1. A bat with moment of inertia I about an axis at one end is swung with angular velocity ω at a stationary ball with mass m so that the point of impact with the ball is at a distance y from the axis of rotation of the bat. After the perfectly elastic collision the ball acquires a speed v and the bat continues with angular velocity ω'.

   (a) Write down an equation showing conservation of energy in this collision.

   (b) Write down an equation showing conservation of angular momentum in this collision.

   (c) Eliminate ω' from the above equations and solve for v as a function of the position of the point of impact y and show that

$$v = \frac{2yI\omega}{I + my^2}$$

(d) In order to achieve a maximum energy transfer at the point y we require that v is as large as possible. Find in terms of I, m and ω the value of y which results in the maximum value of v and show that this agrees with the expression for y_{maxE} in your Sweet Spot lab hand out.

(e) Hence find the maximum speed of the ball v_{max} and observe that this is equal to the speed of the bat at the sweet spot. Also show that $\omega' = 0$ indicating that the bat comes to rest when you strike a ball at rest at this sweet spot.

(f) For a ball of mass 200 g and a uniform bat (ie a rigid rod) of mass 500 g and length 1 m swung with angular velocity 40 rad/s find the sweet spot and the speed of the ball after the collision.

(g) How far would this ball go if its velocity is directed at an angle of 45^o to the horizontal? Is it a home run?