

Math

- 2.1 Assume that a rectangular coordinate system has its origin at the center of an elliptical planetary orbit and that the coordinate system's x axis lies along the major axis of the ellipse. Show that the equation for the ellipse is given by

The hard way;

(2.1)
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$$r + r' = 2a$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

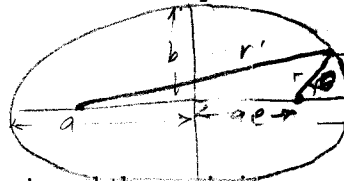
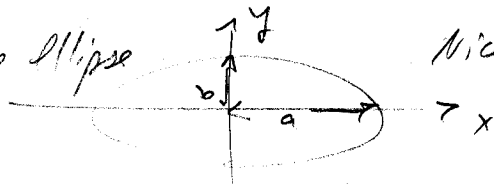


Fig. 2.4

where a and b are the lengths of the semimajor axis and the semiminor axis, respectively.

Matt's way: Parametrize the ellipse

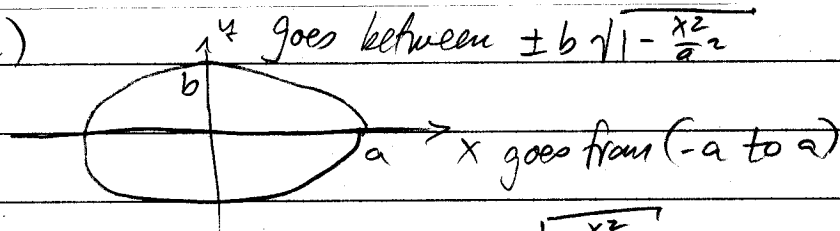


Nicer.

$$x = a \cos \theta, \quad y = b \sin \theta$$

2.2 Using the result of Problem 2.1, prove that the area of an ellipse is given by $A = \pi ab$.

$$y^2 = b^2 \left(1 - \frac{x^2}{a^2}\right)$$



$$\text{Area} = \iint dA = \int_x \int_y dy dx = \int_{-a}^a \int_{-b\sqrt{1-\frac{x^2}{a^2}}}^{+b\sqrt{1-\frac{x^2}{a^2}}} dy dx$$

- 2.7 (a) Assuming that the Sun interacts only with Jupiter, calculate the total orbital angular momentum of the Sun-Jupiter system. The semimajor axis of Jupiter's orbit is $a = 5.2$ AU, its orbital eccentricity is $e = 0.048$, and its orbital period is $P = 11.86$ yr.

① Total orbital angular momentum L of Sun + Jupiter is
 (2.32) $L = \mu \sqrt{GMa(1-e^2)}$ where $M_J = 2 \times 10^{27} \text{ kg}$
 51 $M_\odot = 2 \times 10^{30} \text{ kg}$

$$M = M_J + M_\odot =$$

$$\mu = \frac{M_J M_\odot}{M_J + M_\odot}$$

$$L =$$

- (b) Estimate the contribution the Sun makes to the total orbital angular momentum of the Sun-Jupiter system. For simplicity, assume that the Sun's orbital eccentricity is $e = 0$, rather than $e = 0.048$. Hint: First find the distance of the center of the Sun from the center of mass.

$$L = mvr \text{ where Sun's orbital speed } v_\odot = \frac{2\pi a_\odot}{P_J}$$

$$P_J = \text{the system's orbital period} =$$

and the distance from Sun's center to system's center of mass

$$a_\odot = \frac{\mu a}{M_J}$$

- 2.7(c) Making the approximation that the orbit of Jupiter is a perfect circle, estimate the contribution it makes to the total orbital angular momentum of the Sun-Jupiter system. Compare your answer with the difference between the two values found in parts (a) and (b).

