1. Find $\dfrac{\mathrm{d}w}{\mathrm{d}t}$ at $t=\pi/4$ if $w=xy+yz+zx$ and $x=\cos t$, $y=\sin t$ and $z=\cos 2t$

   $\dfrac{\mathrm{d}w}{\mathrm{d}t}=(y+z)\dfrac{\mathrm{d}x}{\mathrm{d}t}+(x+z)\dfrac{\mathrm{d}y}{\mathrm{d}t}+(x+y)\dfrac{\mathrm{d}z}{\mathrm{d}t}=(y+z)(-\sin t)+(x+z)\cos t+(x+y)(-\sin 2t)$

   When $t=\pi/4$ then $x=y=1/\sqrt{2}$ and $z=0$ so $\dfrac{\mathrm{d}w}{\mathrm{d}t}=-2\sqrt{2}$

2. A cylindrical can of radius r and height h is heated. The radius increases at a rate of 0.0015 cm/min and the height increases at a rate of 0.0025 cm/min.

   (a) Find the rate of increase of the volume of the can when r=2 cm and h=6 cm.

   $V=\pi r^2 h$ so $\dfrac{\mathrm{d}V}{\mathrm{d}t}=2\pi rh\dfrac{\mathrm{d}r}{\mathrm{d}t}+\pi r^2\dfrac{\mathrm{d}h}{\mathrm{d}t}=24\pi\times 0.0015+4\pi\times 0.0025=0.046\pi$ cm^3/min

   (b) Find the rate of increase of the surface area of the can when r=2 cm and h=6 cm.

   $S=2\pi r^2+2\pi rh$ so $\dfrac{\mathrm{d}S}{\mathrm{d}t}=(4\pi r+2\pi h)\dfrac{\mathrm{d}r}{\mathrm{d}t}+2\pi\dfrac{\mathrm{d}h}{\mathrm{d}t}=(8\pi+12\pi)\times 0.0015+4\pi\times 0.0025=0.03\pi$ cm^2/min

3. A charged particle moves along a path given by $x=5\cos t-3$ and $y=4\sin t$ in the presence of a potential, $V=-\dfrac{1}{\sqrt{x^2+y^2}}$

   (a) Find the rate of change in the potential when $t=\pi/2$

   $\dfrac{\mathrm{d}V}{\mathrm{d}t}=\dfrac{\partial V}{\partial x}\dfrac{\mathrm{d}x}{\mathrm{d}t}+\dfrac{\partial V}{\partial y}\dfrac{\mathrm{d}y}{\mathrm{d}t}=\dfrac{x}{(x^2+y^2)^{3/2}}\cdot(-5\sin t)+\dfrac{y}{(x^2+y^2)^{3/2}}\cdot 4\cos x$

   When $t=\pi/2$ then $x=-3$ and $y=4$ so $\dfrac{\mathrm{d}V}{\mathrm{d}t}=-\dfrac{3}{125}\cdot -5=\dfrac{3}{125}$ Volts/sec

   (b) Find the maximum and minimum value of the potential along the particle path.

   $\dfrac{\mathrm{d}V}{\mathrm{d}t}=0\ \Rightarrow\ \dfrac{x}{(x^2+y^2)^{3/2}}\cdot(-5\sin t)+\dfrac{y}{(x^2+y^2)^{3/2}}\cdot 4\cos x=0\ \Rightarrow 5x\sin t=4y\cos t$

   So $5(5\cos t-3)\sin t=16\cos t\sin t\ \Rightarrow\ 9\cos t\sin t-15\sin t=0\ \Rightarrow\ 3\sin t(3\cos t-5)=0$

   So $\sin t=0\ \Rightarrow\ t=0$ or $t=\pi\ \Rightarrow (x,y)=(2,0)$ or $(-8,0)\ \Rightarrow V=-\frac{1}{2}$ or $V=-\frac{1}{8}$.

   Note that the other equation $3\cos t-5=0$ has no real solutions.