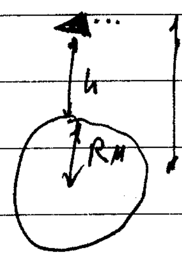
Phys A - week 6 - Giancoli Ch 6 - Gravity + Orbits
6, 11, 36, 52, 54, 60
done for wk 3: ✓ ✓ ✓

G.6.52 / 153 Apollo orbited Moon at $h = 100\,km = 10^5\,m$.
How long did it take to go around Moon once?



$R = R_M + h =$

$F = ma \quad \frac{G\cancel{m}M}{R^2} = \frac{mv^2}{R} = \frac{\cancel{m}}{R}\left(\frac{2\pi R}{T}\right)^2$

Solve algebraically for $T^2 = \frac{4\pi^2 R^3}{GM}$

$R = R_M + h = 1.84\times10^6\,m$

Use $M_{moon} = M = 7.35\times10^{22}\,kg$ $\quad R_{moon} = R_M = 1.74\times10^6\,m$

Numbers for $T^2 = \frac{4\pi^2(1.74\times10^6\,m + 10^5\,m)^3}{6.67\times10^{-11}\frac{m^3}{\cancel{kg}\cdot s^2}(7.35\times10^{22}\,\cancel{kg})} = 5\times10^7\,s^2$

$T = 7\times10^3\,s \left|\frac{hr}{3600\,s}\right| = 2\,hrs$

G-6.54 (a) Calculate F of Sun on Earth, and of Moon on Earth.

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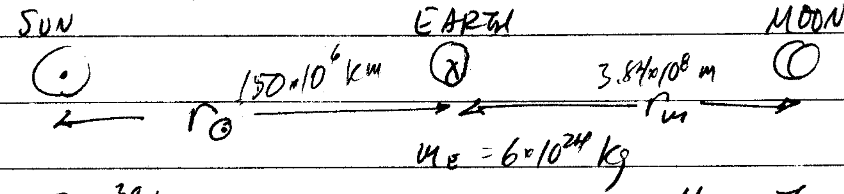


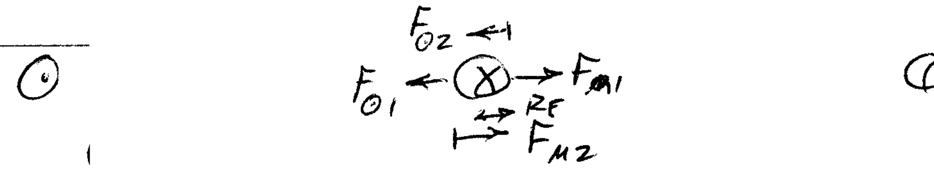
$M_\odot = 2 \times 10^{30}$ kg $\qquad M_M = 7.35 \times 10^{22}$ kg

$$F_\odot = \frac{G M_\odot m_E}{r_\odot^2} = \frac{6.67 \times 10^{-11} \frac{m^3}{kg \cdot s^2} \, 2 \times 10^{30} kg \cdot 6 \times 10^{24} kg}{(1.5 \times 10^{11} m)^2} = 3.6 \times 10^{22} \, N$$

$$F_M = \frac{G M_M m_E}{r_M^2} = \frac{6.67 \times 10^{-11} \frac{m^3}{kg s^2} \, 7.35 \times 10^{22} kg \cdot 6 \cdot 10^{24} kg}{(3.84 \cdot 10^8 \, m)^2} = 2 \times 10^{20} \, N$$

(b) If the Sun's $F_\odot \gg F_M$, how come Moon has a greater effect on Earth's tides?

