but the direction was not always clear. For some of the math reformers, the remedy lay in bringing advanced mathematics (e.g. topology, set theory, group theory) into the lower grades (Kilpatrick & Stanic, 1995). For others the issue was how mathematics should be taught: students should discover as many as possible of the mathematical principles they need to know. The general public had perceived that the New Math produced students that understood mathematics, but it was not clear to the public exactly what their students understood. They had the impression that American students could not compute, and that practical skills had all but been abandoned. The New Math movement was labeled a failure because of these public perceptions (Fay & Graeber, 2003). Despite these perceptions, large-scale, thorough, longitudinal empirical studies of how teaching changed during the implementation of New Math and the relationship between those changes and what students learned (or did not learn) were never done. The 1975 NACOME report argued that the New Math had never even existed, given its weak implementation in schools. This lack of data leaves the New Math reformers vulnerable to claims based on personal experience, rumor, or myth. (Kilpatrick, 1992). As a result, Americans entered the 1970's with a joint national outcry for mathematics education to move "Back to the Basics". The Back to the Basics movement in American mathematics instructional practice was characterized by computational skill, teacher lectures, strict discipline and student's performance on standardized tests.

Mathematics Educational Research is Born

Despite this new call for mathematics instruction to move back to a nineteenth century fact method orientation, the mid 1960's and 1970's experienced a rapid acceleration in the volume of research in mathematics education. Prior to 1960, research was mainly a spare-time amateur's field, with the exception of a few prominent and influential mathematics educators: David Eugene Smith, William A. Brownell, Henry Van Engen, and E. G. Gibb (Kilpatrick, 1992). For the most part, research findings were isolated and easily ignored. That began to change in the sixties and seventies. Public funding of educational research experienced rapid growth through the sixties and into the seventies (Parshall, 2003). New researchers were following lines of study outside the previously studied areas. These studies focused on organizational patterns of classrooms, research on pedagogical techniques, psychological studies of learning and teaching, and research on problem solving. This movement of research professionals continued through the seventies, eighties and nineties with further development of topics investigated as well as differentiated approaches for studying these topics. Mathematics educators began to take notice of the findings of the new research movement. Influential reports like those by NACOME, the National Council of Supervisors of

Mathematics, and Stake and Easley (1978), provided a foundation for new calls of Mathematics curriculum reform. The NCTM's Agenda for Action, a set of recommendations for school mathematics in the 1980's was a direct outgrowth of this newly formed research community (NCTM, 1980). When, during the 1980's, The National Commission on Excellence in Education declared that the nation was "at risk" because it's schoolchildren seemed ill prepared to meet the challenges of the modern age, the first group to formulate sets of standards was the NCTM (Kilpatrick, 1992). The NCTM first responded to the national call for improving mathematics classroom instruction by issuing statements of values, and benchmarks of quality, directed at improving and changing the condition of mathematics teaching and learning in American public schools (Kilpatrick & Stanic, 1995). The first of these standards documents was for curriculum and evaluation, published in 1989, next came one for teaching in 1991, one for assessment in 1995, and finally an updated document that brought these three together in one statement, as the Principles and Standards for School Mathematics in 2000 (NCTM, 1989; 1991; 1995; 2000). Based on the growing field of empirical evidence gathered in the last four and half decades, the NCTM purposed for Principles and Standards for School Mathematics to be a definitive statement that brings under one umbrella the theories and practices of those going back as far as Cohen and Pestilozzi. This most current reform movement has pushed emphasis toward what is called “mathematical power,” and the previously mentioned "mathematical proficiency", which both involve reasoning, solving problems, connecting mathematical ideas, communicating mathematics to others and having a positive disposition toward the discipline. This emphasis has been well received, yet due to past failures in mathematics reform, and the division of beliefs about the goals of instruction between educational progressives and traditionalists in the area of mathematics, there still exists in this current debate much misunderstanding, and contention.

These debates between progressives and traditionalists have reflected different goals for school mathematics held by different groups of people at different times. Is this question merely a matter of personal preference? How does the development of the mathematics educational research community change the nature of this dialectical division between the two opposing view points? Clearly, this is only a cursory overview of the history of debates in mathematics education. Nonetheless, one thing remains clear, the consistent, insistent call for reform, present throughout the span of centuries and decades, remains a vibrant issue in our modern society.

Summary

This chapter provided the historical background of two widely divergent views of the goals of American mathematics education as they pertain to mathematics curricula and instruction in our nation's public schools. At the very onset of mathematics becoming a core school subject, various groups of people have disagreed as to what constitutes successful mathematics teaching and learning. These two divergent philosophies were traced from our nations Colonial roots up to the present debate surrounding standards based educational practices. This chapter also provided a look at the roots of the mathematics educational research community, and showed its significance in directing the current reform efforts. Now I turn to the current research literature to determine what relationship curriculum and instructional practice has on student achievement. As stated in chapter 1, student achievement is the domain of successful mathematics which entails five aspects: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition toward mathematics as a discipline. I will be analyzing what are the strengths and weaknesses of the empirical evidence based on research that meets standards of relevance, soundness, and generalizability. I will weigh these strengths and weaknesses and analyze what research has found about the effects of instructional practices and curricula on student achievement.

CHAPTER 3: CRITICAL REVIEW OF THE LITERATURE

Introduction

This paper is an inquiry into the effect of instructional practices and curricula upon student achievement in mathematics. It will be important to substantiate a supposition that my inquiry is based upon. If the research supports the claim that qualitatively different instructional practices and curricular materials work together to produce differential learning outcomes, it will then be possible (and my main interest), to understand what learning outcomes relate to specific critical attributes of mathematics classrooms. In the first section I review studies that seek to establish empirically how qualitatively different curricular practices work together to produce differential learning outcomes. Following that are a group of studies that look at specific instructional practices and their influence on promoting conceptual understanding with procedural skill in student learning outcomes in math. I then turn to a series of studies that utilize discourse analysis in order to determine what qualitative differences exist from one reform oriented classroom environment to another, and how do those degrees of difference impact student learning. I finish my review with a group of studies that measure instructional practices, and their effects on students' motivation and attitude toward the discipline of mathematics.

Curricula and Student Learning Outcomes

This first group of studies compares the learning outcomes of students who experienced characteristically divergent curricula in math class. These studies seek to learn how the opportunities to learn mathematics with conceptual understanding in mathematics curricula affect student-learning outcomes.

Moss and Case (1999) were interested in knowing how opportunity to learn with conceptual understanding was influenced by mathematics curriculum. They designed a special curriculum for elementary school students that focused on building conceptual understanding and computational proficiency in the area of rational numbers. These researchers compared the achievement of students using their rational numbers curriculum with that of students using a more traditional curriculum that emphasized procedural manipulation of rational numbers. Moss and Case controlled for differences in instructional practice by selecting two groups with the same patterns of instructional activity. They were both considered "child-centered", and utilized manipulative, group work, and discussions. Twenty-nine fourth-grade students participated in this study. Sixteen treatment group students who attended a laboratory school at the University of Toronto and 13 control group students from a private school serving the same population. The experimental group received instruction during 20, 40-minute sessions spread over a 5-month period. The control group received 25, 40-minute lessons spread over a slightly shorter time. Researchers assessed student learning in an interview format that was designed specifically for this study, containing items geared toward the instruction of the experimental group and items geared toward the control group. This assessment was used as both pre- and posttest and both contained percentage items, fraction items, and decimal items.

Moss and Case (1998) reported that the results showed a significant improvement in favor of the experimental group ($p < .001$) although both groups improved from pre- to posttest. Results were the same when analyzed separately for fractions, decimals and percents, as well as for five of six subcategories: nonstandard computation, compare and order, misleading appearance, word problems, and interchangeability of representations. Results for the sixth category, standard computation, were relatively similar for both groups, with results in favor of the experimental group, though statistically insignificant.

Moss and Case (1999) determined that these findings were the result of the opportunities to learn with conceptual understanding that the experimental group's curriculum afforded them. They thought the most significant elements of their curriculum was that (a) it placed a greater emphasis on the meaning of the rational numbers, than on procedures for manipulating them, (b) a greater emphasis on the proportional nature of rational numbers, highlighting rather than glossing over the difference between rational and whole numbers; (c) a greater emphasis on children's natural ways of viewing problems and their spontaneous solution strategies; and (d) the use of an alternative form of visual representation as a mediator between proportional quantities and their conventional numeric representations (i.e. an alternative to the standard pie chart). These elements taken together, gave the experiment students an opportunity to move beyond the understanding of any single form of rational number representation toward a deeper understanding of the rational number system as a whole. This emphasis was most clearly demonstrated on the problem subset that focused on the interchangeability of representations.

Two aspects of consideration when thinking about the generalizability of this study both pertain to the sample. The size of the sample is small, and both classrooms were in schools that do not represent the general public. Even so, the results of the study show a positive relationship between opportunity to learn and students' conceptual understanding.

Cramer, Post, & del Mas (2002) conducted a similar study, but with a much larger sample and more generalizability power. They used a posttest-only control group design in order to study differences in students' achievement on initial fraction learning based on whether they underwent their school district's traditional-based commercial curriculum or the researcher's Rational Number Project (RNP) curriculum, which placed greater emphasis on the use of multiple physical models and translations within and between modes of representation. These different modes of representation were the use of manipulative and pictorial, verbal, real world and symbolic contexts as well as students interacting with one another in group situation. This experiment consisted of 1600 fourth- and fifth grade students from 66 classrooms in 17 different schools throughout the Minneapolis area. 33 treatment group teachers and 33 control group teachers were randomly assigned. The treatment group teachers kept daily logs over the course of the 30 day experiment, as well as conducted interviews with their students periodically throughout the experiment. Two tests were administered to both groups of children. A posttest and retention test were develop to assess student learning in six strands: (A) fraction concepts, (B) fraction equivalence, (C) fraction order, (D) concept of unit ideas, (E) operations (+,-) and estimation, and (F) transfer. These items were designed to measure students' understanding of the part-whole model for fraction and the relative size of fractions.

Cramer et al. (2002) determined that the RNP students significantly outperformed the CC students on both the post-tests and retention tests ($p < .0083$). The subscales that RNP students tested particularly well in included conceptual understanding, ordering, and transferability of their understanding to tasks not directly taught to them, and estimations of sums and differences of fractions. Despite the CC groups having spent much more instructional time working with operations, the two groups showed no significant difference. Data from teacher interview items revealed that RNP students mainly used conceptual approaches to solving the problems, and 76% of the RNP students successfully completed the items. The CC students used procedural methods as often as the RNP student's conceptual methods, but their success rate was only 47%.

The results of Cramer et al. (2002) showed a greater number sense for fractions, documented in the RNP groups success in transferability of their understanding to tasks not directly taught to them. Cramer et al. (2002) concluded that this greater flexibility with number was afforded to the RNP group by the curricular goal of spending much more time developing a flexible concept of unit and quantitative concept of fraction. These results are similar to Moss and Case (2002), and show the importance of providing students with instruction that involves multiple representations (particularly multiple manipulative models, but also opportunities to discuss these models with peers) to help them develop conceptual understanding in initial fraction learning. Though this experiment only had a posttest and retention test, this design offered controls for one of the sources of external invalidity-the interaction of the pre-testing process with the treatments. In addition, the large sample size and random assignment methodology are strengths, which allow for generilability.

Reys, Reys, Lapan, Holliday, & Wasman (2003) measured differences in achievement on the Missouri Assessment of Performance (MAP) test of students from three school districts using Standards-based mathematics curricula and three school districts using traditional curricula. Both sets of district groups were chosen to match prior student mathematics achievement, socioeconomic level and geographical area. The MAP exam was designed around six content strands: number sense (including computation); geometric and spatial sense; data analysis, probability, and statistics; algebra; mathematical systems; and discrete mathematics. Reports of MAP performance included achievement level, a national percentile score and percent correct by content strand. Reys et al. (2003) analyzed prior mathematics achievement between each of the sets of districts using 1997 archived MAP data. MAP scores in spring of 1999 provided a basis to compare the mathematics achievement of students who had a standards-based curriculum for at least two years with that of students who had not had such curriculum. Their findings showed significantly higher scores in the areas of data analysis, probability, statistics, and discrete mathematics ($p < .005$) in favor of the Standards-based groups. No significant differences were found between groups in the area of procedural computation.

The content and format of assessment instruments can influence estimates of student achievement. It seems that the MAP mathematics exam was not developed to be advantageous to students using any particular set of curriculum materials. The MAP was focused on skills, concepts, and problem solving and also included open-ended and multiple-choice response formats. Though significant differences were found in mathematics performance in favor of schools using standards-based mathematics curricula, the impact of textbooks on student achievement may not have been directly established. It seems that the only known factor that was different across the matched pairs of school districts was the mathematics curriculum materials used to guide teaching and learning.

Yet other variables, which these researchers did not control for, including quality of teaching, could have some confounding effect on the results of the study. These results would have heightened strength had the researchers included controls for quality of instructional implementation across test groups.

Boaler (1998) assessed the mathematics achievement of two groups of students based on their exposure to different curricular approaches. In a qualitative 3-year case study, Boaler followed the mathematics education of two same grade cohorts, each at a different school in the United Kingdom. Her aim was to compare and contrast the achievement of each cohort on two types of tests: an applied activity test, which was designed to measure conceptual understanding and procedural skill, and a short written test, which was a measure of computational skill. Her findings suggest that curricula with an emphasis on integrating procedural skill with practical application have positive effects on conceptual understanding but are merely comparable to traditional curricula on affecting procedural skills.

The two schools from which the case study groups were chosen had opposite curricular goals for mathematics instruction. At Amber Hill school Boaler (1998) followed 110 students starting in year 7 when the students were 11 years old. Amber Hill had a strict emphasis on formal explanations of mathematical procedures followed by practice of these methods. Every lesson from this cohort group was done through this textbook style format. At Phoenix Park School Boaler followed 163 students of the same age and year as Amber Hill. Boaler described this school's curriculum as open-ended projects that mixed procedural skills with contextualized formats.

