11.1 Using Fig. 11.2, verify that the change in the Sun's effective temperature over the past 4.5 billion years is consistent with the variations in its radius and luminosity.
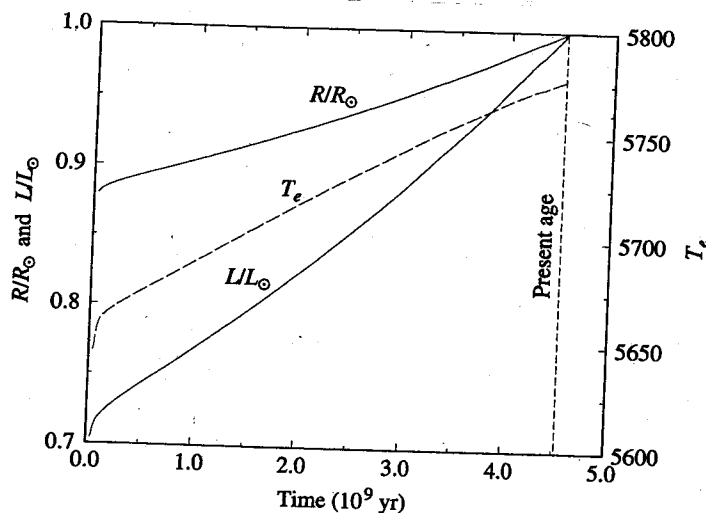

Figure 11.2 The evolution of the Sun from its birth to the present. As

Recall $L = 4\pi\sigma R^2 T^4 \rightarrow T =$

Let T_i = temp 4.5 billion years ago. Read from chart

$$\frac{R_i}{R_\odot} = \qquad\qquad \frac{L_i}{L_\odot}$$

Calculate $\dfrac{T_i}{T_\odot} =$

If $T_\odot = 5770\,K$ now then $T_i =$

Compare to T_i on chart =

11.2 (a) At what rate is the Sun's mass decreasing due to nuclear reactions? Express your answer in solar masses per year.

(b) Compare your answer to part (a) with the mass loss rate due to the solar wind.

(c) Assuming that the solar wind mass loss rate remains constant, would either mass loss process significantly affect the total mass of the Sun over its entire main-sequence lifetime?

$$\text{Power} = \frac{\text{Energy}}{\text{time}}$$

$$L = \frac{\delta m \, c^2}{\delta t}$$

(a) Rate of mass change $= \frac{\delta m}{\delta t} =$

(b) Ex 1 found the solar wind causes a mass loss rate of $40 \, g$ $\frac{d M_\odot}{dt} =$

Compare:

(c) Sun's main sequence lifetime $\tau \sim 10^{10} \, yrs = \underline{\qquad} \, s$

Total Mass lost $\delta m = \frac{\delta m}{\delta t} \cdot \tau =$

$$V_{turb} = 0$$

11.5 (a) Using Eq. (9.58) and neglecting turbulence, estimate the full
width at half-maximum of the hydrogen H_α absorption line due
to random thermal motions in the Sun's photosphere. Assume
that the temperature is the Sun's effective temperature.

$$(\Delta\lambda)_{1/2} = \frac{2\lambda}{c}\sqrt{\left(\frac{2kT}{m} + v_{turb}^2\right)\ln 2}, \qquad (9.58)$$

where mass per atom $m \simeq m_H \simeq m_p = 1.67 \times 10^{-24}$ g

$\lambda = \lambda_{H_\alpha} = 6562.80$ Å

$T = 5770$ K $\qquad\qquad k = 1.38 \times 10^{-16} \frac{erg}{K}$

$$\frac{2kT}{m} =$$

$$\frac{2\lambda}{c} =$$

$$(\Delta\lambda)_{1/2} =$$

(b) Using H_α redshift data for solar granulation, estimate the full
width at half-maximum when convective turbulent motions are
included with thermal motions.

If $V_{turb} \simeq 0.4 \frac{km}{s}$

Then $(\Delta\lambda)_{1/2} =$

(c) What is the ratio of v_{turb}^2 to $2kT/m$?