Consider the DIFFERENCE in each F across the Earth:



Sun's force: strongest $F_{\odot 1} = \frac{G M_\odot m_E}{(r_\odot - R_E)^2}$, weakest $F_{\odot 2} = \frac{G M_\odot m_E}{(r_\odot + R_E)^2}$

$$\frac{F_{\odot 1}}{F_{\odot 2}} = \frac{(r_\odot + R_E)^2}{(r_\odot - R_E)^2} = \frac{r_\odot^2 + 2 r_\odot R_E + R_E^2}{r_\odot^2 - 2 r_\odot R_E + R_E^2} \simeq \frac{r_\odot^2 + 2 r_\odot R_E}{r_\odot^2 - 2 r_\odot R_E} = \frac{1 + 2R_E/r_\odot}{1 - 2R_E/r_\odot}$$

$$\frac{R_E}{r_\odot} = \frac{6.4 \times 10^6 \, m}{1.5 \times 10^{11} \, m} \sim 4 \times 10^{-5} \quad \text{So} \quad \frac{F_{\odot 1}}{F_{\odot 2}} = \frac{1 + 8 \times 10^{-5}}{1 - 8 \times 10^{-5}} \approx 1$$

SUN　　　　　　　　EARTH

$\leftarrow F_{\odot 2}$

6.54 Continued: $\odot$ - $F_{\odot 1} \leftarrow \otimes$　　　$\frac{F_{\odot 1}}{F_{\odot 2}} \approx 1$

Sun's gravitational pull on the far side of the Earth is nearly the same as on the near side, so Sun exerts little TIDAL force on Earth.

On the other hand, consider the Moon's effect

EARTH

$\otimes \rightarrow F_{M1} = \frac{GM_M M_E}{(r_M - R_E)^2}$ stronger on near side ☾

$\longmapsto F_{M2} = \frac{GM_M M_E}{(r_M + R_E)^2}$ weaker on far side

$$\frac{F_{M1}}{F_{M2}} = \frac{(r_M+R_E)^2}{(r_M-R_E)^2} \approx \frac{r_M^2 + 2R_E r_M}{r_M^2 - 2R_E r_M} = \frac{1 + 2R_E/r_M}{1 - 2R_E/r_M}$$

$$\frac{R_E}{r_M} = \frac{6.4\times10^6\,m}{3.84\times10^8\,m} \sim 2\times10^{-2}$$ (1.67×10^{-2}, using calculator)

(3%, using calculator)

$$\frac{F_{M1}}{F_{M2}} \approx \frac{1+0.02}{1-0.02} = 1.04$$: 4% stronger on near side than on far side of Earth

This non-negligible DIFFERENCE in grav forces from one side of Earth to the other means the MOON has a STRONGER TIDAL FORCE than the Sun does, on Earth.

G.6.60

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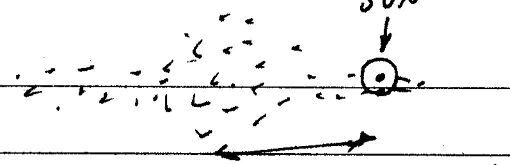
MILKY WAY GALAXY

SUN orbits center of MW with:



$$r = 30{,}000\,ly \left|\frac{10^{16}\,m}{ly}\right| = \underline{3\times10^{20}\,m}$$
3×10^4

$$T = 200\times10^6\,yr\left|\frac{\pi\times10^7\,sec}{1\ \ yr}\right| \approx \underline{2\pi\times10^{15}\,sec}$$

(a) How much mass in the MW inside Sun's orbit?

$F=ma$

$$\frac{GmM}{r^2} = m\left(\frac{v^2}{r}\right) = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 \longrightarrow$$ Solve for M algebraically (then insert numbers)

$$M = \frac{4\pi^2 r^2}{r T^2}\ \frac{r^2}{G} = \frac{4\pi^2 r^3}{G T^2} = \frac{4\pi^2 (3\times10^{20}\,m)^3}{\frac{20}{3}\times10^{-11}\frac{m^3}{kg\,s^2}(2\pi\times10^{15}s)^2}$$

$$= \frac{3\cdot3\cdot3\cdot3\times10^{60-30+11}}{20} = \frac{27\cdot3}{20}\times10^{30+11} = \underline{4\times10^{41}\,kg}$$

(b) How many N solar masses is this? $M_\odot = 2\times10^{30}\,kg$

$$M = N M_\odot \rightarrow N = \frac{M}{M_\odot} = \frac{4\times10^{41}\,kg}{2\times10^{30}\,kg} = 2\times10^{11}$$

There are approx 200 billion Suns worth of matter in the Milky Way Galaxy (inside our Sun's orbit – and more beyond that)