

## Exercises and Problems for Section 2.5

### Exercises

- The temperature,  $H$ , in degrees Celsius, of a cup of coffee placed on the kitchen counter is given by  $H = f(t)$ , where  $t$  is in minutes since the coffee was put on the counter.
  - Is  $f'(t)$  positive or negative? Give a reason for your answer.
  - What are the units of  $f'(20)$ ? What is its practical meaning in terms of the temperature of the coffee?
- The temperature,  $T$ , in degrees Fahrenheit, of a cold yam placed in a hot oven is given by  $T = f(t)$ , where  $t$  is the time in minutes since the yam was put in the oven.
  - What is the sign of  $f'(t)$ ? Why?
  - What are the units of  $f'(20)$ ? What is the practical meaning of the statement  $f'(20) = 2$ ?
- The cost,  $C$  (in dollars) to produce  $g$  gallons of ice cream can be expressed as  $C = f(g)$ . Using units, explain the meaning of the following statements in terms of ice cream.
  - $f(200) = 350$
  - $f'(200) = 1.4$
- The time for a chemical reaction,  $T$  (in minutes), is a function of the amount of catalyst present,  $a$  (in milliliters), so  $T = f(a)$ .
  - If  $f(5) = 18$ , what are the units of 5? What are the units of 18? What does this statement tell us about the reaction?
  - If  $f'(5) = -3$ , what are the units of 5? What are the units of  $-3$ ? What does this statement tell us?
- After investing \$1000 at an annual interest rate of 7% compounded continuously for  $t$  years, your balance is  $\$B$ , where  $B = f(t)$ . What are the units of  $dB/dt$ ? What is the financial interpretation of  $dB/dt$ ?
- Investing \$1000 at an annual interest rate of  $r\%$ , compounded continuously, for 10 years gives you a balance of  $\$B$ , where  $B = g(r)$ . Give a financial interpretation of the statements:
  - $g(5) \approx 1649$ .
  - $g'(5) \approx 165$ . What are the units of  $g'(5)$ ?
- Suppose  $C(r)$  is the total cost of paying off a car loan borrowed at an annual interest rate of  $r\%$ . What are the units of  $C'(r)$ ? What is the practical meaning of  $C'(r)$ ? What is its sign?
- Suppose  $P(t)$  is the monthly payment, in dollars, on a mortgage which will take  $t$  years to pay off. What are the units of  $P'(t)$ ? What is the practical meaning of  $P'(t)$ ? What is its sign?
- Let  $f(x)$  be the elevation in feet of the Mississippi river  $x$  miles from its source. What are the units of  $f'(x)$ ? What can you say about the sign of  $f'(x)$ ?
- An economist is interested in how the price of a certain item affects its sales. At a price of  $\$p$ , a quantity,  $q$ , of the item is sold. If  $q = f(p)$ , explain the meaning of each of the following statements:
  - $f(150) = 2000$
  - $f'(150) = -25$

### Problems

- A laboratory study investigating the relationship between diet and weight in adult humans found that the weight of a subject,  $W$ , in pounds, was a function,  $W = f(c)$ , of the average number of Calories per day,  $c$ , consumed by the subject.
  - Interpret the statements  $f(1800) = 155$ ,  $f'(2000) = 0$ , and  $f^{-1}(162) = 2200$  in terms of diet and weight.
  - What are the units of  $f'(c) = dW/dc$ ?
- A city grew in population throughout the 1980s. The population was at its largest in 1990, and then shrank throughout the 1990s. Let  $P = f(t)$  represent the population of the city  $t$  years since 1980. Sketch graphs of  $f(t)$  and  $f'(t)$ , labeling the units on the axes.
- If  $t$  is the number of years since 1993, the population,  $P$ , of China, in billions, can be approximated by the function
 
$$P = f(t) = 1.15(1.014)^t.$$
 Estimate  $f(6)$  and  $f'(6)$ , giving units. What do these two numbers tell you about the population of China?
- For some painkillers, the size of the dose,  $D$ , given depends on the weight of the patient,  $W$ . Thus,  $D = f(W)$ , where  $D$  is in milligrams and  $W$  is in pounds.
  - Interpret the statements  $f(140) = 120$  and  $f'(140) = 3$  in terms of this painkiller.
  - Use the information in the statements in part (a) to estimate  $f(145)$ .
- Let  $f(t)$  be the number of centimeters of rainfall that has fallen since midnight, where  $t$  is the time in hours. Interpret the following in practical terms, giving units.
  - $f(10) = 3.1$
  - $f^{-1}(10) = 16$
  - $f'(8) = 0.4$
  - $(f^{-1})'(5) = 2$
- Let  $p(h)$  be the pressure in dynes per  $\text{cm}^2$  on a diver at a depth of  $h$  meters below the surface of the ocean. What do each of the following quantities mean to the diver? Give units for the quantities.
  - $p(100)$
  - $h$  such that  $p(h) = 1.2 \cdot 10^6$
  - $p(h) + 20$
  - $p(h + 20)$
  - $p'(100)$
  - $h$  such that  $p'(h) = 20$

## Exercises and Problems for Section 2.6

## Exercises

1. For the function graphed in Figure 2.52, are the following quantities positive or negative?

(a)  $f(2)$       (b)  $f'(2)$       (c)  $f''(2)$

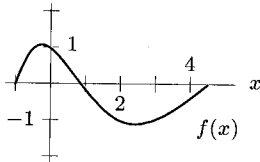


Figure 2.52

2. The graph of a function  $f(x)$  is shown in Figure 2.53. On a copy of the table indicate whether  $f$ ,  $f'$ ,  $f''$  at each marked point is positive, negative, or zero.

