1. (a) Show that the function $y = Ae^{kx} + Be^{-kx}$ satisfies the differential equation $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = y$ for a particular value of k and state this value.

   (b) Suppose $y = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$ when $x = 0$. Find the constants A and B. Hence write down the solution of the differential equation with these boundary conditions and call the function $y = S(x)$

   (c) Suppose $y = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $x = 0$. Find the constants A and B. Hence write down the solution of the differential equation with these boundary conditions and call the function $y = C(x)$

   (d) Simplify as far as possible the expression $(C(x))^2 - (S(x))^2$.

2. Find the solution to the following system of partial differential equations

$$\frac{\partial u}{\partial x} = ye^{xy} + 2x$$

$$\frac{\partial u}{\partial y} = xe^{xy} + 2y$$

   Given $u(0, 0) = 0$

3. Using the method of separation of variables find the eigenfunctions and eigenvalues for the wave equation on a rectangular drum with sides of length 1 unit and 2 unit. That is find solutions of the PDE

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -k^2 u$$

   satisfying the boundary conditions $u(0, y) = 0$, $u(1, y) = 0$ and $u(x, 0) = 0$ and $u(x, 1) = 0$. For each possible solution give an expression for the allowed values of k.