From data she collected Boaler (1998) was able to make these conclusions. On a paper and pencil test, which assessed students with a math activity based on architecture, 55% of the Amber Hill students could successfully complete the architectural task, compared with 75% of the Phoenix Park students. This is in spite of the fact that Amber Hill tracks their mathematics students based on ability, and the group the researcher was following was the highest achieving of Amber Hill's mathematics students. Despite the fact that the Amber Hill students scored significantly higher on the NFER entry tests than the Phoenix Park students, the Phoenix Park students significantly outperformed the Amber Hill students in an applied activity. When assessing the paper and pencil tests that Boaler devised herself, there were no significant differences in the performance of the two groups. In addition, on their national assessments taken at the end of the year, 11 Phoenix Park students significantly outperformed the Amber Hill students, with 88% of Phoenix Park students passing the exam compared to 71% of the Amber Hill students ($p < .001$). Based on interviews conducted with students after the test were scored, Boaler concluded that the reason the Amber Hill students did not perform as well as the Phoenix Park students was that they had developed an "inert" knowledge that they found difficult to apply because they couldn't identify what the questions were asking and which procedures were appropriate to use. Phoenix Park students felt that their success on the tests was due to their abilities to think about unfamiliar situations and determine what was required.

In a quantitative study of 10-to 12-year-old children in Brazil, Saxe (1988) sought to understand the differences between three different groups of children's mathematical understanding as a function of the role it played in their everyday lives. Saxe found that children construct mathematical understanding and skills as a result of the kinds of practical use that emerge in their everyday cultural practices. This study assessed and compared the mathematical ability of identifying and comparing large numerical values, between 3 groups of children: urban children ($n= 23$) who were not in school but bought and sold candy to earn income, urban non-sellers ($n= 20$) in first or second grade, and rural nonsellers ($n= 17$) in first or second grade. The study was conducted in Brazil because of its inflated currency, resulting in very large calculations when buying and selling.

Each group of students was interviewed and assessed so as to determine their relative mathematical understandings in the areas of number representation, arithmetic problem solving, and ratio comparison. In these interviews researchers assessed the qualitative differences of each group's answers. For example, in arithmetic, students who solved a subtraction problem wrong received points if the problem was within Cr$200 of the answer. They also collected data about the kinds of strategies students used to solve the various problems. Results showed that significantly more sellers (70%, $p< .005$) and urban nonsellers (75%, $p< .005$) achieved passing scores than did rural nonsellers in the area of number representation. For arithmetical calculations, sellers performed significantly better than both urban nonsellers ($p < .05$) and rural nonsellers ($p < .001$). For ratio problems students were analyzed for their judgments about which ratio would provide the greatest profit and their use of appropriate justifications. Sellers significantly outperformed both groups, averaging scores 30% higher than the other two groups.

From these data, Saxe (1988) Concluded that the cultural context of the sellers and their everyday experiences had presented them with an opportunity for creating new problem-solving procedures and understandings in the mathematical areas of number representation, arithmetic, and ratios. Although these children were from Brazil, the findings have some relevance for students of mathematics in all countries; these results suggest that students will be more likely to construct procedural skill with conceptual understanding if they are given the opportunity to do so. The children in the Saxe study found that opportunity as a part of their everyday life.

Taken as a whole, these findings suggest that when curriculum includes opportunities for students to learn mathematics with conceptual understanding, they do not do so at the sacrifice of procedural skill. In fact, Boaler (1998) showed that the curriculum at Phoenix Park lead to a greater ability to work with a flexible knowledge, that the students at Amber Hill obviously lacked. Reys et al. (2003) reported similar findings that Standards-based groups showed significantly higher scores in the areas of data analysis, probability, statistics, and discrete mathematics ($p < .005$), with no significant differences found between groups in the area of procedural computation. Cramer et al. (2002) reported that Despite the CC groups having spent much more instructional time working with operations, the two groups showed no significant difference. Further more, RNP students mainly used conceptual approaches to solving the problems, and 76% of the RNP students successfully completed the assessment were as the

CC students used procedural methods as often as the RNP student's conceptual methods, but their success rate was only 47%. Moss and Case linked the success of their experimental group with distinct qualitative differences found in their curriculum. Most notably were the opportunities their curriculum provide for students to study the meaning of rational numbers as opposed to just procedures for manipulating them. These findings have established evidence that the nature and scope of curricula has a direct impact on student achievement.

Instructional Practice and Student Learning Outcomes

The next group of studies expands the investigative lens a bit to include in the analysis not only curricular differences, but also the nature of instructional practices. This group of studies all utilizes similar methods to compare the learning outcomes of two groups of students who experience different instructional practices in their math classes. Fennema, Carpenter, Franke, Levi, Jacobs, and Empson (1996) and Hiebert and Wearne (1996), use quantitative longitudinal studies to explore the relationship between differential instructional practices and student learning outcomes over time. Hiebert and Wearne (1993), and Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti, & Perlwitz, (1991) both conduct qualitative studies to explain this relationship. Carpenter, Fennema, Peterson, Chiang, and Loef (1989) used a mixed qualitative / quantitative method to do so. Carpenter, Franke, Jacobs, Fennema and Empson (1998) and Schoen, Cebulla, and Finn (2003) both use quantitative methods to explore this relationship.

In their quantitative study Hiebert and Wearne (1993) tracked 2 non-traditional classrooms, and 4 traditional oriented classrooms, all of which received instruction on place value, and multidigit addition and subtraction, in order to find differences and similarities between instructional tasks, classroom discourse and their connections with academic performance outcomes. Their results showed that differential instructional practices and learning outcomes for place value and multidigit addition and subtraction is likely to be related. Of the six classes in the study, changes in performance differed most in classrooms that differed most in the nature of instructional tasks and discourse.

The six classrooms in this study were in one school and all of the 2nd grade students were regrouped for mathematics instruction, creating two upper division classes and four lower division classes. Researchers chose one upper level class (classroom F) and one lower level class (classroom D) to be the project classrooms in which specially trained treatment teachers taught the children only with the topics of place value and multidigit addition and subtraction and only at the time and in the sequence these topics were covered in the more conventional programs carried out in the other classrooms. When compared with their same level counter parts, Students in classrooms D and F scored higher, and showed greater improvement on each of four different test indicators. The lower group classes A, B, and C had relatively similar patterns of instructional tasks and discourse, while classroom D was in sharp contrast. An example is the amount of problems each class worked on in a mathematics lesson. Students in classrooms A, B, and C, worked from 955 to 965 place-value problems during the year, whereas students in classroom D worked on 501 place-value problems. When researchers looked at how students in all classrooms improved on the place value section of the test, from the beginning of the year to the end, classroom D's mean improvement was between 45% and 65% greater then the mean improvement of Classrooms A, B, and C. Results of this study provide other evidence that suggests instructional practice is likely to influence learning outcomes. Students in classroom D did not just work fewer problems, they spent more time on each problem (50% more time with an average of 4.5 minutes per problem), and were asked more questions from their teachers requesting them to describe and explain alternative strategies (these questions were almost non existent in classrooms A, B, and C).

Overall the measures of this study did not produce significant results, which conclusively prove a specific instructional program produces a specific learning out- come. This research does show a likely relationship that when two instructional programs are sufficiently different, learning outcomes will also be different.

In a mixed method comparative study, Carpenter et al. (1989) investigated how coaching teachers with explicit knowledge on children's thinking in a specific content domain influenced the teacher's instruction and how this instruction influenced their student's achievement. Like Hiebert and Wearne (1993) these researchers set out to measure instructional differences and their resulting student learning outcomes. Twenty first grade teachers assigned randomly to an experimental treatment, participated in a month-long workshop in which they learned about the research on children's solutions of addition and subtraction problems. These teachers learned to classify problems, to identify the processes that children use to solve different problems, and to relate processes to the levels and problems in which they are commonly used. Other first grade teachers (n = 20) were assigned randomly to a control group, and received no instruction on the research of children's solutions of addition and subtraction problems. When these researches compared instructional practices of both groups of teachers, although instructional practices were not prescribed for any of the teachers, experimental teachers spent significantly more time teaching problem solving in the form of word problems (55% of total teach time for project teachers, versus 36% for non-project teachers, $p< .01$) and number facts significantly less (25% versus 47%, $p< .01$), than did control teachers. Experimental teachers knew more about individual students' problem-solving processes, being able to accurately predict a student's solution strategy for specific problems half of the time. The control group teachers were able to accurately predicted student solution strategies only a third of the time ($p< .01$). These significant differences in instructional practice also produced significant differences in student's learning outcomes. Students in experimental classes demonstrated a higher level of recall of number facts, averaging 20% more correct answers than control group ($p< .05$). Experimental students scored significantly higher on a measure of problem solving for complex addition and subtraction problems (their average score was 8.6 out of 12, compared to 7.8 for control students, $p< .05$). Experimental students reported significantly greater understanding of mathematics than did control students, reporting that the last time they were in math class they understood the math that they were doing most of the time ($p< .01$). They also scored higher in reported confidence of problem-solving abilities, answering on average 3 quarters of a point higher on a twelve-point scale ($p < .01$).

The researchers measured in-class instruction with two classroom observers who observed each teacher and class for four week-long periods from November through April, during the course of one school year. Two observers created two sets of data, one recording teacher data and the other recording student data. Observation categories allowed this data to be coded in a reliable way; the coding was confirmed reliable by high inner rater reliability percentages. The observation categories in table 1 represent all of the actions and behaviors raters coded during each lesson. The data from this measure was compiled as the mean proportion of time spent on that activity within the total time spent on addition and subtraction instruction. This is a sufficient measure to determine if one instructional group is different from another.

The results they found from this measure were that during addition and subtraction instruction, experimental group teachers spent significantly more time on word problems than did control teachers. In contrast the data shows that control teachers spend significantly more time on number facts problems than did CGI teachers.

Table 1

Action and Behavior Observation Category

Teacher Observation Category	Student Observation Category
Setting	Setting
Whole Class	Whole Class
Medium Group	Medium Group
Small Group	Small Group
Teacher/student	Teacher/student
Alone	Alone
Content	Content
Number fact Problem	Represented Problem
Represented Problem	Word Problems
Word Problems	Other addition/subtraction
Other addition/subtraction	Nonengaged with content
Expected Strategy	Strategy Used

Direct Modeling	Direct Modeling
Advanced counting	Advanced counting
Derived Facts	Derived Facts
Recall	Recall
Multiple	Not clear
Not clear	

Teacher Behavior	Lesson Phase
Poses Problem	Review
	Development
	Controlled practice
	Student work (seatwork)

Student achievement measures were also found to be adequate for producing the stated outcomes. The researchers used a standardized achievement test as a pretest to control for prior mathematics achievement. A highly detailed picture of student computational skills and problem-solving skills, where gathered with three written tests and individual interviews. In all there were two posttest measures of computational skills-one based on a written test and one from the interview-and five posttest measures of problem solving-the standardized test, the three problem-solving scales, and the interview of problem solving strategies.

The measure used to determine teacher's knowledge of their students problem solving strategy and ability was also found to be reliable and adequate to produce given results. Teachers were asked in an interview to predict each of their students' solution strategies to five items on three student interview assessment measures (number fact strategies, Problem-solving strategies, and Problem-solving abilities).

The experimental treatment teachers learned how to recognize when a student was using direct modeling, advanced counting, derived facts or recall strategies to solve problems. They were shown major categories of addition and subtraction content such as, number facts, represented problems, and word problems. With this knowledge teachers were more inclined to spend more time on problem solving, and these teachers had a greater understanding of student's problem solving strategies. This experiment shows empirically that classroom practices do have a direct effect on student learning. This learning was shown to be the result of students spending more time on problem solving strategies, and teachers having a greater knowledge of their student's abilities and strategies for solving problems.

This study reported two student learning outcomes that were differentially influenced by differing instructional practices. Student's use of invented strategies for solving problems relates to the NCTM & NRP learning goals of mathematical proficiency in that it shows strategic competence: the ability to formulate, represent, and solve mathematical problems. The experimental student's ability to recall number facts reflects aspects of procedural skill: the ability to recall number facts fluently. One other element of mathematical proficiency reported in this study's findings was the student's disposition toward mathematics.

These researchers also derived an index based on student's answers to several questions. This index was proven to have a low internal consistency for student beliefs on individual scales. Although the experimental group students were shown to have a significantly higher reported confidence in problem solving abilities than the control group, this evidence alone dose not allow me to conclude that these instructional elements lead experiment students to a positive disposition toward mathematics. The critical attributes of experimental classrooms that were found to develop these aspects of mathematical proficiency were found in the experimental treatment teacher's knowledge of how to recognize when a student was using direct modeling, advanced counting, derived facts or recall strategies to solve problems. They were shown major categories of addition and subtraction content such as, number facts, represented problems, and word problems. With this knowledge teachers were more inclined to spend more time on problem solving, and these teachers had a greater understanding of student's problem solving strategies.

As a continuation of Carpenter et al. (1989), Fennema et al. (1996) sought to find relationships between teacher's developing understanding of student's mathematical problem solving strategies in single and multiple digit addition, subtraction, multiplication, division word problems, and place value concepts with their student's thinking and learning over time. In a 4-year longitudinal study Fennema et al sought to investigate changes in 21, 1st, 2nd and 3rd grade teachers' instructional practices as they learned about children's thinking and the impact of these changes on the learning outcomes of their students. These 21 teachers participated in workshops in each of the project's 3 years and received in class support throughout the course of the study. In the workshops instruction emphasized several common themes: (a) Children can learn important mathematical ideas when they have opportunities to engage in solving problems in a variety of ways; (b) individuals and groups of children will solve problems in a variety of ways; (c) children should have many opportunities to talk or write about how they solve problems; (d) teachers should elicit children's thinking; and (e) teachers should consider what children know and understand when they make instructional decisions. The researchers found that when project teachers increasingly demonstrated the workshop's characteristics of instruction, their student's gains on conceptual and problem-solving performance was directly related to these changes in teachers' instruction.

