1. Find, in the form $y = f(x)$, the general solution to the differential equation

$$x^2 \frac{\mathrm{d}y}{\mathrm{d}x} + xy = x + 1$$

2. Find the solution to the differential equation

$$\sqrt{x^2+1}\,\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{2y}\ ,$$

3. Solve the differential equation $x\,\dfrac{\mathrm{d}y}{\mathrm{d}x} + 2y = \cos x$, given that the $y = \frac{2}{\pi}$ when $x = \frac{\pi}{2}$.

4. Solve the differential equation $x^2\,\dfrac{\mathrm{d}y}{\mathrm{d}x} + e^{1/x} = 0$, given that $y = 0$ when $x = -1$.

5. A element A decays to another element B which is also radioactive. A is called the mother nucleus and B is called the daughter nucleus (why radioactive elements are considered feminine is open to question!). Suppose the half life of element A is 30 minutes and the half life of element B is 10 minutes and that you start off with 1.0 moles of A and zero moles of B. Your task in this problem is to find the number of moles of B as a function of time t in minutes. This of course depends on the number of moles of A at time t.

   (a) Show that the number of moles of A at time t is $A = 1.0e^{-k_A t}$ where $k_A = \ln 2/30 \text{ min}^{-1}$. (This follows from John's worksheet question on radioactive decay – ie use the fact that the rate of decay of A is proportional A.)

   (b) The rate of change in the number of moles of B equals the rate of increase in B due to the decay of A minus the rate of decrease in B due to the decay of B. That is

$$\frac{\mathrm{d}B}{\mathrm{d}t} = k_A A - k_B B$$

   where $k_B = \ln 2/10 \text{ min}^{-1}$. Substitute for A in the above differential equation and solve it for B as a function of time. Sketch the function using your graphical calculator.

   (c) What is the maximum number of moles of B and when is it at a maximum?