Astrophysics Ch 3: Light & Spectra
1, 6, 8, 13

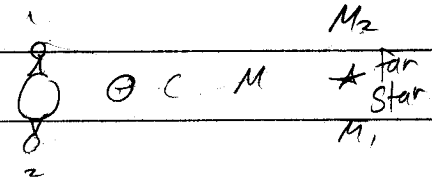
E17
due ~~Feb 04~~ 12 Feb

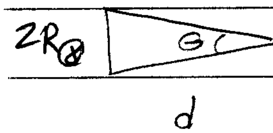
3.1 In 1672, an international effort was made to measure the parallax angle of Mars at the time of opposition, when it was closest to Earth; see Fig. 1.6.

(a) Consider two observers who are separated by a baseline equal to Earth's diameter. If the difference in their measurements of Mars' angular position is 33.6″, what is the distance between Earth and Mars at the time of opposition? Express your answer both in units of cm and AU.

$\theta =$

(b) If the distance to Mars is to be measured to within 10%, how closely must the clocks used by the two observers be synchronized? *Hint:* Ignore the rotation of Earth. The average orbital velocities of Earth and Mars are 29.79 km s^{-1} and 24.13 km s^{-1}, respectively.



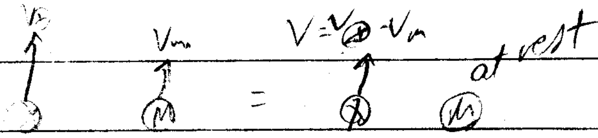


$R_\otimes = 6.38 \times 10^6$ m

$$2R_\otimes = d\,\theta \quad \text{where} \quad \theta = 33.6'' \left|\frac{1'}{60''}\right|\frac{1^\circ}{60'}\left|\frac{2\pi\ \text{rad}}{360^\circ}\right| = \frac{1.63\cdot 10^{-4}}{\text{rad}}$$

$$d = \frac{2R_\otimes}{\theta} = \frac{R_\otimes}{\theta/2} = 7.83\cdot 10^{10}\,\text{m}\left|\frac{AU}{1.49\cdot 10^{11}\,\text{m}}\right| = 5.3\ AU$$

(b) The motion of Mars & Earth will both contribute to the uncertainty:



$V = 29.79 - 24.13 = 5.66\cdot 10^3\ \frac{m}{s}$

$$\frac{\Delta d}{d} = \frac{1}{10} = \left|\frac{d' - d}{d'}\right| \quad \text{where the } d \text{ is wrong; actually,} \quad d' = \frac{2R_\otimes + V\Delta t}{\theta}$$

$$0.1 = 1 - \frac{d}{d'}$$

$$\frac{d}{d'} = 0.9 = \frac{9}{10} \rightarrow d' = \frac{10}{9}d = \frac{10}{9}\left(2R_\otimes/\theta\right)$$

$$d' = \frac{2R_\otimes + V\Delta t}{\theta} = \frac{10}{9}\left(\frac{2R_\otimes}{\theta}\right) \rightarrow 2R_\otimes\left(\frac{10}{9} - 1\right) = V\Delta t$$

$$\Delta t = \frac{2R_\otimes}{V}\frac{1}{9} = \frac{2}{9}\,\frac{6.38\cdot 10^6\,\text{m}}{5.66\cdot 10^3\,\frac{\text{m}}{\text{s}}} = \underline{250}\ \text{s}$$

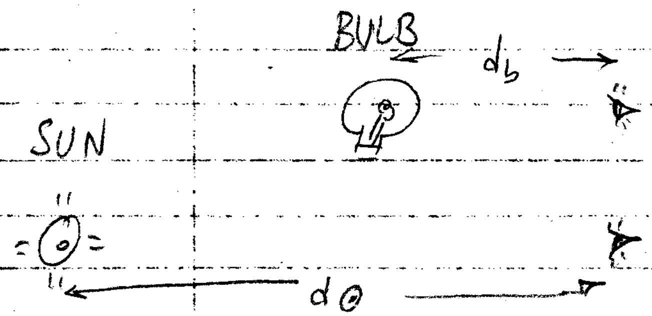
Power = Luminosity $= L_b$

3.2 At what distance d_b from a $P_b = 100$ watt light bulb is the radiant flux equal to solar constant?

p.77 $L_\odot = 3.826 \cdot 10^{33} \frac{\text{erg}}{\text{s}} \left| \frac{J}{10^7 \text{erg}} \right| = \underline{3.826 \cdot 10^{26}}$ watt

p.67 $\underset{\text{received}}{\text{Flux}} = \frac{L}{4\pi d^2}$ where d = distance from source