Where the first study (Carpenter et al. 1989) measured instructional practices in terms of time spent on problem solving, this study included measures capable of categorizing instructional practices with much greater detail. They created a scale with four categories of instructional practice, which include opportunities for children to solve problems, children sharing their thinking with peers and teacher, teacher's elicitation and understanding of children's thinking, and teacher's use of children's thinking as a basis for making instructional decisions. Teachers were rated in each category upon a continuum from least to most like, based on audio and video recordings of classroom instruction, classroom observation and post lesson interviews with researchers. The continuum consisted of five levels: 1, 2, 3, 4-A, and 4-B. Instruction of teachers categorized as level 1 usually involved activities that were focused on the learning of procedures and guided by an adopted textbook. Level 1 teachers usually practiced direct instruction by demonstrating the steps in a procedure as clearly as they could and then having the children practice repeating the steps. By the end of the study 90% of the teachers had made increases on the instructional practice scale and where categorized at level 3 or higher (level 3 = 12, level 4-A = 2 and level 4-B = 5). At level 3, children spent most of mathematics class engaged in solving and reporting their solutions to a variety of problems. There was a decreased emphasis on learning standard mathematical procedures. The level 3 teachers talked less than the level 1-2 teachers and spent more time listening. These classes were organized so that children were either talking or writing about how they solved the problems. Level 3 teachers based the curriculum on the problem types explored in the workshops. The teachers continually elicited children's thinking and could report it to the class. The teachers usually recognized the various strategies used by the children. However, their understanding was at times incomplete, and understanding children's thinking appeared to be an end in itself rather then a means by which to plan instruction. The factor that distinguished level 4 teachers from level 3 teachers was that they ascertained what their students knew by questioning and listening to responses, and then used what they learned to decide on future mathematical experiences. There was some variation in the instruction of the teachers that were categorized as level 4. The distinctions were subtle and were a matter of degree. Level 4-A teachers appeared to consider groups of children as they made instructional decisions about mathematical activities. In contrast level 4-B teachers had more detailed knowledge of each child's thinking than level 4-A teachers and seemed always to be aware of the impact that instruction would have on each individual.

Student learning was measured before the first year, and during spring of year 1, 2, and 3 for each teacher's class. Two measures were used to determine student learning for concepts, problem solving and computation. The concepts and problem solving test consisted of a range of problem-types in single digit and multiple digit addition, subtraction, multiplication, and division word problems, and items that measured place value concepts. This was a valid measure for conceptual problem solving. An example of a concept and problem-solving item for grades 2 & 3 was: "Fay has 4 packages of gum. There are 13 pieces of gum in each package. How many pieces of gum does Fay have altogether?" For first grade teachers, student scores increased from 43% correct in Year 0 to 65% correct in year 3; for 2nd grade teachers student performance increased from 59% correct responses in year 0 to 69% correct in year 3; and 3rd grade teacher's students increased from 75% in year 0 to 86% correct in year 3. Although the changes for the computational skills test were in a positive direction, the overall changes were too small to be significant.

In order to find out how student learning responded to increases in teacher instructional practices, the researchers measured all instances in which a teacher gained an instructional level or more from one year to the next and determined whether that change was followed by an increase in achievement by the students of those teachers in the following year. Eight of the 11 teacher's instruction was categorized at least one level higher in year 1 then it was in year 0. For 7 of those 8 teachers, mean student achievement in concepts and problem solving improved significantly either that year or the following. Five teachers changed a level in instruction during year 2. In three of these five cases, student achievement increased the following year. Researchers also considered to what degree changes in student achievement for specific teachers were preceded by changes in teachers' level of instruction. For 6 of the 11 teachers, student achievement in concepts and problem solving was significantly higher at the end of year 1 than it was at the end of year 0. There were eight instances in which student's concepts and problem-solving achievement for a particular teacher increased significantly from year 1 to 2 or from year 2 to 3. In each case the teacher's instruction was categorized at a higher level at the end of the preceding year than it was at the end of the year before.

These results suggest that a change in a teacher's level of instruction was reflected in the achievement of her students. This finding further supports the claim that instructional practices have a direct influence on student learning outcomes. This study also showed a direct link between elements of instructional practices that produce mathematical proficiency when the content is focused on single and multiple digit addition, subtraction, multiplication, division word problems, and place value concepts. Those instructional elements focus on giving children opportunities to solve problems, facilitating children in sharing their thinking with peers, eliciting children's mathematical thinking so as to understand their solution strategy and teacher's use of children's thinking as a basis for making instructional decisions.

In another quantitative longitudinal study, Hiebert and Wearne(1996) tracked 53 children over the course of the first 3 years of their schooling to measure how the instructional goals of teaching for understanding, or teaching for procedural knowledge, influenced emerging relations between children's understanding of multidigit numbers and their procedural skill. These researchers found that there was indeed a significant relationship between instructional practice and learning outcomes for place value and multidigit addition and subtraction in students. Researchers found that two qualitatively different forms of instruction differentially influenced students' level of understanding of place value and multidigit addition and subtraction over time, where the students who had received three consecutive years of alternative instructional practices developed proficient use of computational procedures, and showed higher levels of understanding after the third year. Conversely, Hiebert and Wearne(1996) also found that when instructional practices involved working from a textbook, where the teacher demonstrated standard algorithms to find the answer to the first problem and then the students worked individually to complete the remaining similar problems, students were incapable of developing appropriate procedures and performing correctly without instruction.

Each year the school moved students from one instructional group to another and this factor presented this research with a substantial interference. The researchers wanted to avoid the confounds of mixed instructional histories, so they used only the data from students who tracked all three years in one group. By the beginning of the third year only 17 students remained from the original 48 who were in the alternative instruction group, and only 9 students remained of the original 24 who were in the Traditional-Oriented group. Despite this attrition, when the two groups mean scores measuring understanding were compared, at the end of 1st and 2nd grade they were not significant, but were significant favoring the reform-oriented instruction group at the end of 3rd grade (3.17 out of 4.00 for the alternative instruction group, and 1.55 for the traditional group. $P < .01$). This means that the two forms of instruction differentially influenced students' level of understanding over time.

Throughout the 3 year study, student learning assessment took place during interviews at the beginning, middle and end of the school years. Each student was given a set of tasks designed to measure understanding of multidigit subtraction and addition. The interview format allowed the researchers to use multiple strategies for assessing student knowledge. These assessments measured student's conceptual understanding, procedural skill and understanding of procedural skill. In one example of the tasks researchers used to assess understanding of computational skills, a study member would ask the student to compute a two-digit addition or subtraction problem with regrouping, written vertically on paper. After the student wrote the answer, the interviewer asked the child to “show how that works” with colored chips, red valued as 10’s and yellow valued as 1’s. This allowed the researchers to determine if the child could represent a standard algorithm in multiple ways, thus proving a certain level of understanding for the computational skill. I use this example as an illustration of the validity of all student assessment measures.

Although there are no described measures for instructional practice in this study, the design of the study created these two groups to be qualitatively different. This study employed specially trained treatment teachers who worked with the children only with the topics of place value and multidigit addition and subtraction and only at the time and in the sequence these topics were covered in the more conventional programs carried out in the other classrooms. The alternative instruction was characterized by presenting students with contextualized problem situations, opportunities to develop their own solution strategies, encouraging students to represent quantities with both written numbers and physical manipulatives, and discussions of multiple solution strategies with other students. Class discussion focused on how and why these nonstandard procedures worked or did not work. Working with specially employed teachers permitted a faithful implementation of the project’s alternative instructional approach, but it confounded the teachers with the treatment group, making that aspect of this study intransferable. The control group classrooms were qualitatively different from the alternative classrooms. A typical lesson in these classrooms involved working thought two textbook pages. The teacher demonstrated or reviewed how to find the answer to the first problems on these pages, and then the students worked individually to complete the remaining similar problems. Physical materials were not used beyond 1st grade. The teacher introduced and prescribed the standard algorithms for addition and subtraction as suggested by the textbook and encouraged students to use them. Students spent most of their time practicing these procedures.

This study empirically showed that two qualitatively different forms of instruction differentially influenced students' level of understanding of place value and multidigit addition and subtraction over time.

The quantitative study of Schoen et al. (2003) sought to determine which teacher practices most strongly correlate to higher achievement of high school students receiving instruction using standards-based curricula. These researchers collected two teacher questionnaires, one during the middle of the year and one at the end, in which teachers commented about their instructional practices; students achievement data, classroom observations, and school demographics information. An index of teacher practices revealed that the single biggest predictor of student achievement in standards-based classrooms was whether or not the teacher had completed a workshop designed to prepare for teaching the curriculum (which explained 30% of the variance in adjusted student achievement). Other factors that were found to be positively associated with student achievement included teacher-teacher cooperation, increased use of group work, a variety of assessment methods, and high teacher expectations.

Cobb et al. (1991) conducted a comparative qualitative study in order to measure the difference in student arithmetical conceptual understanding and computational fluency between students in ten project classrooms and students in eight control classrooms. Project Classrooms were characterized by instructional practices that gave students opportunities to discuss critique, explain, and when necessary, justify their interpretations and solutions. This approach engaged students in small-group collaborative mathematical activity and then in teacher-orchestrated class discussions of their problems, interpretations, and solutions. The control classroom's instruction was textbook guided and followed a very standard traditional pattern. All students were in the 2nd grade, from three schools. Cobb et. al. (1991) administered two tests of arithmetical competence to both groups of students. The first was the state-mandated multiple choice standardized achievement test (ISTEP), the mathematics portion of which is composed of two subtests, Computation, and Concepts and Applications. The second arithmetic test called the Project Arithmetic Test was developed by project staff. This test utilized two different scales, one designed so that student could use computational algorithms without conceptual understanding, and the other a relational scale designed to measure conceptual understanding of place-value numeration and computation in non-textbook formats. Comparisons of the two groups of student's performance on the two tests, the ISTEP, and the Project Arithmetic Test, indicated that project students developed a higher level of reasoning in arithmetic than non-project students did. The results also indicated that project students' had similar levels of computational performance as non-project students.

The measures of student performance were adequate measures in this study because the test provided a variety of problem types requiring the students to have both computational fluency and conceptual understanding. For example, the ISTEP had two sections, the first measuring computational skills, presented students with one and two digit addition and subtraction computations with or without regrouping, in the standard textbook vertical column format. The other section measured conceptual understand and required students to identify the numerical value of representations. For example a typical item required students to identify seven 10-valued cubes and two individual cubes as 72. The second test this study used, The Project Arithmetic Test was developed by project staff, and adequately measured student's conceptual understanding of arithmetical computation in non-textbook formats. These items were two-digit addition and subtraction tasks presented in an "everyday language" format. Examples included: 21 more than 49 is___, or What number do 12 ones and 3 tens make? All of these items required regrouping. This test required students to show how they knew what the answer was.

This study did not have any measures for teacher instructional practice, but described the typical instructional practice, which took place in project classrooms. This was described as beginning with the teacher posing a problem and students working in pairs to solve it. A variety of manipulative materials were made available, but it was primarily the children's responsibility to decide if the use of a particular manipulative might help them solve their mathematics problem. The teachers observed and interacted with the children as they engaged in mathematical activity in pairs. After the children had worked together for perhaps 20 minutes, the teacher orchestrated a whole-class discussion of the children's interpretations and solutions. The study does not describe what kind of instructional practice that took place in non-project classrooms only describing it as textbook oriented.

The ten project teachers in this study all volunteered to be part of a one week summer institute conducted by the researchers with the goal of building understanding in these teachers about the differences between traditional textbook instruction, which produces an emphasis on correct procedures, and "pragmatic" everyday mathematical problem solving, which has as its aim building student's conceptual understanding. These ten teachers all volunteered to be the project teachers throughout the year long study and received instructional materials and support throughout the year. These methods ensure that the project classrooms will have a faithful application of reform oriented instructional practice. What is interesting is the level of support these teachers were supplied with, and the learning outcomes of their students. A nice addition to this study would be the comparison of teachers enacting the same instructional practices with and without support from the researchers.

In their 3 year longitudinal study of students going through first, second, and third grade, Carpenter et al. (1998) documented the development of students' inventive strategies for solving multidigit addition and subtraction problems, and compared this data with students who instead relied on standard algorithms to solve problems. Carpenter et al. (1998) selected a sample of 82 students distributed among 27 teachers classes in three different schools, and interviewed them each individually five times over the course of 3 years. In the one hour audio taped interview researchers asked students to complete a set of tasks designed to assess student's knowledge of base-10 number concepts, their strategies for solving addition and subtraction problems and computation exercises, their abilities to use specific invented strategies, and their abilities to extend and use addition and subtraction procedures flexibly. Instructional practices were consistent throughout all project teachers' classes, and emphasized word problems, giving students opportunities to solve problems using a variety of strategies, and alternative strategies were discussed with the whole class or in small groups.

Results showed that students in invented strategy groups demonstrated significantly fewer systematic errors than students in the algorithm group. These children demonstrated conceptual understanding using inventive strategies flexibly to transfer their use to new situations as demonstrated by the fact that students in the invented-strategy groups were significantly more successful in solving extension problems than the students in the algorithm group. Students who initially used inventive strategies demonstrated knowledge of base-ten number concepts before students who relied primarily on algorithms, indicating that inventive strategies draw upon prior knowledge and understanding within students. In most classes, standard algorithms were introduced to the students between the fall and spring interviews of 2nd grade, and the researchers noted how the use of invented strategies versus use of algorithms changed once formal algorithms had been explicitly taught. Carpenter et al. (1998) found that during the first two interviews 65% of students used an invented addition strategy, and after algorithms were introduced an additional 23% came up with invented addition strategies in subsequent interviews, giving a total of 88% of students having invented strategies by the end of the study. At the end of the study, students were divided into different groups based on their approaches to the problems and their success with the interview tasks was analyzed. The students who had invented their own strategies demonstrated richer understandings of the concepts, and the students who used invented strategies first also made fewer systematic errors when using the algorithms than other students did.

Taken as a group, Hiebert and Wearne (1993), Carpenter et al. (1989), Fennema et al. (1996), Hiebert and Wearne (1996) and Cobb et al. (1991) offer empirical evidence that qualitatively different forms of instruction do differentially influence students' learning outcomes. These studies also provide evidence of a general pattern of instructional practices that lead to elements of mathematical proficiency. This general pattern can be described as consisting of: (A) presenting students with a variety of contextualized problem situations (Carpenter et al., 1989; Cobb et al., 1991; Fennema et al., 1996; Hiebert & Wearne, 1993; Hiebert and Wearne, 1996) (B) Children developing multiple solution strategies (Carpenter et al. 1989, Hiebert & Wearne, 1993; Hiebert & Wearne, 1996) (C) representing quantities in multiple ways (Hiebert & Wearne, 1993; and Hiebert & Wearne, 1996) (D) sharing and discussing their solution strategies with the class (Hiebert & Wearne, 1993; Hiebert & Wearne, 1996) (E) teachers using knowledge of what children know to make decisions about mathematics instruction, thus developing instruction that builds on students prior knowledge (Carpenter et al., 1989; Fennema et al., 1996).

