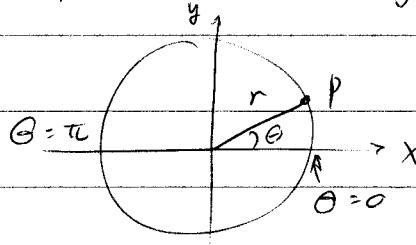


POLAR & SPHERICAL COORDS

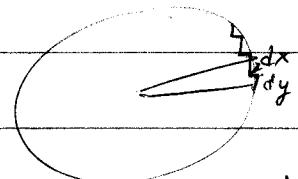
Physics of Astronomy - week 2 - 15 Jan 2004 - E/12

POLAR COORDINATES = 2D

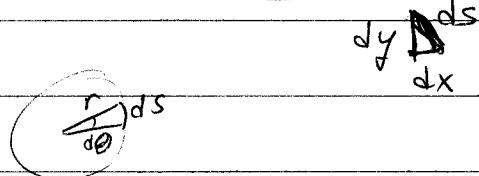
$$\begin{array}{c} r \\ \theta \\ \text{---} \\ x \\ y \end{array}$$



Cartesian arclength element $ds = \sqrt{dx^2 + dy^2}$



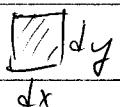
Polar arclength element $ds = r \cdot d\theta$



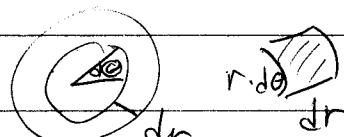
Arclength $S = \int ds$

Circumference $= \int_{\theta=0}^{\theta=2\pi} ds =$

Cartesian Area element $dA = dx \cdot dy$



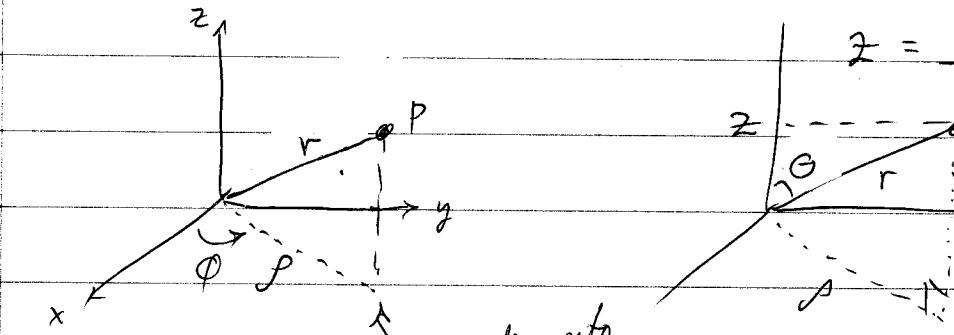
Polar Area element $dA = r d\theta \cdot dr$



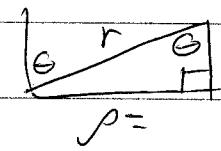
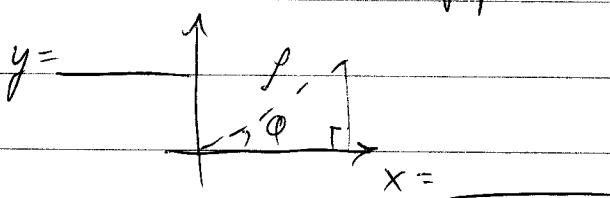
Area of disk = $\iint dA = \int r d\theta / dr = \int r dr / d\theta$

assign limits & find area:

SPHERICAL COORDINATES = 3D

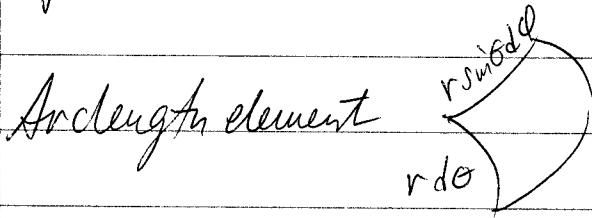


projection onto
x-y plane



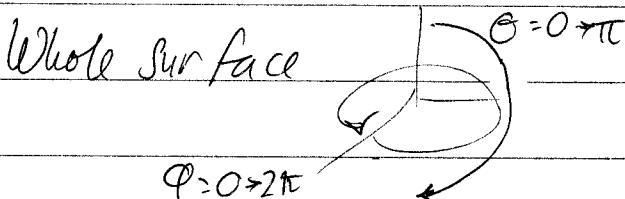
Sub $\rho(r, \theta)$ into $x(\rho, \phi)$

$$y = \underline{\hspace{10mm}} \quad x = \underline{\hspace{10mm}}$$



$$ds^2 = dr^2 + (r d\theta)^2 + (r \sin \theta d\phi)^2$$

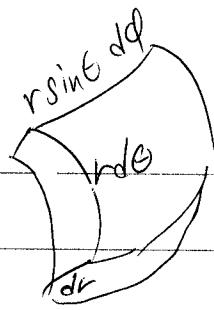
$$\text{Surface area element } dA_{\text{surface}} = \int r d\theta \int r \sin \theta d\phi = r^2 \sin \theta d\theta d\phi$$



Assign limits & evaluate

Area of sphere =

$$\text{Volume element } dV = dr \cdot r d\theta \cdot r \sin\theta d\phi$$



$$\text{Volume of Sphere} = \iiint$$

set up limits and evaluate:

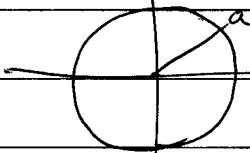
1(a, d), 2(a), 4(a, b), 24, 25
Boats HW Ch 5 f4 p. 225

E12

Physical Systems - weeks 2+3 - due Mon 19 Jan 04

Given a circle of radius a , find by integration using polar coordinates

- (a) its area;
- (b) the centroid of one quadrant;
- (c) the moment of inertia of a circular lamina about a diameter;
- (d) its circumference;
- (e) the centroid of a quarter circle arc.

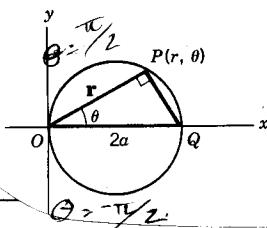


(a) $A = \int \int dA =$

(b) $C = \int ds =$

Using polar coordinates:

- (a) Show that the equation of the circle sketched is $r = 2a \cos \theta$. Hint: Use the right triangle OPQ .
- (b) Find its area by integration.
- (c) Find the centroid of the first quadrant semicircular area.



HINT:

$$\begin{aligned} & \int r \cos \theta \sin \theta d\theta \\ &= +\frac{1}{2} 8r^2 \theta \end{aligned}$$

(c) $A = \int \int dA = \int r dr d\theta \quad dr =$