(d) Determine the relative change in the full width at half-maximum due to Doppler broadening when turbulence is included. Does turbulence make a significant contribution to $(\Delta\lambda)_{1/2}$ in the solar photosphere?

$$\frac{V_{\text{turb}}^2}{2kT/m} =$$

(d) Let $\Delta\lambda_{\text{turb}} = (\Delta\lambda)_{1/2}$ with $V_{\text{turb}} = 0.4 \frac{\text{km}}{\text{s}}$, and $\Delta\lambda_o = (\Delta\lambda)_{1/2}$ with $V_{\text{turb}} = 0$.

Then $\frac{\Delta\lambda_{\text{turb}} - \Delta\lambda_o}{\Delta\lambda_o} =$

Estimate the thermally Doppler-broadened line widths for the hydrogen Lyman α, C III, O VI, and Mg X lines given on page 401; use the temperatures provided. Take the masses of H, C, O, and Mg to be 1 u, 12 u, 16 u, and 24 u, respectively.

$$\text{Eq. } \frac{(2.57)}{296} \, (\Delta\lambda)_{1/2} =$$

	Lyman α	C III	O VI	Mg X
	H	C	O	Mg
mass (u)	1	12	16	24
λ (Å)	1216	977	1032	625
T (K)	20,000	20,000	300,000	1.4×10^6

$\Delta\lambda_{1/2}$

11.8 Suppose that you are attempting to make observations through an optically thick gas that has a constant density and temperature. Assume that the density and temperature of the gas are 2.5×10^{-7} g cm^{-3} $= \rho$ and 5770 K, respectively, typical of the values found at the base of the Sun's photosphere. If the opacity of the gas at one wavelength (λ_1) is $\kappa_{\lambda 1} = 0.26$ cm^2 g^{-1} and the opacity at another wavelength (λ_2) is

$\kappa_{\lambda 2} = 0.30$ cm^2 g^{-1}, calculate the distance into the gas where the optical depth equals 2/3 for each wavelength. At which wavelength can you see farther into the gas? How much farther? This effect allows astronomers to probe the Sun's atmosphere at different depths (see Fig. 11.17).

$T =$

$\dfrac{(9,13)}{266} \qquad d\tau_\lambda =$

$\text{distance} \quad d = \displaystyle\int_0^d ds = -\int_0^{\tau_{0,\lambda}} \dfrac{1}{\kappa_\lambda \rho} \, d\tau_\lambda =$

$d =$

Let $\tau_{0,\lambda} = \dfrac{2}{3}$. Then the depth observed at λ is d:

for λ_1, $\quad d_1 =$

for λ_2, $\quad d_2 =$

11.9 (a) Using the data given in Example 11.2, estimate the pressure scale
            height at the base of the photosphere.

       (b) Assuming that the mixing length to pressure scale height ratio
            is 2.2, use the measured Doppler velocity of solar granulation to
            estimate the amount of time required for a convective bubble to
            travel one mixing length. Compare this value to the characteristic
            lifetime of a granule.

$$P = 5 \times 10^4 \frac{dynes}{cm^2}$$

$$\rho = 2.5 \times 10^{-7} \frac{g}{cm^3}$$

Pressure scale height (10.63)

What is g at surface of sun (base of photosphere)?

$$f = ma$$

$$\frac{GmM}{r^2} = mg$$

$$\frac{GM_\odot}{R_\odot^2} = g =$$

(a) So $H_\rho =$

(b) Let α = $\dfrac{\text{mixing length}}{\text{pressure scale height}}$ $= \dfrac{l}{H_\rho} = 2 \rightarrow l =$ ___

velocity $= \dfrac{\text{length}}{\text{time}}$: $V = \dfrac{l}{t} \rightarrow t =$

If $V = 0.4 \times 10^3 \frac{m}{s}$, then convection time $t =$

11.10 Show that Eq. (11.7) follows directly from Eq. (11.6). p.410-11

✓ assume $T = $ const locally

(11.6) $\frac{d}{dr}(2nkT) = -\frac{GM_\odot n \mu_p}{r^2}$

410

$a \frac{dn}{dr} = b \frac{n(r)}{r^2}$: $a = \underline{\qquad}$, $b = \underline{\qquad}$

$a \int_{n_0}^{n(r)} \frac{dn}{n} = b \int_{r_0}^{r} \frac{dr}{r^2}$ Solve both sides.