Qualitative Differences in Reform Oriented Classrooms

In the following sections I look at groups of studies that loosely share a common goal of understanding how qualitative differences in reform oriented instructional practices create and support student mathematical proficiency. Wood, Williams, and McNeil (2006) analyzed various classrooms for qualitative differences in discourse patterns, and associated these discourse patterns with student cognitive activity. McClain, and Cobb (2001), McCrone (2005), Willams and Baxter (1996) all investigated how a classroom culture of inquiry/argument develop over the course of a school year. Two research studies Stein, Grover, and Henningsen (1996) and Henningsen, and Stein, (1997) sought to understand more clearly the relationship between instructional practices, mathematical tasks and the demands for cognitive challenge that are associated with variances in the development, and implementation of mathematical tasks. I then look at a group of research that focused on qualitative differences in classrooms that all share the five characteristics of reform oriented classrooms, in order to see in greater detail what are the differences in student learning outcomes. I finish with a group of articles that looked at the nature of the teacher's role as a facilitator of student dialogue, and what are the effects on student learning.

Wood et. Al. (2006) investigated how patterns of discourse interactions within mathematics classrooms affected student's verbalized mathematical thinking. These researchers compared interaction patterns and children's mathematical thinking as verbalized in class discussions within four classroom cultures, conventional textbook, conventional problem solving, report strategies, and inquiry/argument. Researchers used videotape and transcripts of 30 lessons each from 5 different 2nd grade classrooms (1 characterized as both conventional textbook and conventional problem solving, 2 report strategies, and 2 inquiry / argument) in order to create two coded data bases. One data base for interaction patterns and the other for children's mathematical thinking. Coding was done separately from interaction patterns and children's thinking, but the same transcripts and video was used to make each data base. The process of coding for the analysis of children's mathematical thinking consisted of categorizing individual children's verbalized statements based on a hierarchy of cognitive activities, which allowed the researchers to describe children's mathematical thinking. The cognitive activities in the hierarchy, starting with the least demanding and successively building one upon another, consist of the following: Comprehension, Applying, Analyzing, Synthetic-analyzing, Evaluative-analyzing, Synthesizing, and Evaluating. Following this analysis, the coded interaction patterns and the coded children's statements were combined to recreate each class discussion in its entirety. Each coded statement of children's mathematical thinking was re-examined within the interaction pattern it occurred. The results of this analysis revealed that a combination of specific interaction patterns exists within each classroom culture and that these specific patterns of interaction reveal a gradation and particular changes in the roles for participation among children and teachers from one classroom culture to the next. Furthermore, this gradation was also seen in terms of the quality of children's expressed mathematical thinking from one classroom culture to the next.

In conventional textbook classes the data revealed 3 predominant interaction patterns. The dominance of these three interaction patterns indicated that children's participation was limited to responding to teacher's questions by giving known answers or predetermined information. Out of 34 mathematical problems analyzed 50% were characterized by the Initiation Response Evaluation pattern, making that type the most prevalent. The IRE pattern consisted of teachers asking a question, students responding either correctly or incorrectly, and the teacher evaluating the student's response. Twenty one percent were characterized by Give Expected Information. This pattern was a less tightly controlled form of the IRE pattern in which students still provided previously taught information in response to teacher's questions. 15% were characterized by the funnel pattern in which the teacher, through a series of questions, led the student to the correct answer.

In the conventional problem-solving class researchers found that students were expected to tell others their strategy for solving problems and there were more opportunities for children to participate in the discourse than in the textbook discussions. Out of 49 mathematical problems analyzed 14% were a more open pattern of interaction called Exploring Methods, which required students to tell others their strategy for solving a problem. However, the most dominant type of interaction pattern occurred 39% of the time. That pattern was the Hint to Solution pattern, characterized by the teacher "hinting" at the solution method in ways that essentially removed the mathematical challenge or complexity of the problem, which was then easily solved by the children. This pattern, along with the IRE (18%) and Give Expected Information (16%) patterns, made up 73% of the types of interaction identified in the conventional problem-solving classroom culture.

In the Strategy Reporting classroom culture, two new patterns comprised 14% out of 85 problems analyzed. These interaction patterns were: Argument (9%), in which children and teacher participate in discourse to resolve their differences or disagreement about strategies or answers, and Inquiry (5%) in which children and teacher ask questions for clarification of the strategy or ideas of the child explaining. These two patterns of interaction reflect an important change in the student participation structure of the Strategy Reporting classroom culture. This change in participation was characterized by students doing the reporting and explaining of their solutions. However, Exploring Methods was the most dominant form of interaction and occurred 44% of the time out of 85 problems analyzed. The next most common interaction pattern was called Teacher Elaboration (11%). This pattern is characterized by the teacher revealing the means by which a student solved a problem. Typically the teacher would elaborate on and extend a child's explanation to ensure that important ideas are conveyed to the other children.

In the Inquiry/Argument classroom culture another important shift occurred in the students participation structure from an emphasis on the child reporting her/his different strategies to the children as listeners taking over the role of the teacher in questioning, clarifying, and validating mathematical ideas; despite the most frequent interaction pattern being the Exploring Methods (35%) pattern. 16% of the interaction patterns involved Argument and 9% Inquiry out of a total of 110 mathematical problems analyzed. Taken together, this was almost double that of the Strategy Reporting classrooms. In addition, Two new interaction patterns occurred in Inquiry/Argument classrooms: Building Consensus (5%) which was characterized by the children and teacher developing common meanings, and Checking for Consensus (7%), which meant that the teacher initiated a final attempt to open the discussion so any child could make comments or ask questions before moving on in the discussion.

This study offers a much more detailed picture of how discourse patterns established in the classroom specifically affect how children construct mathematical knowledge in that classroom. The findings from Wood et al. (2006) show that those interaction patterns that required greater involvement from the students were related to higher levels of expressed mathematical thinking by children. Children in the conventional textbook classroom culture were most often engaged in recalling previously taught information. Although the goal of the conventional problem-solving classroom was to create an open discussion of students' solutions and to engage them in problem-solving, the teacher's efforts to make the problem understandable unintentionally created a situation in which the mathematical challenge of the problem was removed, taking away the opportunity for students to engage in higher-level thinking.

The most important results of Wood et al. (2006) were the findings that revealed the nature of the differences that existed between reform-oriented classroom cultures. The strategy reporting classroom culture established patterns of interaction between the student explaining and the teacher agreeing or disagreeing, and thus eliminated the opportunity for the entire class to collaborate in justifying mathematical ideas. Only in the inquiry/argument classroom culture were there opportunities for all children to be involved in meaning making. Children's thinking was extended to include whether a method or result is reasonable, connecting ideas for making judgments, and identifying flaws in mathematical arguments, and strengthening arguments by considering the mathematics from a different perspective.

The results of this study are based on an empirical analysis of a small sample of reform classes selected from one specific approach to reform. Although there is a possibility that the results only represent processes found in the classes of this study, the results could be corroborated by studying other classes both reform and conventional. Other questions remain about how race, socioeconomic status, and gender factors influence discourse patterns in various classroom cultures. More research is needed to address these important areas.

Kazemi and Stipek (2001) analyzed video taped mathematics lessons of 4 low-income 4th and 5th grade classrooms from 3 schools. Each classroom was described as having a reform-minded instructional orientation. These researchers analyzed individual the same lesson on the addition of fractions, to find instructional practices that pressed students for conceptual thinking to a high or low degree. A coded rating was created for each lesson based on two composite variables. These two composite variables were “Press for Learning”, which meant instruction that: a. emphasized student effort, b. focused on learning and understanding, c. supported students autonomy and d deemphasized student’s getting answers right, and “Positive Affect” which meant the teacher: a. displayed a positive demeanor toward students, b. displayed enthusiasm and interest in mathematics and c. fostered a non threatening environment. This study did not have a design for measuring gains in student conceptual understanding, but relied instead on the quantitative findings reported in Stipek, Salmon, et al. (1998) which showed a significant positive correlation between the degree of press in the observed lessons and growth in student’s conceptual understanding of factions ($r = .51$, $p < .05$). The researchers selected two lessons that scored the highest in press (Ms. Carter, 4.4 of a 5 point scale; Ms. Martin, 4.63), and two lessons that were lower in press (Ms. Andrew and Ms. Reed, both equaling 3.31). By comparing the high cases with the cases that were not coded lowest in press, these researchers were able to capture differences among classrooms that appeared to be very similar. In other words, all study classrooms shared similar components of instruction such as students explaining their thinking, discussing different strategies, and working in small groups. These four lessons all rated high on the positive affect scale, eliminating the possible confound of a differing motivational environment (Ms. Carter, 4.00; Ms. Martin, 4.33; Ms. Andrew, 4.25; Ms. Reed, 4.17).

Kazemi and Stipek (2001) reported finding that classroom practices, which create a high press for conceptual understanding, consist of four key sociomathematical norms. First these are classrooms where an explanation consists of a mathematical argument, not simply a procedural description. In high press situations where students demonstrated explaining with a mathematical argument, they linked their problem-solving strategies to mathematical reasons. This was an expected aspect of whole class discussion; students were required to present their solution strategies with both explanations and justifications. In low-press examples, students were seen giving descriptions or summaries of steps to solve a problem.

These classrooms showed that mathematical thinking involves understanding relations among multiple strategies. Researchers noted that in all classrooms students were expected to share strategies and the teachers who created a high press for conceptual understanding engaged students in conversations that examined the mathematical similarities and differences among multiple strategies. In low-press exchanges, strategies were offered one after the other, with discussion limited to nonmathematical aspects of student work.

In high-press classrooms, errors provide opportunities to reconceptualize a problem, explore contradictions, and pursue alternative strategies. Although both high- and low-press classrooms viewed mistakes as a normal part of the lesson and learning, only in the high-press exchanges did teachers press students to critically analyze their strategies and solutions, conveying clearly that the goal was to understand mathematical concepts. In the low-press cases teacher s precluded further mathematical inquiry by giving the answer themselves.

The final difference Kazemi and Stipek (2001) found between the sociomathematical norms of high- and low-press classrooms was that collaborative work involves individual accountability and reaching consensus through mathematical argumentation. In all four observed lessons, students worked in groups of two to four for most of the instructional time, as recommended by the shared curriculum. Students in high-press classrooms were observed communicating using mathematical language, describing and defending their differing mathematical interpretations and solutions. These students held each other accountable for thinking through the mathematics involved in a problem until consensus was reached through argumentation. In low-press classrooms students were seen working together and agreeing on a solution without debating the mathematics involved, with members of the group often deferring to a student perceived to be the most skilled.

These researchers were able to demonstrate how key sociomathatical norms were enacted in classroom discourse, and how classrooms with the appearance of reform-oriented instruction failed to produce conceptual understanding in their students. The differences among the high- and low-press classrooms provide evidence for a superficial treatment of reform-oriented instructional practices. This data suggests and supports other findings that in order for instructional practices to develop a high level of cognitive demand, teachers must do more than merely implement social norms, but they must pay close attention to sociomathematical norms and how they support each student's conceptual thinking.

Kazemi and Stipek (2001) give clear data about what it means for these sociomathematical norms to be in place during classroom conversations. This study's sample size is small, but I don't think sample size limits this study's findings. The study does not demonstrate how these norms are established. In order to understand how sociomathematical norms are established in classrooms I turn to Mclain (2001).

McClain (2001) conducted a classroom based designed teaching experiment in which the research team worked collaboratively with a third grade teacher, to plan classroom sessions, and analyze the subsequent classroom events. They analyzed this data for the purpose of determining critical attributes of the teacher's proactive role in the establishment of classroom sociomathematical norms (normative aspects of classroom actions and interactions that are specifically mathematical). The researchers developed instructional activities for mental computation and estimation with numbers up to 100. This experiment took place over the course of one school year. The researchers gathered data in the form of videotape recordings from two cameras of 103 mathematical lessons, copies of student's written work, three sets of daily field notes that summarize classroom events, notes from the daily and weekly debriefing and planning sessions in which the teacher participated and videotaped clinical interviews conducted with each student in September, December, January, and May. The researchers were only concerned with the establishment of the classroom's sociomathematical norms so they analyzed data from just the first four months of the school year. Results of student learning, which are reported out in Cobb et al. (1997), showed that these students made significant progress in their mathematical proficiency, and as a result the data collected in the course of this teaching experiment constitute an appropriate case in which to investigate the process of supporting student's mathematical development.

McClain et al. (2001) reported that the teacher's role in symbolizing students' offered solutions was a significant factor in the development of sociomathematical norms. At the beginning of the year, the teacher would use the chalk board to make a representation of the solution strategy offered by a student. This action directly affected many aspects of how students came to participate in argumentation. It gave the students a base from which to refer to as they began to establish the sociomathematical goal of what is an acceptable explanation. This also contributed to the development of a shared understanding of different forms of solutions: sophisticated solutions, and easy, simple, or clear solutions. This helped form the sociomathematical norm of focusing discussions around the mathematical difference of one solution to another. Students were required to share a solution that was different from one already shared, and then work toward explaining the mathematical aspects of their differences. The representations on the chalk board assisted students begin to use prior strategies as a begging point to start explaining their current strategy.

From these norms, researchers determined that whole class discussions in which all students understood to be normal across the year included: (1) The students were expected to explain and justify their reasoning. (2) When a student's contribution was judged to be invalid in some way by the classroom community, the teacher explained that the student had acted appropriately by attempting to explain his or her thinking. (3) The students were expected to listen to and attempt to understand other's explanations. (4) The teacher often commented on or redescribed students contributions. (5) Students were expected to indicate nonunderstanding and if possible, to pose questions that would help clarify the issue.

This study is fascinating, it provides clear direction for teachers who wish to plan, and implement the sociomathematical norms of the classroom. Quantitative analysis of the developments in this study would have allowed for a more clear understanding of how these norms developed throughout the year. I now turn to a study that is interested in understanding how teachers can be successful in implementing these practices.

McCrone (2005) wanted to investigate how mathematical discussions developed in an elementary mathematics classroom and explore the teacher's role in that development. The investigation focused on one fifth-grade classroom in a school located in a suburban neighborhood with a small percentage of minorities. Most children came from homes primarily in the middle class.

