

Boas (12) 4 - Partial Differentiation & Series

Cu 1.12
 22 Power series expansions: write an analytic function $f(x)$ as a series of polynomials.

Given $f(x) = \sin x$, find $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n + \dots$

$$x=0: \sin(0) = \underline{\hspace{2cm}} \quad f(x)^0 = \underline{\hspace{2cm}} \quad \rightarrow a_0 = \underline{\hspace{2cm}}$$

$$\text{Differentiate: } f'(x) = \frac{df}{dx} = \underline{\hspace{2cm}}$$

$$x=0: f'(0) = \underline{\hspace{2cm}} \quad \rightarrow a_1 = \underline{\hspace{2cm}}$$

$$\text{Differentiate again: } f''(x) = \frac{d^2f}{dx^2} = \frac{d}{dx} \left(\frac{df}{dx} \right) = \underline{\hspace{2cm}}$$

$$x=0: f''(0) = \underline{\hspace{2cm}} \quad \rightarrow a_2 = \underline{\hspace{2cm}}$$

$$\text{One more: } f'''(x) = \frac{d}{dx} \left(\frac{d^2f}{dx^2} \right) = \underline{\hspace{2cm}}$$

$$x=0: f'''(0) = \underline{\hspace{2cm}} \quad \rightarrow a_3 = \underline{\hspace{2cm}}$$

$$\text{Finally } f(x) = a_0 + a_1x + a_2x^2 + \dots = \underline{\hspace{2cm}} \quad \text{5 of 11}$$

Alternating Series (+ and - terms alternate)
always CONVERGE to $f(x)$ if the terms get
successively smaller: $|a_{n+1}x^{n+1}| > |a_nx^n|$

Practice 3 find the power series expansions for

$$g(x) = \cos(x) \quad \text{and} \quad h(x) = e^x.$$

Notice similarities and differences!

Binomial Series $f(x) = (1+x)^p = a_0 + a_1x + a_2x^2 + \dots$

If x is small, you can often just use the
first two terms of the series.

Ch 1 § 15 #27 Accelerate electrons with voltage V

39

Work done = Kinetic energy

$$U = qV$$

$$E_{\text{tot}} = \gamma mc^2 = mc^2 + K$$

$$K = \underline{\hspace{2cm}}$$

$$U = K$$

Show that $\frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}}$ where $\frac{1}{\gamma} = \frac{mc^2}{qV}$

Then use the Binomial Series to expand $\frac{v}{c} = (1+x)^p$

$$x = \underline{\hspace{2cm}}$$

$$p = \underline{\hspace{2cm}}$$

$$\frac{v}{c} =$$

1.15 #28

39

28) The energy of an electron at speed v in special relativity theory is $mc^2(1 - v^2/c^2)^{-1/2}$, where m is the electron mass, and c is the speed of light. The factor mc^2 is called the rest mass energy (energy when $v = 0$). Find two terms of the series expansion of $(1 - v^2/c^2)^{-1/2}$, and multiply by mc^2 to get the energy at speed v . What is the second term in the energy series? (If v/c is very small, the rest of the series can be neglected; this is true for everyday speeds.)

$$E = mc^2 \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = mc^2 (1+x)^p$$

$$p = \underline{\hspace{2cm}}$$

$$x = \underline{\hspace{2cm}}$$

$$\text{Use } (1+x)^p = 1 + px + \frac{p(p-1)}{2!} x^2 + \dots$$

$$\text{To find } \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \underline{\hspace{2cm}}$$

$$E = \underline{\hspace{2cm}}$$

Ch 4. Partial Differentiation: if $f = f(x, y)$ then

145

91. $\frac{\partial f}{\partial x}$ = change of f with respect to x , HOLDING y constant

$\frac{\partial f}{\partial y}$ = change of f w.r.t. y , HOLDING x constant.

148 PARTIAL DIFFERENTIATION

Ch. 4

PROBLEMS, SECTION 1

1. If $u = x^2/(x^2 + y^2)$, find $\partial u/\partial x$, $\partial u/\partial y$.
2. If $s = t^u$, find $\partial s/\partial t$, $\partial s/\partial u$.
3. If $z = \ln \sqrt{u^2 + v^2 + w^2}$, find $\partial z/\partial u$, $\partial z/\partial v$, $\partial z/\partial w$.
4. For $w = x^3 - y^3 - 2xy + 6$, find $\partial^2 w/\partial x^2$ and $\partial^2 w/\partial y^2$ at the points where $\partial w/\partial x = \partial w/\partial y = 0$.
5. For $w = 8x^4 + y^4 - 2xy^2$, find $\partial^2 w/\partial x^2$ and $\partial^2 w/\partial y^2$ at the points where $\partial w/\partial x = \partial w/\partial y = 0$.
6. For $u = e^x \cos y$, (a) verify that $\partial^2 u/\partial x \partial y = \partial^2 u/\partial y \partial x$;
(b) verify that $\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 = 0$.

