

Astrophysics HW

Carroll + Ostlie

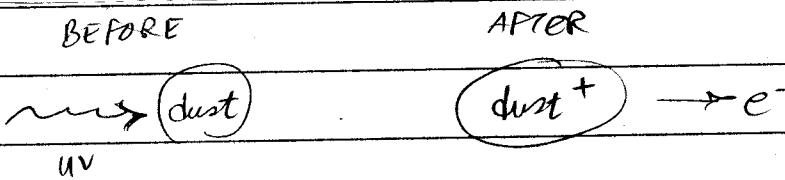
Gianncoli Ch 40 # 45, 46

Ch 39 # 7, 9

 $\alpha_5 \# 4, 9, 14, 17,$ (16)

- (5.4) The photoelectric effect can be an important heating mechanism for the grains of dust found in interstellar clouds (see Section 12.1). The ejection of an electron leaves the grain with a positive charge, which affects the rates at which other electrons and ions collide with and stick to the grain to produce the heating. This process is particularly effective for ultraviolet photons ($\lambda \approx 1000 \text{ \AA}$) striking the smaller dust grains. If the average energy of the ejected electron is about 5 eV, estimate the work function of a typical dust grain.

KE



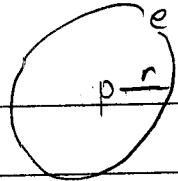
Photon energy = work function + electron Kinetic energy

$$\frac{hc}{\lambda} = \Phi + KE$$

$$hc = 6.63 \times 10^{-34} \text{ J.s} \times 3 \cdot 10^8 \frac{\text{m}}{\text{s}} = 1.99 \times 10^{-25} \text{ J.m} \quad \begin{array}{|c|c|} \hline \text{eV} & \text{\AA} \\ \hline 1.602 \times 10^{-19} \text{ J} & 10^{-10} \text{ m} \\ \hline \end{array}$$

$$\Phi =$$

- 5.9 To demonstrate the relative strength of the electrical and gravitational forces of attraction between the electron and the proton in the Bohr atom, suppose the hydrogen atom were held together *solely* by the force of gravity. Determine the radius of the ground-state orbit (in units of Å and AU) and the energy of the ground state (in eV).



$$F = ma$$

m_e = electron, M_p = proton

$$\frac{Gm_e M_p}{r^2} = m_e v^2 \rightarrow v^2 = \frac{GM}{r}$$

Angular momentum quantization $L = m_e v r = n \hbar \rightarrow v^2 = \frac{(n\hbar)^2}{mr^2}$

$$v^2 = v^2$$

U1

U2

U3

U4

GO

EMP

5/6

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Each quantum state of the H atom is labeled by a set of four quantum numbers $\{n, l, m_l, m_s\}$
 (QN)

② List the sets of QN for the H atom with $n=1, n=2, n=3$

③ Show that the degeneracy of energy level n is $2n^2$

$$l = 0, 1, 2, \dots (n-1); \quad m_l = 0, \pm 1, \pm 2, \dots, \pm l; \quad m_s = \pm \frac{1}{2}$$

Degeneracy means SAME ENERGY : Same n .

- 5.14 A white dwarf is a very dense star, with its ions and electrons packed extremely close together. Each electron may be considered to be located within a region of size $\Delta x \approx 1.5 \times 10^{-10}$ cm. Use Heisenberg's uncertainty principle, Eq. (5.18), to estimate the minimum speed of the electron. Do you think that the effects of relativity will be important for these stars?

$$\Delta x \Delta p \geq \hbar \rightarrow \Delta p = \frac{\hbar}{\Delta x} = \frac{1.06 \times 10^{-34} \text{ J.s}}{1.5 \times 10^{-10} \text{ cm}} / \frac{10^2 \text{ cm}}{\text{m}} =$$

- 5.17 The members of a class of stars known as *Ap stars* are distinguished by their strong magnetic fields (usually a few thousand gauss).¹⁹ The star HD215441 has an unusually strong magnetic field of 34,000 G. Find the frequencies and wavelengths of the three components of the H _{α} spectral line produced by the normal Zeeman effect for this magnetic field.

H _{α} is the red line emitted by e⁻ transitions from n=3 to 2
 $\lambda_0 = 656.28 \text{ \AA}$

This is almost the same as Giancoli Ch 40 #46
 which we did in class last Thurs, Feb. 26.

$$E = \frac{hc}{\lambda} = h\nu \quad \Delta E = h\nu' = \frac{hc}{\lambda'} \frac{\Delta\lambda}{\lambda} = E_0 \frac{\Delta\lambda}{\lambda}$$

For reference, (O eqn 5.20) p. 151 $\nu = \frac{eB}{4\pi c \mu}$
 is the [cgs] version of the
 electron cyclotron frequency derived next in [m/s]

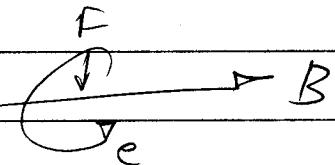
Deriving the cyclotron frequency (Giancoli 27 #66)