Data was gathered through videotapes, audiotapes as well as written field notes on a daily basis for the first 6 months of school, after which occasional follow up visits occurred for one full week each remaining month of the school year. The researcher and teacher logged weekly entries in a shared reflective journal. Data analysis focused on development of discussions over the first 6 months of the school year, as well as the teacher's pedagogical choices and connections between the pedagogy and the nature of the discussions. The researcher and a colleague independently coded the data after it was broken into subsets, and then compared and contrasted those results to develop a reliable coding scheme. Frequency tables were established to more clearly demonstrate changes in the nature of discussions as the year progressed. Categories of these tables were Nature of contributions, nature of interactions, and level of response.

Results documented two parallel changes through the course of the year. One was that student participation moved from non-attentive listening to active listening and use of other's ideas to develop new conjectures. In September and October, out of 63 observable contributions, 24 were giving facts and 18 were describing steps in a process. Only 18 of the contributions during this period of time were sharing reasoning or justification, questioning others ideas or offering new observations. During the January observation students had contributed 63 times sharing reasoning or justification, questioning others ideas, or offering new observations. The parallel change was that of the teacher's role in the classroom, moving from modeling and explaining appropriate discursive responses, to the more facilitative role of redirecting and suggesting, as a means to encourage students to respond to each other.

McCrone (2005) found interesting differences in the discourse patterns from September to January. Although most students could explain steps they took to arrive at an answer in September, a couple of months passed before they began to include justifications as well as the solution process. In January, students were more likely to take intellectual risks by sharing unfinished thoughts and unchecked ideas. In addition, students would build on the ideas and observations of others.

McCrone documented the importance of shared responsibility within developing sociomathematical norms. Teachers and students together worked to develop ways of working with and communicating mathematical concepts and as a result the nature and level of discussion is enhanced. This study documented the changes that took place in one successful mathematics classroom, but further studies should document changes in both successful and less productive classrooms to gain a clearer picture of all the relationships that effect establishing sociomathematical norms.

Willams and Baxter (1996) conducted a qualitative case study, reviewing data from a middle school mathematics class whose teacher had previously been acknowledged as being successful in instituting discourse oriented teaching. These researchers sought to discover themes and patterns relevant to discursive elements, which contributed to or detracted from the success of the mathematics lessons.

The data reported on in this study was gathered as part of the QUASAR project. Project staff documented how classroom instruction occurred at each of the project's classrooms. Staff visited each school numerous times informally, and three times a year, made formal visits in which they would gather videotape lessons, observation journals, as well as interviews with teachers, students, and school personnel. Each formal visit spanned three consecutive days, and was conducted by two observers. One observer focused on the mathematics content of the lesson, and the other focused on the target students in the class. Observations were also informed by post observation interviews.

For this study, researchers analyzed the data of one teacher who taught sixth, seventh, and eighth grades. Her classroom was located in an economically disadvantaged section of a large urban city. The student population was very transient and only one-third of students who entered the sixth grade graduated from the eighth grade. The lessons reviewed were over a three year period, and a total of nine documentation visits. The researchers identified critical elements in each lesson as they pertained to three areas: (1) mathematical content; (2) The student's view of the lessons; and (3) the classroom environment.

In their analysis Williams and Baxter (1996) found that the teacher spent a great deal of time helping children understand her expectations for mathematics lessons. They called these efforts social scaffolding and described it happening when the teacher explained what kinds of questions the students should ask and when they are expected to ask them. The researchers also describe the teacher's efforts of eliciting student thinking. The most direct technique the teacher used to encourage students to discuss their thinking was to ask them to explain their thoughts to the class. When students had difficulty expressing their thinking the teacher used Socratic questioning to lead the student to their misunderstanding. Other findings revealed challenges of such a instructional environment. Some students' explanations suggested that they viewed the discourse as an end in itself rather than a means to an end. Some groups of students were observed discussing mathematics and when asked why an answer was chosen they didn't have any mathematical justification, and instead offered the reply that they had arrived at the answer as a group. Students also were observed to offer procedural and ritualized explanations when the teacher wanted them to explain how the answer was obtained.

Willams and Baxter's (1996) case study provided detailed description of one teacher's implementation of reform oriented instructional practices. Though the study focuses on only one class and one teacher, it adds one more piece to the puzzle of how to implement reform oriented instructional practices.

Stein, Grover and Henningsen (1996), and Henningsen and Stein (1997) looked at the nature of student's thinking processes in the classroom and how those processes were altered when teachers attempt to implement various problem types. They measure student learning in terms of the capacity for mathematical thinking and reasoning students are required to do for successful completion of the problem. Their examination of the effects of classroom instruction on student learning was framed by the concept of mathematical tasks. Student learning was measured in terms of the student's success at completing the mathematical task. A mathematical task is defined as a classroom activity that focuses students attention on a particular mathematical idea, the products that students are expected to produce, the operations that students are expected to use to generate those products, and the resources available to students while they are generating the products. These researchers viewed mathematical tasks through two categories. The first was the features of the task. Task features included the number of possible solution strategies, the number and kind of potential representations that could be used to solve the problem, and the communication requirements of the task (i.e., the extent to which students were required to explain their reasoning and/or justify their answers). The other category was the level of cognitive demands the task required for children to complete the task. The cognitive demands were classified with respect to four levels, starting with the least demanding and progressing to the most demanding. The levels are: (A) memorization, (B) the use of formulas, algorithms, or procedures without connection to concepts, understanding, or meaning; (C) the use of formulas, algorithms, or procedures with connection to concepts, understanding, or meaning; and (D) cognitive activity that can be characterized as "doing mathematics," including complex mathematical thinking and reasoning activities such as making and testing conjectures, framing problems, looking for patterns, and so on. These researchers described how a task can be viewed as passing through three phases: the first stage is the planning of curricular or instructional materials; the second stage happens when the teacher explains to students about the task, setting up the activity in the classroom; and the third stage happens when the task is implemented by students as they progress through the lesson. Cognitive demands of a task may be different during each of these phases and can be potentially heightened or lowered between any two successive phases. Factors that influence such transformations are classroom norms, task condition, teacher instructional habits, and student learning habits and disposition.

Stein et al. (1996) analyzed a stratified random sample of 144 mathematical tasks used during alternative mathematics instruction of 6th 7th and 8th grade children. The researchers found that mathematics tasks that produced high-level learning outcomes were ones in which students worked from innovative materials and/ or from teacher-developed materials, rather than from a textbook series; high-level tasks frequently required students to work in pairs or groups. The majority of these tasks encouraged the use of multiple-solution strategies, multiple representations, and required that they explain or justify how they arrived at their answers.

This study analyzed narrative summary classroom observations, coupled with video recordings of three teacher's mathematics classrooms. The data base consisted of 620 tasks, which were then used as the source of a random selection based on stratification dimensions of year, site, and teacher. The stratified random sample created a data base of 144 observed tasks which was an equal distribution of tasks across seasons (48 per season), across sites (36 per site) and across years (48 per year) of the study. These 144 tasks were organized the into four coded categories of task description, task set up, task implementation, and factors associated with decline or maintenance of high-level tasks.

Stein et al. (1996), sought to understand how the set-up and implementation of mathematics tasks worked to influence student capacity for mathematical thinking and reasoning to a high degree, and what factors were associated with task changes from the set-up phase to the implementation phase as well as ways in which high-level tasks declined. This study analyzed narrative summary classroom observations and organized the data into four coded categories of task description, task set up, task implementation, and factors associated with decline or maintenance of high-level tasks. The researcher’s found that mathematics tasks that produced high-level learning outcomes were ones in which students worked from innovative materials and/ or from teacher-developed materials than from a textbook series; tasks were likely to engage students with a reform-inspired topic like statistics, geometry, or algebra, and frequently had students working in pairs or groups. The majority of these tasks encouraged the use of multiple-solution strategies, multiple representations, and required that they explain or justify how they arrived at their answers. Finally, three quarters of the tasks were set up to demand that students engage in rather sophisticated mathematical thinking and reasoning.-either connecting procedures to underlying concepts and meaning or tackling complex mathematical problems in novel ways. The results also described that the level of cognitive demand consistently declined during implementation phase and the higher the cognitive demands of the tasks at the set-up phase, the lower the percentage of tasks that actually remained that way during implementation.

The sample mathematics tasks (144) used in this study was chosen from a larger body of tasks (620). The methods these researchers used to select those tasks builds strength in the validity of the findings because of the stratification variables (site, year, and teacher) and the random selection.

Building on the work of Stein et al. (1996) Henningsen & Stein (1997) examined tasks which were set up to engage students in mathematical thinking and reasoning at the level of doing mathematics, with a desire to learn what were the classroom based factors which allowed for students to remain at that high-level of mathematical thinking and reasoning throughout the duration of the task. This investigation could offer important insight into what attributes of a mathematics classroom support reasoning and high level thinking (two elements of my definition of mathematical proficiency).

Henningsen & Stein used the data set from their earlier investigation (Stein et al. 1996), and selected the 58 tasks from the initial study's 144 tasks, that were identified as being set up to encourage thinking and reasoning at the doing mathematics level. During the implementation stage of these 58 task, students were observed to engage actively in doing mathematics in 22 of the tasks. In the other 36 tasks students' engagement with the task during implementation was not observed to characterize doing mathematics and those tasks declined into a number of various lower levels of cognitive demand. This data set allowed Henningsen & Stein to examine the factors associated with maintenance or decline of the doing-mathematics tasks.

Hennignsen and Stein (1997) categorized all the ways that tasks declined or maintained a high level of cognitive demand. The number of tasks for which each factor was judged to be an influence was calculated. From this information frequency graphs were constructed in order to be able to identify the sets of factors judged to be predominant influences in the largest percentage of tasks within each pattern.

Their results found that the factors influential in assisting students to engage at high levels of mathematical reasoning are: (A) High level tasks must be appropriate for the particular group of students, building on students' prior knowledge (factor found in 82% of data set); and (B) supportive action from teachers must include, giving an appropriate amount of time(77%), scaffolding students in their understanding(73%), and consistently pressing students to provide meaningful explanations or make meaningful connections(77%).

In this study, the patterns of student engagement and profiles of influential factors, provide a useful framework within which qualitative portraits were interpreted. Student engagement patterns and the factor profiles had qualities of generalizability because they had been suggested by patterns of findings from Stein et al. (1996).

Wood (1999) investigated what classroom contexts lend themselves to engaging children in the resolution of disagreements through a process of argumentation. In this study the researchers described the chronology of major instructional events, which composed the general social norms of the classroom.

These major events consisted of a brief orientation to the lesson (2 min), children working in pairs, using their heuristic procedures and varied strategies to solve problem-centered instructional activities (20 min); followed by class discussions, where children were involved in the mathematical activities of explaining, justifying, and confirming their mathematical thinking (20 min). It is in the context of the whole class discussion that children engaged in argument. This investigation is a report of what happened after pair work and during the whole class discussions.

This was an investigation of classroom international patterns and was situated in a data resource of 50 videotaped lessons of one teacher collected over a period of 18 months in the teacher's second-grade mathematics classroom. The data were analyzed in two sets. The first set comprised lessons that occurred during the first 4 weeks of a school year and twice monthly thereafter. They were used primarily to determine how the teacher initiated and established the social norms for interactions that occurred during disagreements. The other set consisted of lessons from the second half of the previous school year; lessons in which instances of confusion or disagreement arose during class discussion. These lessons were used primarily to analyze the interplay between the teacher and students during these discussions.

From the analysis of the second set, the lessons that occurred during the second half of the year, the researchers found the most prevalent context for argument consisted of a pattern of interaction and discourse that always involved a challenge. A challenge was defined as a statement or question of disagreement about the explanation given. The general pattern of all incidents involving argument was as follows:

1. A child provided an explanation of her or his solution to the problem.
2. A challenge was issued from a listener who disagreed with the solution presented. The challenger might or might not tell why he or she disagreed.
3. The explainer offered a justification for her or his explanation.
4. At this point, the challenger might accept the explanation or might continue to disagree by offering a further explanation or rationale for his or her position.
5. The explainer continued to offer further justification for her or his solution.
6. This process continued and other listeners sometimes contributed in an attempt to resolve the contradiction.
7. The exchange continued until the members of the class (including the

teacher) were satisfied that the disagreement was resolved.

This was the general pattern researchers found to be how disagreements in thinking arose and were resolved in this class. Notice the active participation of the listeners. Clearly listeners were expected to examine and evaluate the reasoning of others and to participate in the resolution of disagreements and the constitution of shared meanings.

Wood (1999) also analyzed the first set of lessons from the beginning of the school year and reported how the teacher created this particular context with the students. Wood noted that the teacher began the school year by working to establish with her students a network of shared understandings and mutual expectations for their behaviors as participants in a mathematics class that focused on disagreements. One of her central points in the beginning of the year was the distinction between criticism that was personal and criticism that was about mathematical ideas. The teacher worked extensively to clarify for the children what was expected from them in such situations. She explicitly stated these expectations as norm statements and then elaborated on the expectations by creating circumstances in the opening discussions to provide an opportunity for the children to experience how to participate in the expected ways. The emphasis at this stage was on the students' role as listeners. She modeled two key terms students should use, agree and disagree, and how to use them in different situations. Once the children began to freely agree and disagree she began to extend her expectations for how the children should interact and talk with one another. She reinforced her expectations that each student should listen to an explanation because she expected the listeners to ask questions of the explainer. She modeled the types of questions students might ask. The following segment illustrates these expectations.

1. Tch: Now, before Russ comes up and explains what he did, I want to explain something to the rest of the class. Your job is to be listening to what he's saying and trying to decide if you have a question, so that you can ask it. You might think, "I'm not sure of what you're saying?" or "I'm not sure how you did it?" or "You didn't count the way I thought you should, " or "could you do it one more time?" okay? Come on up, Russ. Remember this is not talk time. This is "listen to Russ" time.

Not all of these modeled statements were questions. "you didn't count the way I thought you should" is not a question by instead a challenge. Thus, the teacher provided an explicit illustration of the way to contest an explanation. These questions modeled by the teacher represent a context in which students actively question in order to understand better, as well as make known an explicit challenge to the reasoning used to solve problems.

Wood (1999) reported that these patterns of discourse were continually supported by the teacher. Wood noted that the teacher's normative comments during this time were most frequently directed to the children as listeners and were often encapsulated by the key phrase "active listener". The teacher's use of explicit normative comments decreased and almost disappeared by mid-October, and her overt maintenance of children's participation in the interaction during the discussion also declined. By the second half of the year, a context for argument had been established.

This section looked at groups of studies that all sought to understand how qualitative differences in reform oriented instructional practices create and support student mathematical proficiency. These studies have provided empirical evidence that merely invoking the social norms of reform oriented instruction is not sufficient to generating valid opportunities for students to develop mathematical proficiency.