$F_{bulb} = F_{sun}$ (at earth) $= 1.36 \cdot 10^6 \frac{\text{ergs}}{\text{s-cm}^2} = 1360 \text{ Watts/m}^2$ = solar constant



Bulb's flux = Sun's flux

$$F_b = \frac{L_b}{4\pi d_b^2} = \frac{L_\odot}{4\pi d_\odot^2} = F_\odot$$

$$\frac{d_b^2}{L_b} = \frac{d_\odot^2}{L_\odot}$$

$$\left(\frac{d_b}{d_\odot}\right)^2 = \frac{L_b}{L_\odot} = \frac{100 \text{ Watt}}{3.826 \cdot 10^{26} \text{ Watt}} = \underline{\qquad}$$

$$d_b = d_\odot \sqrt{\frac{L_b}{L_\odot}} = 1.5 \cdot 10^{11} \text{ m} \sqrt{\qquad} = \underline{7.65 \cdot 10^{-2} \text{ m}}$$

3.5 - Derive $m - M_\odot = -\frac{5}{2}\log_{10}\left(\frac{F}{F_{10,\odot}}\right)$ $\frac{(3.9)}{68}$

$= +\frac{5}{2}\log_{10}\left(\frac{F_{10,\odot}}{F}\right)$

"Where $F_{10,\odot}$ is the radiant flux received from the Sun at a distance of $d = 10\,pc$" (p.69)

$\frac{(3.6)}{68}$ $m - M = 5\log_{10}\left(\frac{d}{10pc}\right)$

distance to source

calibration distance

absolute magnitude @ calibration distance

apparent magnitude of source

$\frac{(3.8)}{68}$ $M - M_\odot = -\frac{2}{5}\log_{10}\left(\frac{L}{L_\odot}\right) = \frac{2}{5}\log_{10}\left(\frac{L_\odot}{L}\right)$

source

abs. mag of Sun

Sun's Luminosity

abs. mag. of source

$$M = M_\odot + \frac{2}{5}\log\left(\frac{L_\odot}{L}\right)$$

$$m - M = m - \left(M_\odot + \frac{2}{5}\log\frac{L_\odot}{L}\right) = 5\log\left(\frac{d}{10pc}\right) \quad (1)$$

$$F = \frac{L}{4\pi d^2} \quad \text{so} \quad \frac{F_\odot}{F} = \frac{L_\odot/4\pi d_\odot^2}{L/4\pi d^2} = \frac{L_\odot}{L}\left(\frac{d}{d_\odot}\right)^2$$

$$\frac{F_{\odot,10}}{F} = \frac{L_\odot}{L}\left(\frac{d}{10pc}\right)^2 \quad (2)$$

$$m - M_\odot = 5\log\frac{d}{10pc} + \frac{2}{5}\log\frac{L_\odot}{L}$$

$$= 5\log\frac{d}{10pc} - \frac{5}{2}\log\frac{L}{L_\odot}$$

3.5 Cont'd

$$m - M_\odot = 5\left[\log\frac{d}{10pc} - \frac{1}{2}\log\frac{L}{L_\odot}\right]$$

$$= 5\left[\log\frac{d}{10pc} + \log\left(\frac{L_\odot}{L}\right)^{1/2}\right]$$

$$\log A + \log B = \log A \cdot B$$

$$\log\frac{d}{10pc} + \log\left(\frac{L_\odot}{L}\right)^{1/2} = \log\frac{d}{10pc}\,\frac{L_\odot}{L}^{1/2}$$

$$= \log\left[\left(\frac{d}{10pc}\right)^2 \frac{L_\odot}{L}\right]^{1/2} = \frac{1}{2}\log\left[\left(\frac{d}{10pc}\right)^2\frac{L_\odot}{L}\right]$$

$$(2):\quad \frac{F_{\odot,10}}{F} = \frac{L_\odot}{L}\left(\frac{d}{10pc}\right)^2$$

$$m - M_\odot = \frac{5}{2}\log\left[\left(\frac{d}{10pc}\right)^2\frac{L_\odot}{L}\right] = \frac{5}{2}\log\left(\frac{F_{\odot,10}}{F}\right)$$

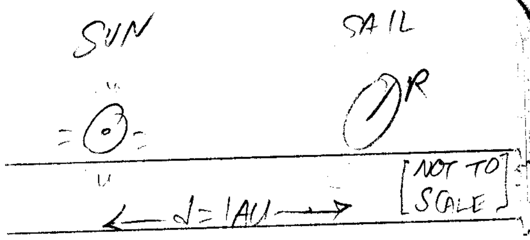
U1 U2 U3 U4 G6

3.6 A 1.2×10^4 kg spacecraft ($M =$) is launched from Earth and is to be accelerated radially away from the Sun using a circular solar sail. The initial acceleration of the spacecraft is to be $1g$ ($= a$). Assuming a flat sail, determine the radius of the sail if it is

(a) black, so it absorbs the Sun's light.