11.11 Calculate the magnetic pressure in the center of the umbra of a large sunspot. Assume that the magnetic field strength is 2000 G. Compare your answer with a typical value for the gas pressure at the base of the photosphere.

(11.13) magnetic $P_m = \frac{B^2}{8\pi} = $

414   pressure

(in cgs)

Ex 11.2 : $P_{gas} = $

11.12 Assume that a large solar flare erupts in a region where the magnetic field strength is 300 G and it releases 10^{32} ergs in one hour.

$B =$ $E =$ $t = 60 \, min \left| \dfrac{60 \, s}{min} \right| = \underline{\qquad} \, s$

(a) What was the magnetic energy density in that region before the eruption began?

$$pressure = \dfrac{energy}{volume}$$

$$p = 2.5 \times 10^{-7} \, \dfrac{g}{cm^3}$$

(b) What minimum volume would be required to supply the magnetic energy necessary to fuel the flare?

(c) Assuming for simplicity that the volume involved in supplying the energy for the flare eruption was a cube, compare the length of one side of the cube with the typical size of a large flare.

(d) How long would it take an Alfvén wave to travel the length of the flare?

(e) What can you conclude about the assumption that magnetic energy is the source of solar flares, given the physical dimensions and time scales involved?

(a) $P_m = \dfrac{B^2}{8\pi} = u_m$

(b) energy density $u_m = \dfrac{Energy}{volume} \longrightarrow vol =$

(c) $Volume \sim x^3 \longrightarrow x =$

(d) Alfven speed (cgs) $V_A = \dfrac{B}{\sqrt{4\pi\rho}} =$

$V = \dfrac{x}{t} \longrightarrow t =$

(e)

11.13 (a) Calculate the frequency shift produced by the normal Zeeman effect in the center of a sunspot that has a magnetic field strength of 3000 G. $= B$

(b) By what fraction would the wavelength of one component of the $\lambda = 6302.5$ Å Fe I spectral line change due to a magnetic field of 3000 G?

(5.20) $\Delta \nu = \text{frequency shift (cgs)} = \dfrac{e B}{4 \pi m_e c}$ [in esu]

$\Delta \nu =$

(b) $c = \lambda \nu \rightarrow \lambda = \qquad \rightarrow \dfrac{d\lambda}{d\nu} =$

$\dfrac{d\lambda}{\lambda} = \dfrac{\Delta \lambda}{\lambda} =$

11.14 Argue from Eq. (11.12) and the work integral that magnetic pressure is given by Eq. (11.13).

From thermodynamics, work $W = \int P \, dV$ [pressure , change in volume]

$P =$

(11.12) Therefore $\dfrac{\text{energy}}{\text{vol}} \; u_m = \dfrac{B^2}{8\pi} =$ (11.13)
412   414

$$e^{x+y} = e^x e^y$$

$$\boxed{P = a T^x}$$

$$\ln P = \ln a T^x$$
$$= \ln a + \ln T^x$$
$$\boxed{\ln P = \ln a + x \ln T}$$

$$x = \text{slope}$$
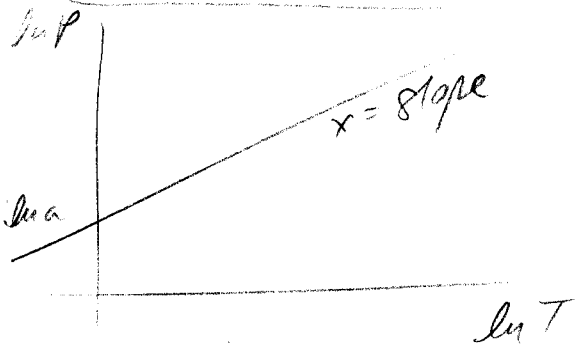
$$\frac{d \ln P}{d \ln T} = x$$

$$P \propto T^2 \quad \text{convection}$$

$$P \propto T^3 \quad \text{radiation}$$

$$\ln ab \overset{?}{=} \ln a \ln b$$
$$\overset{?}{=} \ln a + \ln b$$
$$e^{\ln ab} = e^{\ln a + \ln b}$$
$$ab = e^{\ln a} e^{\ln b}$$
$$= ab \quad \checkmark$$

$$\ln ab = \ln a + \ln b$$