Student Disposition Toward Mathematics

In this section I review research studies that examine the effects of instruction, curriculum upon mathematical proficiency that pertains to the disposition students have toward the discipline. This topic is approached from many angles. Turner et al. (1998) measured teacher discourse patterns and related this data to their students corresponding self-reports of intrinsic motivation as defined by student involvement. Turner et al. (1998) used the lens of students' involvement as a measure of a student's effort and persistence in accomplishing a set of activity-related processes. They measure involvement as the perception of students during mathematics classes that is characterized by focused concentration, attention, and deep comprehension, as well as positive affect, goal clarity, and intrinsic motivation. Stipek et al. (1998) first examined continuity in motivation across two time periods, the beginning of the year and later in the year, as to assess how motivation orientations changed as a result of their current teacher's classroom. Second, they made associations between students' motivation and their fractions learning, investigating if motivation has any relationship to learning outcomes. Third, they measured how qualitatively deferential instructional practices affected student motivation related to learning of fractions. Nicholls et al. (1990) and Turner et al. (2002) both framed their research within the lens of the classroom goal structure. These researchers looked at how instructional practices and teachers attitudes influenced students' beliefs about the reasons for engaging in achievement behaviors. This influence was characterized as the Classroom Goal Structure. Nicholls et al. (1990)

reported the effects of two different instructional environments on students' motivation to learn mathematics. These researchers employed the terms Task Orientation and Ego Orientation when referring to the classroom goal structure. Task orientation was characterized by a student-referenced definition of success as the gaining of insight or skill or accomplishing something that is personally challenging. Students were deemed Task Oriented when they felt pleased when working hard, learning, and understanding, and when they expressed beliefs that to do well in mathematics one must work hard and try to make sense of things. Ego Orientation was characterized by a view that to experience success in mathematics students must establish his or her ability as superior to that of others. Turner et al. (2002) examined the relation between mathematics instruction and curriculum and students' perceptions of the goal structure in the classroom and their use of avoidance strategies. Turner et al. (2002) thought classroom goal structures exist on a spectrum between two poles. On one end is a performance goal structure, where students' reasons for engaging in academic behavior is to demonstrate ability and outperform others. The other end is a Mastery goal structure where the reason for engaging in achievement behavior is for understanding, intellectual development, and improvement. Turner et al. (2003) investigated teacher discourse patterns in two high-mastery /

high-performance classrooms and related them to patterns in students' achievement-related affect (negative affect following failure), approach (self-regulated learning and positive coping) and avoidance behaviors (self-handicapping, or purposefully withdrawing effort).

Stipek et al. (1998) conducted a quantitative study in which classroom instructional practices were correlated with student's achievement motivation in order to determine to what degree instructional practice influenced achievement motivation and conceptual understanding. Stipek et al. (1998) first examined continuity in motivation across two time periods, the beginning of the year and later in the year, as to assess how motivation orientations changed as a result of their current teacher's classroom. Second, they made associations between students' motivation and their fractions learning, investigating if motivation has any relationship to learning outcomes. Third, they measured how qualitatively deferential instructional practices affected student motivation related to learning of fractions.

The year-long study consisted of 24 teachers and their 624 fourth-through sixth-grade students from large, ethnically diverse (over half the students were Latino), urban areas that were predominantly low-income. Teachers were divided into three groups. Two groups of teachers expressed commitment to reform oriented mathematics instruction and taught a fractions unit from Seeing Fractions, a curriculum developed to be consistent with NCTM standards. One of these groups of teachers were a part of a year-long intervention designed to assist them in their efforts to implement instructional reforms. Another group of teachers taught a fractions unit in a more traditional style with textbooks and expressed no interest in reform-oriented practices. Teachers' classroom practices were coded on nine dimensions on the basis of analysis of videotapes and field notes of instructional periods. The nine coded areas were motivational influences in the instructional practices and included items that measured the degree to which teachers emphasized things like, effort produces results, or performance (getting answers right or wrong), or encouraged students to focus on learning, understanding and mastery, or the degree to which they made comments about students relative performance. Researchers coded periods of whole class instruction separately from times when students where working independently or in groups, to account for possible changes in teacher practice based on unique teacher actions in those contexts- data showed that teacher actions where consistent in all dimensions with the exception of social comparisons made. Pre- and post-tests of fractions understanding and motivational level were administered at the start of the school year and at the end of the fractions unit. Student's attitudes and motivation were also measured through videotape encoding. This combined classroom practice data was correlated with changes in scores in both fraction understanding and motivation assessment areas from pre- to post-tests.

Regression analysis was conducted to assess the degree to which teacher practices predicted student behavior, achievement, and motivation. Stipek et al. (1998) reported that the degree to which teachers conveyed that mistakes are okay, and provided scaffolding for a child having difficulty, the more students were focused on learning and understanding, the more they reported that they sought help when having difficulty, and the more they felt positively while learning about fractions. The same result occurred when teacher actions encouraged effort and autonomy and focused student's attention on learning and mastery. For conceptual items, the average gain from pre- to post- test for students with reform-oriented teachers was 4.25 (out of the 13 conceptual items on the tests) and 2.39 for students with traditionally-oriented teachers. The more teachers emphasized the number wrong on papers, the less students claimed to enjoy working on fractions (r = -.42, p< .08), and the more teachers put check marks on papers to indicate completeness, the less mastery-oriented students claimed to be (r = -.49, p< .05) and the less they enjoyed fractions (r =

-.51, $p < .05$). In contrast, substantive written comments on papers were positively correlated to higher self-ratings of ability ($r = .42$, $p < .09$), mastery orientation ($r = .67$, $p < .05$) and positive emotions ($r = .42$, $p < .05$). Teacher practices that were significantly associated with gains on conceptually-oriented items were characterized by the teacher staying with one child for a substantial length of time in an effort to get a clear explanation or help provide an alternative solution. Also, these teachers encouraged students to keep working or thinking about a problem, gave them instrumental help that facilitated their progress, allowed plenty of time for students to complete their work, required students to go back and try again when they had reached inadequate solutions, or encouraged them to come up with multiple strategies. These teachers neither embarrassed students nor ignored wrong answers in whole-class instruction. Rather, they used students' inadequate solutions and mistakes to enhance the instruction. They commented on the problem-solving process or the strategies students were employing, often making reference to the particular mathematical concepts that students were learning, and they held students to high standards, asking them to explain their thinking in writing as well as verbally.

The sample's demographic and size are two strengths of this research. The students are in sixth grade, and these findings may be more generalizable to middle to high school age children.

Nicholls et al. (1990) sought to assess how instructional practices influenced students' disposition toward the discipline of mathematics. These researchers used 3 questionnaires to compare the effect of two different instructional environments by measuring the motivational orientation of 102 second grade students, from six classrooms at one elementary school. Students were from lower, middle, and upper-middle class homes. The questionnaires measured two dimensions of students' motivation orientation, Task Orientation or Ego Orientation. Task orientation was used to refer to students who had a positive disposition, and was characterized by a student-referenced definition of success as the gaining of insight or skill or accomplishing something that is personally challenging. Students who had a less productive disposition toward mathematics were said have an Ego Orientation. This orientation was characterized by a view that to experience success in mathematics students must establish his or her ability as superior to that of others.

Students were randomly assigned to the six classes at the beginning of the school year. Two different instructional environments were represented by the six classes. Five control classes were described as traditional instructional practices, and 1 treatment class was described as being marked by an atmosphere of dialogue and collaborative problem solving. Patterns of actively in the treatment class consisted of students being introduced to problems, and then working cooperatively in pairs to complete them, followed by teacher-orchestrated whole-class discussions of children's solutions. The teacher facilitated dialogue among different students and did not try to steer students to preconceived solutions. Frequently, students offered two or more conflicting solutions, and the teacher framed the situation as a problem for the students to resolve by justifying and explaining their solutions.

Questionnaires were administered to intact classes in sessions of about 25 minutes during the third-to-last week of the school year. Questionnaires were administered in a comparable way to opinion polling or voting, where individual's identities are not revealed but their positions are counted. Students were encouraged to express their own unique and personal beliefs. Questionnaires were segmented into different scales; each scale consisted of statements that reflected either a Task orientation or Ego orientation. A five point likart scale was used to measure students' responses to statements like: "I solve a problem by working hard", Or "I finish before my friends."

When researchers compared the target class to the others, it was shown that the target class was significantly higher than the others on Task Orientation

($p < .01$).and lower on Ego Orientation ($p < .05$). Only one the treatment classes were remotely similar to the target class.

Nicholls et al. (1990) provided clear qualitative data of the instructional practices that are related to students' disposition toward mathematics. This study would have been benefited by including two reform oriented groups. With two reform oriented groups, qualitative difference in the same instructional social norms would have provided even more of an illuminating picture of the practices that effect students' disposition toward mathematics.

When students are uncertain about their ability to achieve competitively they may purposely withdraw effort as way to protect themselves (Covington & Omelich, 1979). This avoidance behavior is called self-handicapping, and is a way students use to deflect attention away from low ability. Working hard can put self-worth at risk because trying hard and failing to do as well as others is compelling evidence of low ability. By not trying, the cause of failure becomes uncertain and the student is able to stave off the public judgment of low ability behaviors to protect them selves. Turner et al. (2002) investigated how teacher discourse related to both students' perceptions of the classroom goal structure, and their use of avoidance strategies.

Turner et al. (2002) administered surveys to 1,092 sixth-grade elementary school students (52% female, 48% male, 65 teachers who mostly taught math and other core subjects in self-contained classrooms) in four ethnically and economically diverse school districts in three Midwestern states. Classroom observational data was obtained from 9 classrooms in 9 different schools in one of these school districts. Students were read survey questions by trained research assistants. The survey included two scales, which assessed student,'s reports of the use of avoidance strategies in the classroom including: self-handicapping and the avoidance of help seeking. One scale that assessed students' preference to avoid novel approaches to doing academic work and two scales that assessed students' perceptions of the mastery and performance goal structure in their sixth-grade classrooms.

Mathematics instruction was observed and audio taped during the same two units of instruction in each of the 9 classrooms, and classroom observations were conducted during two periods of 5 days during the fall on a unit of factoring and in the spring for a unit of geometry. Observers recorded classroom discourse with audiotape and made notes to provide context for the recordings so that the intent or consequences of a teacher statement or action could be discerned. Teacher discourse was coded into three categories: instructional, organizational and motivational discourse.

Turner et al. (2002) reported that when discourse percentages were analyzed for the 9 observed classrooms, across all coding categories, 52-68% of the discourse fell within the instructional discourse category, 20%-30% fell with the organizational category and 10%-28% fell within the motivational category. When students' perceptions of the mastery goal structure were compared with their reports of avoidance strategies across all classrooms, two classrooms were found to be high-avoidance and low mastery (Ms. Parsons, Ms. Anderson), and two classrooms were found to be low-avoidance, and high mastery (Ms. Robinson, Ms. Davis). Turner et al. described the discourse patterns in the two low-avoidance/high mastery classrooms as being characterized by instructional discourse that was directed at focusing student learning on understanding procedures and concepts rather than asking students to provide "correct" answers. Interestingly, these instructional discourses combined a focus on instructional scaffolding (negotiation and transfer of responsibility) with opportunities to clarify, review, and summarize important concepts (non-scaffolding). Nineteen percent of this discourse was non-scaffolding sequences of right answer questions at particular points in the lesson to ensure that students had the basic understandings necessary to learn new and more complex concepts. These known-answer sequences were used to review at the beginning of a lesson, to reinforce a point during a lesson, or as a summary at the end. Only 13 % of instructional discourse required students to explain or evaluate their answers (transfer of responsibility). Motivational discourse was said to be a hallmark of these classrooms' instructional practices. This discourse was described as the teacher frequently emphasizing student could learn and that they expected all students to contribute to the learning.

The two high-avoidance/ low-mastery classrooms were characterized by a pattern of cognitive support and high demands but low motivational support. The main features of these instructional discourse patterns were for students to explain, defend or evaluate mathematical ideas. The researchers commented that the time these classes spent building understanding (negotiation, 26%) and checking factual knowledge (non-scaffolding, 13%) was low in comparison with the high level of accountability in the discourse. The low use of motivational support (8%) indicated that these teachers did not provide frequent encouragement or reassurance to their classes that they were learning.

These findings are interesting because they show qualitative differences between classrooms that share the same social norms. Much like Kazemi (2001), these findings point to the sociomathematical norms that influence students' learning outcomes. Interestingly this study did not include a scale for student perception of motivational support. Had the study included one it might have offered triangulation of the seemingly important findings that motivational support played in this study.

Turner et al. (2003) investigated how students in two high-mastery/high-performance classrooms differ in their reports of achievement-related affect and approach and avoidance behaviors. With this information they wanted to understand how teacher discourse patterns related to patterns in students' achievement related affect and approach and avoidance behaviors in the two high-mastery/high-performance classrooms.

This study's sample was part of a larger longitudinal study of nine sixth-grade classrooms. This study focused on two of these nine classrooms in two schools in a Midwestern urban school district. The teachers were Ms. Robinson who had 18 students complete surveys (55% female, 44% African American, 44% Euro-American, 6% Hispanic, 6% Asian) and Ms. Clarke who had 16 students complete surveys (56% female, 79% Euro-American, 21% African-American). Researchers audio recorded teacher discourse during the first 2 days of school and for 10 days during both the fall and spring semesters. Observer notes were also taken as a means to provide nonverbal information so that the intent or consequences of a teacher response could be discerned.

Teacher discourse was coded into three categories: (a) instructional discourse that focused on student understanding and autonomy; (b) motivational discourse that focused on student effort, affect, and collaboration; and (c) organizational discourse that focused on management and procedures. During the later two observations students completed surveys which measured students' perceptions of the mastery and performance goal structure, students' self-handicapping, academic self-regulation, positive coping, and negative affect for failure in mathematics.