708

An electron moving ^{with} \vec{v} to a \vec{B} field experiences a Lorentz force:

$$\vec{F} = q\vec{v}\times\vec{B}$$

$$qVB = \frac{mv^2}{r}$$



Solve for angular frequency

$$2\pi\nu = \omega = \frac{V}{r} =$$

$$\dot{\theta} = \frac{r}{s} = r\nu$$

$$\frac{ds}{dt} = v = r\frac{d\theta}{dt} = r\nu$$

Solve for frequency $\nu = \frac{\omega}{2\pi} =$

Compare this to normal Zeeman frequency,

Frequency shifts: $\Delta\nu = \frac{\Delta E}{h}$ in Giancoli 40 #46:

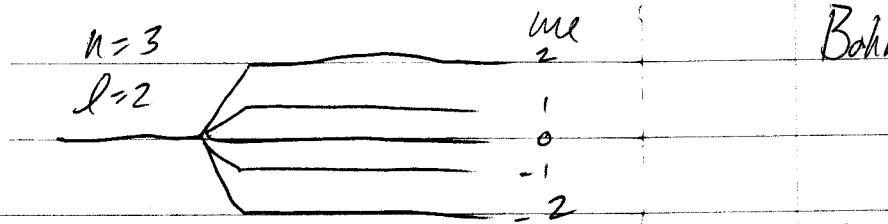
$$\nu_0 = \frac{E_0}{h} = \frac{6.63 \times 10^{-34} \text{ J.s}}{6.63 \times 10^{-34} \text{ J.s}} = \text{Hz}$$

$$\nu_{+} = \frac{E_{+}}{h} = \frac{6.63 \times 10^{-34} \text{ J.s}}{6.63 \times 10^{-34} \text{ J.s}} = \text{Hz}$$

$$\nu_{-} = \frac{E_{-}}{h} = \frac{6.63 \times 10^{-34} \text{ J.s}}{6.63 \times 10^{-34} \text{ J.s}} = \text{Hz}$$

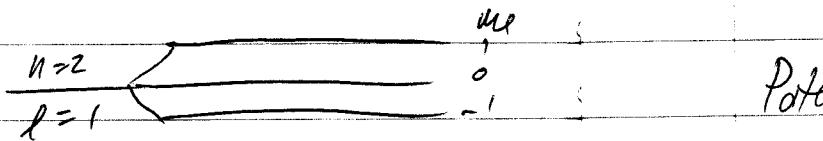
45. (I) Verify that the Bohr magneton has the value $\mu_B = 9.27 \times 10^{-24} \text{ J/T}$ (see Eq. 40-12).

46. (II) Suppose that the splitting of energy levels shown in Fig. 40-4 was produced by a 2.0-T magnetic field. (a) What is the separation in energy between adjacent m_l levels for the same l ? (b) How many different wavelengths will there be for 3d to 2p transitions, if m_l can change only by ± 1 or 0? (c) What is the wavelength for each of these transitions?



$$\textcircled{a} \quad U = -\mu_B B = -\mu_B \mu_0 B$$

$$\Delta U = \Delta m_l \mu_B B$$



\) Transitions are allowed from $\Delta m_l = 0, \pm 1$

$$\textcircled{c} \quad \frac{1}{\lambda_0} = R \left[\frac{1}{n^2} - \frac{1}{n'^2} \right] = \quad n=2, \quad n'=3$$

$\frac{40-2}{1005}$ Orb

$$E = \frac{hc}{\lambda}, \quad \frac{dE}{d\lambda} = hc \frac{d}{d\lambda} (\lambda^{-1}) =$$

$$\Delta E = \frac{hc}{\lambda} \frac{1}{\lambda}$$

$$E_0 = \frac{hc}{\lambda_0}$$

$$E_+ = \frac{hc}{\lambda_0 - D\lambda}$$

$$E_- = \frac{hc}{\lambda_0 + D\lambda}$$

mag

Orb
For

JT

Do

39-3
981

Heisenberg Uncertainty Principle

Uncertainty in position $\Delta x \approx \lambda$ of light used to locate
at particle.

Photon can change particle's momentum by $\Delta p \approx \frac{h}{\lambda}$

$$\Delta x \Delta p \approx \underline{\quad}$$

$$\Delta x \Delta p \approx \Delta t = \frac{h}{2\pi} \quad \text{Do } \frac{\# 7}{1000}$$

The more precisely we measure position (with light of small λ)
the less precisely we can know momentum.

Time uncertainty $\Delta t \approx \frac{\Delta x}{c} \approx \frac{\lambda}{c}$ (travel time of photon
across object)

Photon can change particle's energy by $\Delta E \approx \frac{hc}{\lambda}$

$$\Delta E \Delta t \approx \underline{\quad}$$

$$\Delta E \Delta t \approx \Delta t \quad \text{Do } \frac{\# 9}{1000}$$