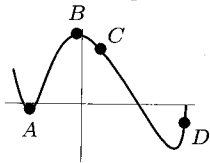


Figure 2.53

Point	$f$	$f'$	$f''$
A			
B			
C			
D			

3. At which of the labeled points on the graph in Figure 2.54 are both  $dy/dx$  and  $d^2y/dx^2$  positive?

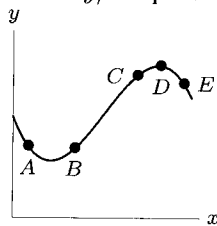
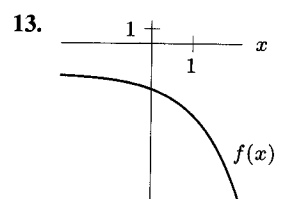
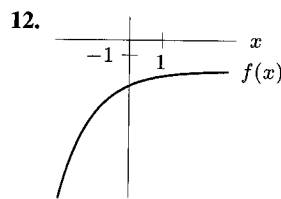
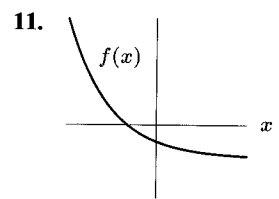
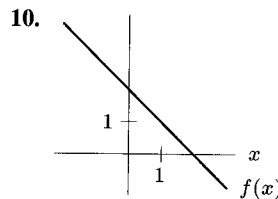
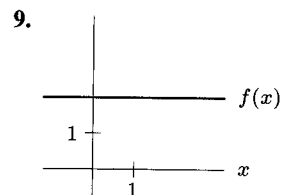
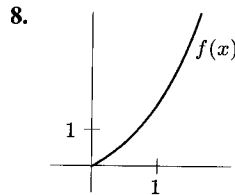


Figure 2.54

5. Sketch the graph of a function whose first and second derivatives are everywhere positive.
6. Sketch the graph of a function whose first derivative is everywhere negative and whose second derivative is positive for some  $x$ -values and negative for other  $x$ -values.
7. Sketch the graph of the height of a particle against time if velocity is positive and acceleration is negative.

For Exercises 8–13, give the signs of the first and second derivatives for each of the following functions.



4. The distance of a car from its initial position  $t$  minutes after setting out is given by  $s(t) = 5t^2 + 3$  kilometers. What are the car's velocity and acceleration at time  $t$ ? Give units.

## Problems

14. The table gives the number of passenger cars,  $C = f(t)$ , in millions, in the US in the year  $t$ .
- (a) Do  $f'(t)$  and  $f''(t)$  appear to be positive or negative during the period 1940–1980?
- (b) Estimate  $f'(1975)$ . Using units, interpret your answer in terms of passenger cars.
15. An accelerating sports car goes from 0 mph to 60 mph in five seconds. Its velocity is given in the following table, converted from miles per hour to feet per second, so that all time measurements are in seconds. (Note: 1 mph is  $22/15$  ft/sec.) Find the average acceleration of the car over each of the first two seconds.

$t$ (year)	1940	1950	1960	1970	1980
$C$ (cars, in millions)	27.5	40.3	61.7	89.3	121.6

Time, $t$ (sec)	0	1	2	3	4	5
Velocity, $v(t)$ (ft/sec)	0	30	52	68	80	88

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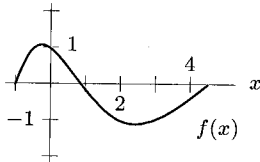


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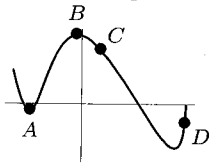


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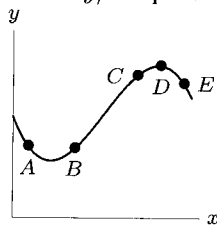
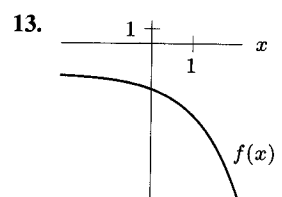
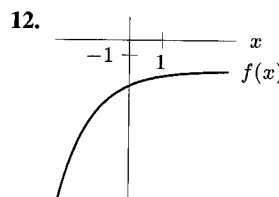
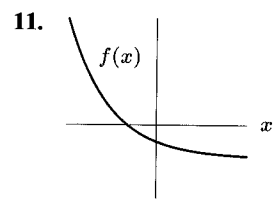
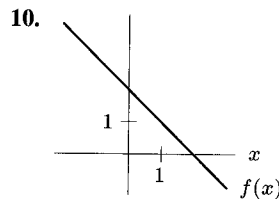
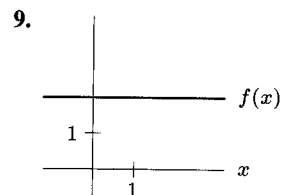
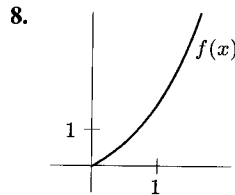


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