Turner et al. (2003) reported that the analysis of students' response surveys showed that students in Ms. Robinson's and Ms. Clark's classrooms did not differ significantly in their reports of self-regulation or positive coping. However, they differed in their reports of negative affect about failure, and use of self-handicapping. Students in Ms. Clark's class averaged 50% more reports of negative affect and self-handicapping after failure than did students in Ms. Robinson's class ($p < .01$). The data on instructional discourse patterns revealed that both teachers' instructional discourse appeared to reflect high support for student understanding and autonomy (Clark 44%, Robinson 47%). However, the quality of their instructional support differed significantly. Ms. Clark's responses supported student understanding 36% of the time, and only supported student autonomy 8%. In contrast, 28% of Ms. Robinson's responses supported understanding and 19% were supportive of autonomy. These teachers appeared to differentially involve students in demonstrating their understanding. Whereas Ms. Clark's discourse reflected more time spent on teacher modeling and demonstration, Ms. Robinson's discourse revealed that she was more likely to ask students to demonstrate a strategy or to evaluate an answer. The two teachers also had differences in their motivational discourse. Ms. Robinson displayed a positive pattern of motivational discourse throughout 21% of her instructional responses, and only 1% of them were categorized as non-supportive. Ms. Clark's motivational discourse reflected somewhat fewer incidences of support (11%) and higher proportions of nonsupport (5%). Within the non-supportive subcategory, Ms. Clark demonstrated almost three times more instances of negative effect ($p< .01$).

Turner et al. (2003) concluded that Ms. Clark's student's higher report of negative affect about failure, and use of self-handicapping may have been related to her relatively frequent use of non-supportive motivational discourse, especially during lessons in which students struggled. Ms. Clark appeared more prone to missing opportunities for students to demonstrate their learning. Such opportunities Turner et al. explained, might have mitigated against the need to self-handicap and to feel negative affect after failure, by providing evidence of developing competence and the experience of taking risks, making mistakes, trying again, and succeeding.

On the other hand, it was reported that Ms. Robinson rarely missed an opportunity to encourage, engage, or compliment her students on whet they were learning. Ms. Robinson's students didn't appeared to be worried about making mistakes because she consistently told them that it is okay to look dumb in class, and focused instead on making new understanding. Ms. Robinson's motivational discourse coupled with support for student autonomy appeared to sustain both mastery goals and in a complementary fashion, the ability of students to strive for competence without putting their self-worth on the line.

The mastery goal is the goal to develop ability. Students with mastery goals focus on the intrinsic value of learning and believe that effort will lead to success. This study focused on classroom goal structures, and how students perceived the classroom goals and their development throughout the school year.

Turner et al. (1998) also examined teacher's discourse patterns and interpreted them in light of student's self-reports of involvement in mathematics classes. Again, Turner et al. defined involvement as the perception of students during mathematics classes that is characterized by focused concentration, attention, and deep comprehension, as well as positive affect, goal clarity, and intrinsic motivation.

Turner et al. (1998) gathered data for 42 students and their fifth-and sixth-grade teachers. Students were in seven classrooms in three elementary schools in a small, mostly white, middle-class town in rural Pennsylvania. Of the seven classrooms, two were high-ability classes, two average-ability classes, and two low-ability classes, as well as a heterogeneous group. Posttest scores from the California Achievement Test (CAT) total battery Normal Curve Equivalent, were 83.9 and 93.8 for the high-ability classes, 87.5 and 79.3 for the average-ability classes, 59.0 and 60.8 for the low-ability classes, and 71.4 for the heterogeneous class. Therefore, four of the classes were above average and two were average-ability classes by national norms. Turner et al. randomly selected 6 student participants by gender from each class making a sample of 21 boys and 21 girls. The research team audio taped classroom discourse during regular mathematics instruction for 5 day periods. In addition, they used classroom observations to complete a Classroom Observation Instrument which provided descriptions of teacher responses which were categorized as being one of six discourse categories: (1) negotiation, adjusting instruction, guiding students to deeper understanding; (2) Transfer of responsibility, supporting development of strategic thinking, of autonomous learning, and holding students accountable for understanding; (3) Intrinsic support, viewing challenge as desirable, advocating risk taking, responding positively to errors, and commenting on progress; (4) Initiation-Response-Evaluation, the routine asking of known-answer question or evaluating a student response as right or wrong; (5) Procedures, giving directions and implementing procedures, or telling students how to act and think; and (6) Extrinsic support, using superficial responses, to encourage students to focus on positive aspects other than learning, or using threats or negative expectations to gain student compliance. Categories 1 through 3 represent elements of scaffolding.

The 42 student informants filled out response logs for each of the study days. These individual logs measured students' perceptions of instruction with 13 semantic differential scales that forced choices between opposing feelings such as happy-sad and open-closed. Other examples of these semantic scales include "Alert-Sleepy, Cheerful-Crabby, Clear-Confused and involved-detached. Each of the scales was measured on a Likert scale from 0 (low) to 9 (high) with the midpoint of "neither". 1 item measured intrinsic motivation with the question "Do you wish you had been doing something else?". Turner et Al.

posited that the ratio between the perceived level of challenge in a task, and the student's perceived skill in accomplishing that task is the primary condition for optimal experience. Therefore, students also answered two questions-"how challenging was math class today?" and "How were your skills in math today?"-by circling a number for each question on a Likert scale from 0 (low) to 9 (high) to derive a measure of the challenge-skill match.

Using the student's self-reports of the match between challenge and skill, the data from the 13 semantic scales as well as their corresponding reports of intrinsic motivation, Turner et al. (1998) concluded that higher levels of involvement were found in the classrooms of Ms. Benjamin, Ms. Carey, and Ms. Adams. The level of each classroom's involvement was determined by comparing the classroom's mean scores for challenge and skill variables. They found that in the classrooms of three teachers (Ms. Benjamin7.4/7.6,; Ms. Carey,6.5/6.0; and Ms. Adams,5.8/5.1) the means for challenge and skill were high and closely matched. While 4 of the teachers (Ms. English, 4.5/6.3; Ms. Ford, 4.1/6.4; Ms. Grant, 3.6/7.9; Ms. Duncan, 2.8/4.9) had average ratings where student's skills exceeded the challenge means. Next, Turner et al. determined which classrooms among the three high-involvement ones did student's more likely report a close match between Challenge and Skill. Students responses were classified into four categories: (1) flow, both challenge and skill>0; (2) boredom, Challenge < 0 and skill >0; (3) apathy (both challenge and skill <0; and (4) anxiety (challenge >0 and skill <0. Scores were treated as situationally dependent such that multiple reports provided by the same student but on different days were considered to be independent of each other. Results of these frequency scores showed that students in Ms. Benjamin's class (27) were in the flow quadrant significantly more often than students in any of the other classes. Students in Ms. Carey’s class(14) were in the flow quadrant significantly more than students in the classes of Ms. Adams(11), Ms. Duncan(1), Ms. English(7), Ms. Ford(5), and Ms. Grant(12), but significantly less than students in Ms. Benjamin’s class(27). Therefore, students in two of the three high-involvement classrooms tended to report significantly more experiences of flow than students in other classrooms.

In the three classrooms rated as more involving by the students, the teacher’s instructional practices shared ratings of challenge that were high and matched with the students reported skills levels, and all rated high on the scaffolding categories. The teachers in high-involvement classrooms pressed the students for understanding and made more provisions for autonomy. Qualitative analyses of classroom discourse showed that scaffolding of classroom instruction through whole-class discussions helped to create a context for and support of student involvement.

Schweinle et al. (2006) measured student motivational belief perceptions of skill, challenge and affect within 7 different mathematics classes and then assessed the qualitative differences of these various instructional environments. The research sample demographic was described exactly the same as Turner et al. (2003). In this study 6 students were selected by gender from 7 fifth- and sixth-grade classrooms. Teachers selected a representative sample of high, average, and low-achieving students from each gender in their classes, for a total of 42 students. Students completed ESF forms on each of 4 days in fall and 4 days in the winter, within a 2-week period each season, resulting in a return of 336 response forms. This survey measured 4 categories of students' perceptions of the instructional environment: Social Affect, the amount of involvement or engagement that students experienced in the social context of the class; Personal Affect, how the individual student felt; Efficacy, students' perceived skills and abilities to perform in class; and Challenge/Importance, how challenging the mathematics tasks were and how important students perceived them to be. Teacher's instructional discourse was audio recorded on the same dates that the students completed ESF's. Observers took detailed notes of the lessons and teacher-student interactions.

Using the classroom observation data Schweinle et al. (2006) coded discourse into the following categories: (a) Affective and Social Support, where the teacher encouraged effort and persistence, alleviated frustration, encouraged cooperation and helped students enjoy mathematics; (b) encouraged autonomy by allowing and encouraging multiple solution strategies and minimizing external control, gave supportive feedback, and used evaluation; and (c) challenge, competence support which gave students opportunity to demonstrate skill, and task importance, where the teacher probed for explanation and justification. Schweinle et al. then examined how teacher discourse patterns might be related to students' perceptions of the classrooms as revealed in the mean factor scores for each classroom. They found specific patterns of instruction that explained the student motivational perception data.

After determining the mean factor scores for each of the 7 classrooms, Schweinle et al. (2006) reported below average levels of Challenge/Importance in two classrooms (Ms. Duncan, -.75 when the mean is 0, and Ms. Ford, -.05; $p < .01$), and above average levels of Challenge/Importance in two other classrooms (Mr. Benjamin, .25 when the mean is 0, and Ms. Grant, .75; $p < .01$). Ms. Duncan's class reported below-average Social (-.75, $p < .01$) and Personal Affect and

Efficacy (-.25, $p < .01$). Mr. Benjamin and Ms. Grant's classes reported above-average Social (Benjamin, .25; Grant, .75; $p < .01$) and Personal Affect (Benjamin, .25, not significant; Grant, .75, $p < .01$) and Efficacy (Benjamin, .25; Grant, .75; $p < .01$).

In classrooms where students reported below average levels of Challenge/Importance, teacher's instructional discourse focused on asking questions, students responding, and either teacher or fellow students evaluating the response. This discourse offered few requests for students to explain mathematical concepts. Most of the questions were for correct answers. When new content was presented these teachers guided their students through activities step-by-step, seeming to emphasize algorithmic approaches.

In classrooms where students reported high levels of Challenge/Importance the instructional discourse was characterized by teachers asking questions and students explaining multiple solution strategies. These teachers expected the students to explain their answers and compare current topics of discussion to previous topics. If a concept was not well understood, the class spent time discussing and developing new ways to present the information until the students demonstrated understanding. This pattern emphasized the importance of understanding mathematical concepts. These teacher's students also reported high levels of autonomy, and may have been a result of them allowing students to develop their own solution strategies and asking them to justify their results. Those practices of teacher support and encouragement of autonomy likely contributed to above average reports of efficacy, because students had many opportunities to demonstrate their competence.

The disposition students have toward the discipline of mathematics is a central part of mathematics achievement. When a student is mathematically proficient, they not only have procedural fluency with conceptual understanding, they see mathematics as sensible, useful, and worthwhile, coupled with the belief that through diligence and hard work they can be successful at learning mathematics. Research presented in this section showed that reform oriented instructional practices consistently produced more healthy dispositions toward mathematics. Instructional practices that were characterized by traditionalist goals were shown to produce more negative goals of students' disposition toward the discipline of mathematics.

Summary

This chapter examined much of the research that is relevant to understanding how instructional practices and curricula effect student mathematical achievement. Comparison studies of the learning outcomes of both traditionalist and reform oriented instructional patterns, revealed that students in the reform groups did no worse than traditional students on measures of procedural knowledge, while the traditional groups did significantly worse on measures of conceptual knowledge. Mathematics classrooms that build students conceptual understanding have been found to share key characteristics, but more importantly is the qualitative differences in instructional practices within these similar classroom environments that work to maintain conceptual development in the learning. Studies that measured the differences in instructional practices and their effects on students disposition toward mathematics as a discipline conclusively determined that when teaching in a manner that is consistent with the goals of reform oriented curricula and instruction; teaching for conceptual understanding and procedural fluency, students develop more healthy dispositions for learning mathematics. While teaching in a manner that is consistent with the goals of traditional curricula and instruction; teaching for memorization and computational accuracy, show negative patterns of students motivation toward the discipline of mathematics.

CHAPTER FOUR: CONCLUSION

Introduction

In Chapter One, my rationale was given for a critical analysis of the current research literature as it pertains to the effects of instructional practices, and curriculum upon student achievement. Chapter two discussed the history of mathematics education throughout our nation's history. Specifically tracing the origins of two opposing philosophies of how mathematics should be taught and learned in our public schools. Chapter Two also revealed the emergence of the mathematics educational research community, which developed simultaneously with the most current efforts at reforming mathematics education in our nations public schools. Chapter Three provided a review of the relevant research literature on the relationship between curricula, instructional practice, and their effects on student achievement. This chapter will summarize those findings, and investigate classroom implications of those findings.

Summary of Findings

This section will summarize the major research findings presented in Chapter Three. It will begin by outlining the general conclusions regarding curricula, instructional practices and their effects on student learning. Then, it will move into the how qualitative differences found among reform oriented classrooms deferentially impact student learning outcomes. Finally, I will summarize how these various forms of reform oriented mathematics instruction and curricula effects students perceived motivation, and disposition toward the discipline of mathematics.

Curricula, Instructional Practice and Student Learning Outcomes

In the first group of studies (Boaler,1998; Cramer et al.,2002; Moss & Case,1999; Reys et al.,2003; and Saxe,1988) it was shown that when curricula includes opportunities for children to develop procedural skill with understanding they are capable of doing so, and demonstrate conceptual understanding without sacrificing computational ability. Specifically, Cramer et al. (2003) found that children who were taught from a curriculum that emphasized looking at various relationships between mathematical ideas, representing those ideas in various forms and discussing those ideas among each other, were able to demonstrate procedural fluency, and computational understanding. These students out performed the control group on the computational ability test even when the control group spent much more time practicing algorithms and computations in a rule method fashion. Further more, it was this same control group who had difficulty transferring knowledge from a known method of computation to unfamiliar method, were the reform group had much better success. Boaler (1998) and Moss and Case (1999) found similar results. In fact, Boaler (1998) showed that the curriculum at Phoenix Park lead to a greater ability to work with a flexible knowledge, that the students at Amber Hill obviously lacked. These Amber Hill student's were exposed to a textbook based curriculum that first introduced students to mathematical procedures and then presented a range of questions for student practices. The advantage the Phoenix Park students received was from a curriculum that was open ended, had multiple points of entry

These studies demonstrate that the benefits of curriculum that is centered on developing mathematical ideas conceptually out weigh curriculum that approaches topic from a disconnected skill orientation.

