Part I

1. If the radius of the path of a body in uniform circular motion is doubled and the speed is kept the same the force needed must be
   (a) half as great as before.
   (b) the same as before.
   (c) twice as large as before.
   (d) four times the size as before.

   Answer (a). Since speed is constant not frequency.

2. A small cube slides in a hemispherical bowl with negligible friction. If the ice cube is released from rest near the top edge of the bowl then at the bottom of the bowl the acceleration is
   (a) zero.
   (b) directed horizontally.
   (c) directed vertically up.
   (d) directed vertically down.

   Answer (c). The cube is moving on a circular path so will have an upward acceleration toward the center.

3. A truck with a ball free to roll around in the back takes a a sharp turn to the right. Which of the following forces cause the ball to move to the left side of the truck?
   (a) gravity.
   (b) normal force.
   (c) centripetal force.
   (d) the absence of a force.

   Answer (d). The ball moves in a straight line and since the truck moves to the right the ball encounters the left side of the truck

4. A body which is traveling around a circle at constant speed
   (a) is not accelerated.
   (b) has constant acceleration.
   (c) is accelerated in the direction of motion.
   (d) none of the above.

   Answer (d). The direction of velocity and acceleration change so neither is constant! (although the magnitude is constant in both cases)

5. On a rainy day the coefficient of friction between the tires of a car and a level road surface is reduced to half its usual value. The maximum safe velocity for rounding a curve is

   (a) reduced to 71% of its original value.

   (b) reduced to 50% of its original value.

   (c) reduced to 25% of its original value.

   (d) unchanged.

   Answer (a). You can travel at more than half the initial velocity because half the velocity requires only 1/4 the force.

Part II

1. A 6.8×10^4 kg spaceship orbits a distant planet circling with a period of 26 hours and an orbital radius of $16{\times}10^6$ m.

   (a) Find

      (i) the velocity of the spaceship.
      $v = \dfrac{2\pi r}{T} = \dfrac{2\pi \times 16 \times 10^16}{26 \times 60 \times 60} = 1070$ m/s

      (ii) the centripetal acceleration of the spaceship.
      For centripetal acceleration $a = \dfrac{v^2}{r} = \dfrac{1070^2}{16 \times 10^6} = 0.072$ m/s^2

      (iii) the mass of the planet.
      The acceleration can be related to the mass of the planet through Newton's 2nd Law. $F_{\text{net}} = ma \;\Rightarrow\; \dfrac{GmM}{r^2} = ma \;\Rightarrow\; M = \dfrac{ar^2}{G} = 2.76 \times 10^{23}$ kg

   (b) (i) What forces if any act on the astronauts inside the spaceship?
      The only force acting on the astronauts is their weight – ie the force of gravity acting on them.

      (ii) Explain why the astronauts feel weightless.
      The astronauts feel weightless because they are in 'freefall'. That is, they are accelerating toward the planet at a rate equal to the acceleration of gravity at that point. (Note: The sensation of weight is actually the normal force of a surface acting on your body in order to limit your acceleration. When acceleration is zero the normal force must equal your actual weight. When your acceleration is equal to the acceleration of gravity the normal force is zero and you feel weightless.)

   (c) The spaceship decreases its orbital radius to get a closer view of the planet. In this lower orbit will its speed be greater or smaller? Explain.
   The speed will be greater since the centripetal force provided by gravity is greater and $F_c \propto v^2$.

2. A ferris wheel with a radius of 15 m makes on complete rotation every 12 seconds

   (a) With what speed are the riders moving?

   $v = 2\pi r/T = 2\pi(15)/12 = 7.85$ m/s

   (b) What is the magnitude of their acceleration?

   $a_c = v^2/r = (7.85)^2/15 = 4.0$ m/s^2

   (c) If a rider has mass 40 kg what is the magnitude of the centripetal force required to keep them moving in a circle?

   $F_c = ma_c = 40(4.1) = 164$ N

   (d) What is the value of the normal force exerted by the seat on the rider at the top of the wheel? What is the value of the normal force exerted by the seat on the rider at the bottom of the wheel? (Hint: Draw free body diagrams.)

   At the top of the loop the acceleration is directed down so the net force is $W - N = 164 \Rightarrow N = W - 164 = 40(9.8) - 164 = 228$ N.

   At the bottom of the loop the acceleration is directed up so the net force is $N - W = 164 \Rightarrow N = 164 + W = 164 + 40(9.8) = 556$ N.

   (e) At what speed would the rider at the top of the wheel feel weightless?

   The rider at the top will feel weightless when the normal force is zero. Then the weight of the rider provide all the necessary force for acceleration. That is the centripetal acceleration equals the acceleration due to gravity.

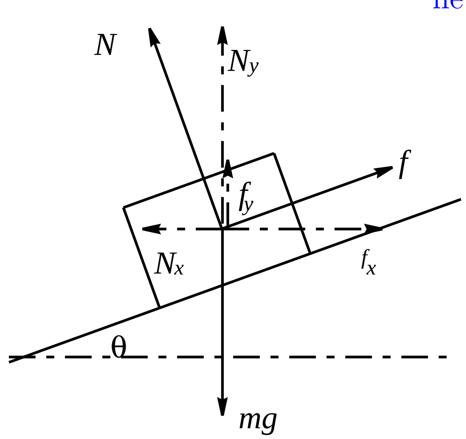
   So $a_c = \dfrac{v^2}{r} = g \Rightarrow v^2 = gr \Rightarrow v = \sqrt{gr} = \sqrt{(9.8)(15)} = 12$ m/s

3. A racing car is turning a corner on a banked track which makes an angle of 26^0 with the horizontal. It moves with a speed v in a horizontal circle of radius $r = 300$ m.

   (a) Draw a free body diagram of the car showing all the forces (including friction) which act on it.

$$F_{\text{net}_x} = N_x - f_x = N\sin\theta - f\cos\theta = \frac{mv^2}{r}$$

$$F_{\text{net}_y} = N_y + f_y - mg = N\cos\theta + f\sin\theta - mg = 0$$



(b) How fast should the car travel so that the frictional force is zero?

$$f = 0 \Rightarrow N\sin\theta = \frac{mv^2}{r} \quad \text{and} \quad N\cos\theta - mg = 0$$

Therefore, eliminating N

$$\frac{\sin\theta}{\cos\theta} = \frac{v^2}{gr} \;\Rightarrow\; \tan\theta = \frac{v^2}{gr} \;\Rightarrow\; v = \sqrt{gr\tan\theta} = 37.9\,m/s$$

(c) The car actually drives around the corner at a speed of 30 ms^{-1} in which direction will the frictional force act?

Friction acts up the slope since the car is moving too slowly to maintain its vertical position without friction

(d) The mass of the car is 1000 kg. Calculate the magnitude of the frictional force and the normal reaction of the road on the car.

$$(1) \quad N\sin\theta - f\cos\theta = \frac{mv^2}{r}$$

$$(2) \quad N\cos\theta + f\sin\theta = mg$$

eliminate N from (1) and (2)

$$(2)\sin\theta - (1)\cos\theta \;\Rightarrow\; f\cos^2\theta + f\sin^2\theta = mg\sin\theta - \frac{mv^2}{r}\cos\theta = 1600N$$

$$\Rightarrow \quad f(\cos^2\theta + \sin^2\theta) = 1600N \;\Rightarrow\; f = 1600N$$

from (1) we solve for N

$$N = \frac{mg - f\sin\theta}{\cos\theta} = 10100N$$