## PROBLEMS, SECTION 4 Ch 6 Boas

1. Verify equations (4.5) by writing out the components.

2. Let the position vector (with its tail at the origin) of a moving particle be $\mathbf{r} = \mathbf{r}(t) = t^2\mathbf{i} - 2t\mathbf{j} + (t^2 + 2t)\mathbf{k}$, where $t$ represents time. $P = r(2)$
   (a) Show that the particle goes through the point $(4, -4, 8)$. At what time does it do this?
   (b) Find the velocity vector and the speed of the particle at time $t$; at the time when it passes through the point $(4, -4, 8)$.
   (c) Find the equations of the line tangent to the curve described by the particle and the plane normal to this curve, at the point $(4, -4, 8)$. $= P$

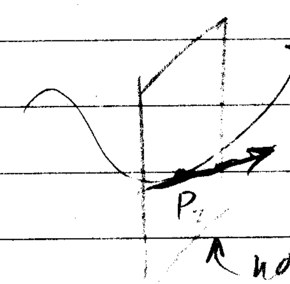
(a) $r_x = t^2 = 4$ when $t = 2$

$r_y = -2t = -4$ when $t = 2$

$r_z = t^2 + 2t = 8$ when $t = 2$ ✓

(b) $\vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{d}{dt}\left(t^2\hat{i} - 2t\hat{j} + (t^2+2t)\hat{k}\right)$

$\vec{v} = 2t\,\hat{i} - 2\hat{j} + (2t+2)\hat{k}$

(c) 


$\vec{r}$ = curve described by particle

$\vec{v}$ = slope of r = tangent to path = direction of normal plane

normal plane (perpendicular)

Tangent line $= P + \vec{v}t = \vec{r}(2) + \vec{v}(2)\,t$

$= (4, -4, 8) + (2t, -2, (2t+2))\big|_{t=2} \cdot t$

$= (4, -4, 8) + (2 \cdot 2, -2, (2 \cdot 2 + 2)) \cdot t$

$= (4, -4, 8) + (4, -2, 6)\,t$

Tangent line $= (4 + 4t)\hat{i} - (4 + 2t)\hat{j} + (8 + 6t)\hat{k}$

p. 108: if $\vec{N} = a\hat{i} + b\hat{j} + c\hat{k}$ is normal to a plane

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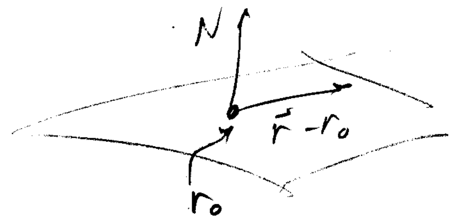



Boas (p.108) and $(\vec{r} - \vec{r}_0)$ is a vector in the plane,

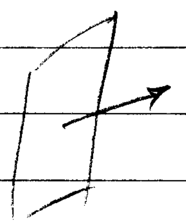
then $\vec{N}$ and $(\vec{r} - \vec{r}_0)$ are perpendicular, so the equation of the plane is $\vec{N} \cdot (\vec{r} - \vec{r}_0) = 0$ or

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 \qquad \underline{(5.10)}$$

p. 108

In our case, $(x_0, y_0, z_0) = (4, -4, 8) = P$

and



$$\vec{V}_P = \vec{N} = (4, -2, 6) = a\hat{i} + b\hat{j} + c\hat{k}$$
$$= (a, b, c)$$

So __tangent plane equation__ is

$$4(x-4) - 2(y+4) + 6(z-8) = 0$$

$$4x - 16 - 2y - 8 + 6z - 48 = 0$$
$$2x - 8 - y - 4 + 3z - 24 = 0$$
$$2x - y + 3z = 8 + 4 + 24$$
$$\underline{2x - y + 3z = 36}$$

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