(b) shiny, so it reflects the Sun's light.

Hint: The spacecraft, like Earth, is orbiting the Sun. Should you include the Sun's gravity in your calculation?



p. 741

$$F = m \cdot a \qquad F_{rad} = \frac{k \langle S \rangle A}{c}$$

$$a = g = 9.8 \frac{m}{s^2} \qquad k = \begin{cases} 1 : \text{absorption} & : \text{BLACK} \\ 2 : \text{reflection} & : \text{SHINY} \end{cases}$$

$$\langle S \rangle = \frac{E_0 B_0}{2\mu_0} = \frac{\text{power}}{\text{area}} = \text{intensity of solar radiation at distance } d$$

$$\text{Intensity} = \frac{\text{power}}{\text{area}} = \frac{L_0}{4\pi d^2} = \langle S_0 \rangle = 1360 \text{ Watts}$$

$$mg = \frac{k \langle S \rangle A}{c} \rightarrow A = \frac{c\, m g}{k \langle S \rangle}$$

$$A = \frac{3.8 \cdot 10^8 \frac{m}{s} (1.2 \cdot 10^4 \text{ kg})\, 9.8 \frac{m}{s^2}}{k \cdot 1360 \left(\frac{\text{Watts}}{m^2} = \frac{\text{Joules}/\text{sec}}{m^2} = \frac{\text{kg}\, m^2 / s^3}{m^2}\right)}$$

$$A = \frac{1}{k}\, 3.3 \times 10^{10}\ m^2 = \pi R^2 \rightarrow R = \sqrt{\frac{A}{\pi}}$$

$$R_{black} = 10^5\ m = 100\ km \qquad R_{shiny} = \frac{1}{\sqrt{2}} R_{black} = 72\ km$$

Need not include Sun's gravity, since sail is always orbiting the Sun, with Earth

$$\frac{\text{Power}}{\text{Area}} = \text{radiant Flux}$$

$$F = \frac{L}{4\pi r^2} = \sigma T^4$$

3.7 The average person has 1.4 m^2 of skin $= A$ at a skin temperature of roughly 92°F (306 K) $= T$. Consider the average person to be an ideal radiator standing in a room at a temperature of 68°F (293 K). $= T_0$

(a) Calculate the energy per second (POWER) radiated by the average person in the form of blackbody radiation. Express your answer both in units of erg s^{-1} and in watts.

Luminosity

(a) Power radiated $= L_{out} = A\sigma T^4$ where $\sigma = 5.67\times10^{-5}\ \frac{\text{erg}}{\text{s cm}^2\text{K}^4}$

$$L = 1.4\,\text{m}^2\left(5.67\times10^{-8}\frac{\text{W}}{\text{m}^2\text{K}^4}\right)(306\,\text{K})^4 = 696\,\text{W}\left(\frac{10^7\,\text{erg/s}}{\text{W}}\right)$$

$$= 6.96\times10^9\ \text{erg/s}$$

(b) Determine the peak wavelength λ_{max} of the blackbody radiation emitted by the average person. In what region of the electromagnetic spectrum is this wavelength found?

$$\lambda_{max} = \frac{0.29\ \text{cm K}}{T} = \frac{0.29}{306\,\text{K}} = 9.5\times10^{-4}\,\text{cm}\left|\frac{\text{m}}{10^2\,\text{cm}}\right|\frac{10^{10}\,\text{Å}}{\text{m}}$$

$= 9.5\times10^4$ Å : INFRARED

(c) A blackbody also absorbs energy from its environment, in this case from the 293-K room ($T_0 =$). The equation describing the absorption is the same as the equation describing the emission of blackbody radiation, Eq. (3.16). Calculate the energy per second absorbed by the average person, expressed both in units of erg s^{-1} and in watts.

$$\text{Power absorbed} = L_{in} = A\sigma T_0^4 = L_{out}\left(\frac{T_0}{T}\right)^4 = 696\,\text{W}\left(\frac{293}{306}\right)^4 = 585\,\text{W}$$

(d) Calculate the net energy per second lost by the average person due to blackbody radiation.

$$\text{Energy lost per second} = \text{Power radiated} - \text{Power absorbed} = L_{out} - L_{in} = 696 - 585$$

$$= 111\ \text{W}$$