If $z = x^2 + 2y^2$, $x = r \cos \theta$, $y = r \sin \theta$, find the following partial derivatives.

- | | | | | | |
|---|---|---|--|---|---|
| 7. $\left(\frac{\partial z}{\partial x}\right)_y$ | 8. $\left(\frac{\partial z}{\partial x}\right)_r$ | 9. $\left(\frac{\partial z}{\partial x}\right)_\theta$ | 10. $\left(\frac{\partial z}{\partial y}\right)_x$ | 11. $\left(\frac{\partial z}{\partial y}\right)_r$ | 12. $\left(\frac{\partial z}{\partial y}\right)_\theta$ |
| 13. $\left(\frac{\partial z}{\partial \theta}\right)_x$ | 14. $\left(\frac{\partial z}{\partial \theta}\right)_r$ | 15. $\left(\frac{\partial z}{\partial \theta}\right)_y$ | 16. $\left(\frac{\partial z}{\partial r}\right)_x$ | 17. $\left(\frac{\partial z}{\partial r}\right)_y$ | 18. $\left(\frac{\partial z}{\partial r}\right)_\theta$ |
| 19. $\frac{\partial^2 z}{\partial r \partial y}$ | 20. $\frac{\partial^2 z}{\partial x \partial \theta}$ | 21. $\frac{\partial^2 z}{\partial y \partial \theta}$ | 22. $\frac{\partial^2 z}{\partial r \partial x}$ | 23. $\frac{\partial^2 z}{\partial r \partial \theta}$ | 24. $\frac{\partial^2 z}{\partial x \partial y}$ |

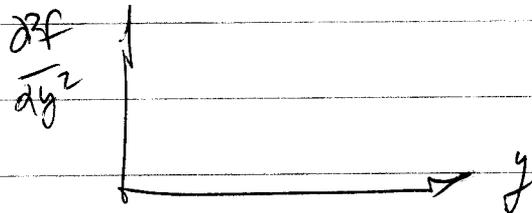
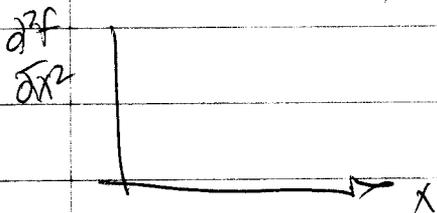
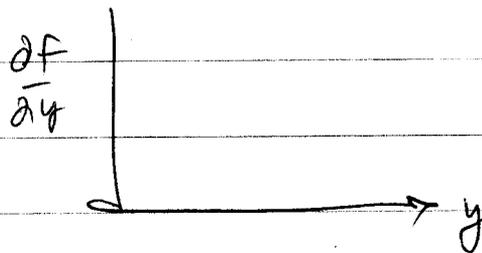
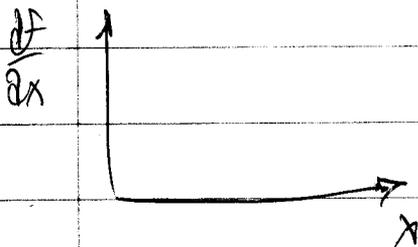
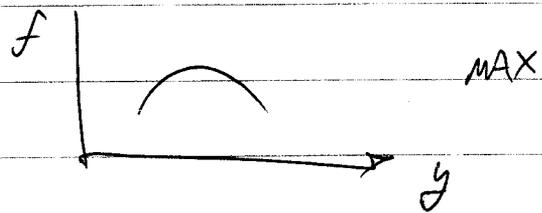
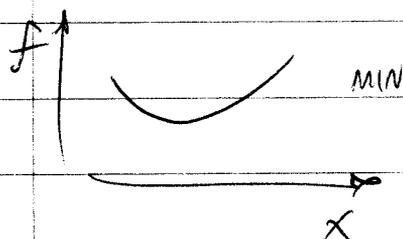
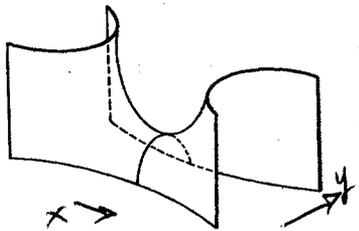
7' to 24'. Repeat Problems 7 to 24 if $z = r^2 \tan^2 \theta$.

4.8 Max-Min of $f(x,y)$ occur where $f' = 0$

169

$$\frac{\partial f}{\partial x} = 0 : \text{slope} = 0 \text{ wrt. } x$$

$$\frac{\partial f}{\partial y} = 0 : \text{slope} = 0 \text{ wrt. } y$$



MIN: f''

MAX: f''