$$L_J = \mu_J a_J v_J$$

$$a_J = \text{Jupiter's distance from C.M.} = \frac{\mu a}{\mu_J} =$$

$$\text{Jupiter's orbital speed } v_J = \frac{2\pi a_J}{P_J} =$$

- 2.7(d) Recall that the moment of inertia of a solid sphere of mass m and radius r is given by $I = \frac{2}{5}mr^2$, and that when the sphere spins on an axis passing through its center, its rotational angular momentum may be written as

$$L = I\omega,$$

where ω is the angular frequency measured in rad s^{-1} . Assuming (incorrectly) that both the Sun and Jupiter rotate as solid spheres, calculate approximate values for the rotational angular momenta of the Sun and Jupiter. Take the rotation periods of the Sun and Jupiter to be 26 days and 10 hours, respectively. The radius of the Sun is 6.96×10^{10} cm, and the radius of Jupiter is 6.9×10^9 cm.

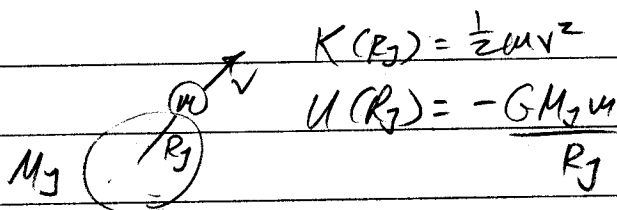
$$\omega_0 = \frac{2\pi}{P_0} =$$

$$I_0 = \frac{2}{5} M_0 R_0^2 =$$

$$L_0 = \omega_0 I_0 =$$

- 2.7(e) What part of the Sun-Jupiter system makes the largest contribution to the total angular momentum?

- 2.8 (a) Using data contained in Problem 2.7 and in the chapter, calculate the escape velocity at the surface of Jupiter.
- (b) Calculate the escape velocity from the solar system, starting from Earth's orbit. Assume that the Sun constitutes all of the mass of the solar system.



$$K(R_J) = \frac{1}{2}mv^2$$

$$U(R_J) = -\frac{GM_J m}{R_J}$$

$$U(r \rightarrow \infty) = 0$$

$$K(r \rightarrow \infty) = 0$$

Conservation of mechanical energy:

$$K(R_J) + U(R_J) = K(r \rightarrow \infty) + U(r \rightarrow \infty) \quad \text{: solve for } v$$

2.11 Cometary orbits usually have very large eccentricities, often approaching (or even exceeding) unity. Halley's comet has an orbital period of 76 yr and an orbital eccentricity of $e = 0.9673$.

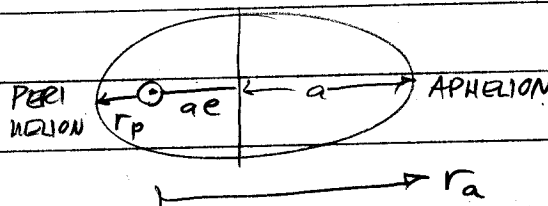
- (a) What is the semimajor axis of Comet Halley's orbit?
- (b) Use the orbital data of Comet Halley to estimate the mass of the Sun.

a $a^3(\text{AU}) = P^2(\text{yr}) =$

b $M_{\odot} = \frac{4\pi^2 a^3}{GP^2} =$

- (c) Calculate the distance of Comet Halley from the Sun at perihelion and aphelion.

$r_{\text{perihelion}} =$



$r_{\text{aphelion}} =$

2.11 (d) Determine the orbital speed of the comet when at perihelion, at aphelion, and on the semiminor axis of its orbit.

$$\frac{(2.30)}{50} \quad v_p^2 = \frac{GM}{a} \left(\frac{1+e}{1-e} \right) \quad \frac{(2.31)}{51} \quad v_a^2 = \frac{GM}{a} \left(\frac{1-e}{1+e} \right)$$

$$\frac{GM}{a} =$$

$$(1+e) =$$

$$(1-e) =$$

$$v_p^2 =$$

$$v_a^2 =$$

(e) How many times larger is the kinetic energy of Halley's comet at perihelion when compared to aphelion?

$$K_p = \frac{1}{2} m v_p^2$$

$$K_a = \frac{1}{2} m v_a^2$$

$$\frac{K_p}{K_a} = \frac{v_p^2}{v_a^2} =$$

$$K_a \quad v_a^2$$