In the second group of studies (Carpenter et al.,1989; Carpenter et al.,1996; Carpenter et al.,1998; Cobb et al.,1991; Hiebert&Wearne,1993; Hiebert&Wearne,1996; & Schoen et al.,2003) specific instructional practices were shown to influence the development of students conceptual understanding and procedural knowledge. Those practices included, presenting students with a variety of contextualized problem situations, children developing multiple solution strategies, representing quantities in multiple ways, sharing and discussing their solution strategies with the class, and teachers using knowledge of what children know to make decisions about mathematics instruction. Further more these elements were

Fennema et al. (1996) reported in their longitudinal study that when teachers instructional practices became more characteristic of these qualities, than their students made significantly higher gains on assessments for computation and understanding. Hiebert and Wearne (1996) found corroborating evidence in their 3 year longitudinal study. They found that when students had undergone 3 consecutive years of instructional practices with these characteristics than, those students performed significantly better on measures of computational fluency and conceptual understanding.

Taken all together, these studies show that both curricular and instructional practices that promote the development of conceptual understanding give students the benefit of learning mathematics with procedural accuracy, fluency, but with the added dimension of conceptual understanding.

Qualitative Differences Among Reform Oriented Classrooms

With regard to the qualitative differences among reform oriented classrooms, the majority of studies show that merely invoking the social norms of reform oriented instruction is not sufficient to generating valid opportunities for students to develop mathematical proficiency. In Wood et al.'s (2006) analysis of classroom interact ional patterns, two different classroom cultures were described as being reform oriented. The main difference between the two deferential forms was the support that the teacher gave to fostering students own mathematical ideas. The teacher did this by encouraging students to take over as questioners of other students reported strategies. This important shift was found to create higher levels of expressed mathematical thinking in students. The students in this classroom were expected to listen carefully to shared strategies and respond by identifying in a method was reasonable or if there were flaws in mathematical arguments, and strengthening arguments by considering the mathematics from different perspectives.

McClain et al. (2001) reported on the development of a successful reform oriented classroom. Important teacher actions which supported this development included symbolizing students offered solutions, setting and modeling norms for engagement in argumentation, and establishing what is valid as an acceptable explanation.

Simon (1995) reported that it is not just a mater of certain skills and methods a teacher must possess to develop a productive reform-oriented classroom, but the teacher must consistently make assessments of student's knowledge, plan a learning trajectory, and based on what happens in class reform the learning trajectory to consistently challenge students.

Henningsen and Stein (1997) added to this discussion showing that when a mathematical task engages students at high-levels of cognitive demand, the mathematical task is appropriate for that particular group of students, building on their prior knowledge; the teacher gives supportive actions like, giving appropriate amounts of time, scaffolding students in their understanding, and consistently pressing students to provide meaningful explanations or make meaningful connections.

Kazemi and Stipek (2001) further elucidated how teachers could move beyond just merely enacting the social norms of reform oriented instruction, to the cognitively productive and fruitful activities of sociomathematical norms. Four key elements were found to be required for such productive classroom interaction. The first is when in a classroom an explanation consists of a mathematical argument, not simply a procedural description. In high press situations where students demonstrated explaining with a mathematical argument, they linked their problem-solving strategies to mathematical reasons. This was an expected aspect of whole class discussion; students were required to present their solution strategies with both explanations and justifications. In low-press examples, students were seen giving descriptions or summaries of steps to solve a problem.

Second these classrooms showed that mathematical thinking involves understanding relations among multiple strategies. Researchers noted that in all classrooms students were expected to share strategies, and the teachers who created a high press for conceptual understanding engaged students in conversations that examined the mathematical similarities and differences among multiple strategies. In low-press exchanges, strategies were offered one after the other, with discussion limited to nonmathematical aspects of student work.

Next, In high-press classrooms, errors provide opportunities to reconceptualize a problem, explore contradictions, and pursue alternative strategies. Although both high- and low-press classrooms viewed mistakes as a normal part of the lesson and learning, only in the high-press exchanges did teachers press students to critically analyze their strategies and solutions, conveying clearly that the goal was to understand mathematical concepts. In the low-press cases teacher s precluded further mathematical inquiry by giving the answer themselves.

The final difference Kazemi and Stipek (2001) found between the sociomathematical norms of high-press classrooms and the mere social norms of low-press classrooms was that collaborative work involves individual accountability and reaching consensus through mathematical argumentation. In all four observed lessons, students worked in groups of two to four for most of the instructional time, as recommended by the shared curriculum. Students in high-press classrooms were observed communicating using mathematical language, describing and defending their differing mathematical interpretations and solutions. These students held each other accountable for thinking through the mathematics involved in a problem until consensus was reached through argumentation. In low-press classrooms students were seen working together and agreeing on a solution without debating the mathematics involved, with members of the group often deferring to a student perceived to be the most skilled.

Student' Disposition Toward Mathematics

In terms of students disposition toward the mathematics discipline and the influence of instructional practices, reform oriented instructional practices consistently produced more healthy dispositions toward mathematics. Stipek et al. (1998) reported that the degree to which teachers conveyed that mistakes are okay, provided scaffolding for children having difficulty, encouraged effort and autonomy and focused students' on learning and mastery, the more students were focused on learning and understanding, the more they reported that they sought help when having difficulty, and the more they felt positively while learning about fractions. These classrooms were characterized by the teacher staying with one child for a substantial length of time in an effort to get a clear explanation or help provide an alternative solution. Also, these teachers encouraged students to keep working or thinking about a problem, gave them instrumental help that facilitated their progress, allowed plenty of time for students to complete their work, required students to go back and try again when they had reached inadequate solutions, or encouraged them to come up with multiple strategies. These teachers neither embarrassed students nor ignored wrong answers in whole-class instruction. Rather, they used students' inadequate solutions and mistakes to enhance the instruction. They commented on the problem-solving process or the strategies students were employing, often making reference to the particular mathematical concepts that students were learning, and they held students to high standards, asking them to explain their thinking in writing as well as verbally.

When Turner et al. (2002) looked at the critical attributes of two classrooms that were found to be low-avoidance and high-mastery, they both shared instructional discourse that was directed toward focusing student learning on understanding procedures and concepts rather than asking students to provide “correct” answers.

Turner et al. (1998) discovered three classrooms where students reported being more involved in mathematics class, and those classrooms were characterized by instructional practices that pressed students for understanding, made more allowance for student autonomy, and providing scaffolding for student learning.

When Schweinle et al. (2006) analyzed the relationship between instructional patterns and students’ perceptions of motivation, they found that students reported below average levels of Challenge/Importance when the teacher’s discourse focused on asking questions which offered few requests for students to explain mathematical concepts. In classrooms where students reported high levels of challenge/importance the instructional discourse was characterized by teachers asking questions and students explaining their answers and comparing topics of discussion to previous topics.

Classroom Implications

“Students learn mathematics through the experiences that teachers provide. Thus, students’ understanding of mathematics, their ability to use it to solve problems, and their confidence in, and disposition toward mathematics are all shaped by the teaching they encounter in school (NCTM 2000)." The teaching that students have encountered for much of our country's history has served to develop practical skills. Yet the school mathematics education popular throughout most of our history is no longer practical for the demands of our modern society. Rote learning of arithmetic procedures are no longer the most beneficial learning goals. People today are much more exposed to numbers and quantitative ideas and need to be able to deal with mathematics at higher cognitive levels than they did in years past. Everyone agrees this is the case with American mathematics education. How best to produce these goals in our students remains to be the point of contention. This review of research literature has shown several findings that suggest it is possible for curriculum and classroom instruction to develop within students computational skills with the ability to think and reason with deep mathematical understanding. The major instructional technique which has pervaded American mathematics education has traditionally involved mathematics teaching and learning, done by direct presentation of rules and algorithms, repetition, and memorization based on the premise that you need to know skills before you apply them to problems, and that by becoming procedurally fluent first you can pull ideas together to attach more complex ideas later. The research literature has not confirmed this premise. Not only has it not confirmed this premise, it has gone so far as to suggest that by focusing on the procedural drill of set rules, teachers make it more difficult for students to learn the deeper conceptual frameworks. Comparison studies presented in this body of research showed that students in conceptual understanding groups did no worse than traditional students on measures of procedural knowledge, while the traditional groups did significantly worse on measures of conceptual knowledge. The evidence from these studies provides power to the argument that American mathematics education needs a new direction for curriculum and instruction.

This body of research has shown many features directed at providing students with the opportunity, support and tools that will enable them to comprehend for themselves the mathematical concepts underlying arithmetical procedures; seeing the relationships between various operations, gaining an understanding of the conceptual frameworks of mathematics; developing skill in communication; skill with computation; all the while gaining a habitual inclination to see mathematics as sensible, useful, and worthwhile; coupled with the belief that through diligence and hard work they can be successful at learning mathematics. One key feature of the curriculum described in these studies was presenting challenging instructional tasks in the form that students can treat problematically. The instructional tasks can not just be exercises on which students are suppose to practice a prescribed procedure. The task must have students focus on the solution of a puzzling situation and require them to question assumptions, challenge ideas, and provide justification for solutions. When students are engaged with mathematics in this way they are not only obtaining specific pieces of knowledge but also a degree of metacognitive knowledge about how to organize mathematical activities. Through these types of challenging problems teachers can encourage high levels of cognitive processing in student thinking; all-the-while building in student's computational skill. As important as computational skill is, these instructional tasks push students to think deeper about mathematics. They challenge students to think logically, communicate clear explanations, and justifications for their own mathematical ideas, as well as evaluate criteria for mathematical arguments generated by others. Student learning can not be expected to deepen or become more conceptually rich, unless students are regularly, actively, and productively engaged with cognitively challenging mathematics.

Whether students operate at these high levels of cognitive demand depends as much on the culture of the classroom as on problematic tasks. Certain Groups in our country have criticized this form of curriculum and instruction, and have claimed that it lacks mathematical depth and rigor. In one sense the research agrees with this criticism. The research has suggested that merely enacting the appearance of reform oriented instruction does not produce the kind of learning that leads to deep mathematical understanding. Through contrasting various reform oriented classrooms, the research has provided evidence that mathematics classes must move beyond the mere enactment of reform oriented social structures, to the deeper and more meaningful sociomathematic norms which govern the discursive and thinking processes of mathematics classes. Key feature described in these studies point to the role of the teacher to establish these sociomathematic norms.

One such norm is that the authority for correct answers and procedures lies with mathematical reasoning. The research suggests that the teacher must continually press students to meet this norm. To often students, and entire classrooms, are organized so that the judgment of right and wrong answers are a domain left entirely to the teacher. Students are not required to think independently about the validity of answers, but look instead to the sovereignty of the teacher's voice. When this is a characteristic of a mathematics classroom, the teaching usually emphasizes the learning of procedures, and computation. Kazemi and Stipek(2001) posit that just presenting cognitively demanding activities in classrooms does not equate into significant mathematics learning. Often when students are required to voice their solutions, they tend to describe the steps they took to solve a problem, without explaining why the solution works mathematically. Through the course of a lesson, teachers are presented with instructional choices which can either lead student thinking to function at high levels of cognitive demand or can decline into merely producing low level procedural knowledge. When instructional tasks have a high level of cognitive demand and require students to think and reason in complex ways they are considered to be at the "Doing Mathematics" level. Although a teacher may set up a lesson to have students grapple with problems at the doing mathematics level, there are many factors that can complicate the lesson and lead to the decline of cognitive demand. One of the most common factors that leads to the decline of a mathematical task from a high level of cognitive demand to a lower level is a shift in the focus of the task from a sustained effort for conceptual explanation of understanding and meaning to a mere recitation of the correct answer or completed problem. This tension is quite a challenge for teachers. The temptation is for the teacher to not allow the students time enough, or autonomy to grapple with the challenges that problems pose to their understanding. Other factors are influential in this process as well. Sometimes the chosen task could be inappropriate for the group of students due to lack of sufficient prior knowledge or the set-up is not sufficiently explained to put the students into the right cognitive mind-frame. Teachers have a significant challenge establishing a classroom culture where sociomathematic norms lead to high levels of cognitive demand. Just having students explain their solution strategies to one another is not enough to further the learning of all students. Deep mathematical understanding is a result of students presenting multiple strategies and then working analytically, logically, and creatively to understand the relationships between each others ideas. This can happen only when the authority for correct answers and procedures lies with mathematical reasoning and students are pressed to engage the mathematics.

These sociomathematic norms and the rules they establish for student behavior convey to students explicit messages about what counts as learning, who they are as learners, and the role of students' interactions with their classmates. When students are presented mathematics in a manner consistent with the reform oriented practices reviewed in this paper, they develop more healthy dispositions for learning mathematics. Rather than focusing on a student's ability in comparison to other students, these mathematics classrooms demonstrate that being unsure, learning from mistakes, and asking questions are necessary parts of learning. When students are regularly actively, and productively engaged with mathematics that is cognitively challenging and in a classroom that is supportive, respectful, and encouraging, they come to see the discipline as important; they show willingness to attempt unfamiliar problems and to develop their perseverance in solving problems without being discouraged by initial setbacks; they learn to enjoy and sense personal reward in the process of thinking, searching for patterns, and solving problems; and they develop confidence in their ability to do mathematics and to confront unfamiliar tasks.

Implications for Further Research

This review of research literature pertained to investigating the effects of curriculum and instruction on student achievement in the area of mathematics. Many important studies have provided clear insights into the qualitative and quantitative differential effects of reform oriented and traditional based instruction and curriculum. Many of these studies sample a broad range of student populations, yet, these particular studies did not purpose to investigate any specific population. Recommendations for further study must include research that seeks to know how these instructional practices and curriculum effect specific demographic populations within our nation's populace. Research should aim for urban schools, rural schools, schools with large migrant populations as well as schools with high levels of immigrant students. Research should also investigate how gender differences interact with these differential forms of instruction and curriculum.

Summary

Chapter One laid out my rationale for a critical analysis of the current research literature as it pertains to the effects of instructional practices, and curriculum upon student achievement. Chapter two discussed the history of mathematics education throughout our nation's history. Specifically tracing the origins of two opposing philosophies of how mathematics should be taught and learned in our public schools. Chapter Two also revealed the emergence of the mathematics educational research community, which developed simultaneously with the most current efforts at reforming mathematics education in our nations public schools. Chapter Three provided a review of the relevant research literature on the relationship between curricula, instructional practice, and their effects on student achievement. Chapter four summarized those findings, and investigated classroom implications of those findings.

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