Giancoli

Ch 9 # 11 (Matt E + Ingrid L)

11. (I) An atomic nucleus at rest decays radioactively into an alpha particle and a smaller nucleus. What will be the speed of this recoiling nucleus if the speed of the alpha particle is 2.5×10^5 m/s? Assume the nucleus has a mass 57 times greater than that of the alpha particle.

$$n \xleftarrow{V_n} x \xrightarrow{V_\alpha} \alpha$$

$\Sigma \vec{F} = 0 = m_n V_n - m_\alpha V_\alpha$

$m_n V_n = V_\alpha m_\alpha$

Find $V_n = \dfrac{V_\alpha m_\alpha}{m_N}$

$m_N = 57\, m_\alpha$

$V_n = \dfrac{2.5 \times 10^5 \frac{m}{s}}{57} = 4.4 \times 10^3\ m/s$ BIGGER → SLOWER

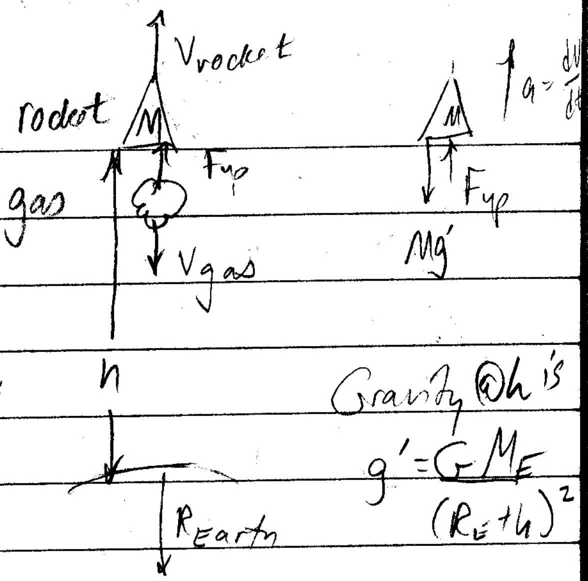
(Jada) 6.4×10^6 m $= R_E$ by coincidence!

$h =$ $= V_r$

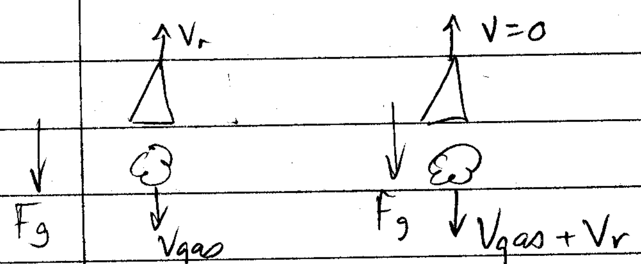
*78. (II) A rocket traveling 1850 m/s away from the Earth at an altitude of 6400 km fires its rockets, which eject gas at a speed of 1200 m/s (relative to the rocket). If the mass of the rocket at this moment is 25,000 kg and an acceleration of 1.7 m/s² is desired, at what rate must the gases be ejected?

V_{gas}

$a = \frac{dV_r}{dt}$ M



Earth's rest frame: Rocket's rest frame:

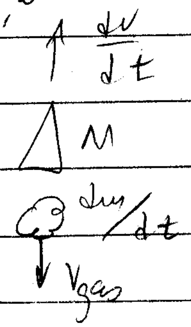


Gravity @h is

$g' = \frac{GM_E}{(R_E+h)^2}$

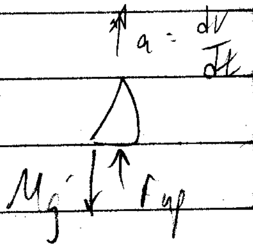
Actually, we are told that $V_{gas} = 1200\,\frac{m}{s}$ relative to rocket, away

Momentum conservation of the rocket-gas system:



$\downarrow P_{gas} = P_{rocket} \uparrow$

$V_{gas}\frac{dm}{dt} = F_{up}$



Forces on rocket: Gravity ↓ and Thrust ↑

$\Sigma F = Ma$

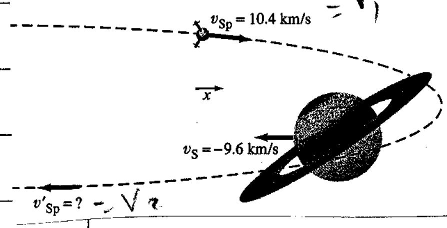
$F_{up} - Mg' = Ma = V_{gas}\frac{dm}{dt} - Mg'$

Gas must be ejected at rate $\frac{dm}{dt} = M(a+g')/V_{gas}$

$g' = \frac{GM_E}{(R_E+R_E)^2} = \frac{GM_E}{(2R_E)^2} = \frac{g}{4} = \frac{9.8\,\frac{m}{s^2}}{4} = 2.45\,\frac{m}{s^2}$

$\frac{dM}{dt} = (2.5\times10^4\,kg)\left(1.7 + 2.45\,\frac{m}{s^2}\right)/1.2\times10^3\,\frac{m}{s} = \underline{86.6}\ \frac{kg}{s}$

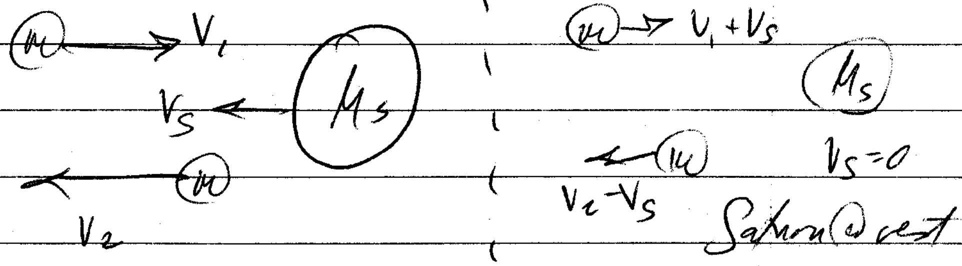
99. The gravitational slingshot effect in Fig. 9–52, shows the planet Saturn moving in the negative x direction at its orbital speed (with respect to the Sun) of 9.6 km/s. The mass of Saturn is 5.69×10^{26} kg. A spacecraft with mass 825 kg approaches Saturn, moving initially in the $+x$ direction at 10.4 km/s. The gravitational attraction of Saturn (a conservative force) causes the spacecraft to swing around it (orbit shown as dashed line) and head off in the opposite direction. Estimate the final speed of the spacecraft after it is far enough away to be nearly free of Saturn's gravitational pull.



We can neglect the change in Saturn's speed due to the momentum transfer, since it is so massive: $dV_{spaceship}\, m_{spaceship} = dV_{Sat}\, M_{Sat}$

$$\frac{dV_{Saturn}}{dV_{spaceship}} = \frac{m_{spaceship}}{m_{saturn}} \sim \frac{10^3 \text{ kg}}{10^{26} \text{ kg}} \sim 10^{-24}$$

Transform to a new frame



$P_{before} = P_{after}$

$$m(V_1 + V_S) = m(V_2 - V_S)$$

$$V_2 = V_1 + 2V_S \,!$$

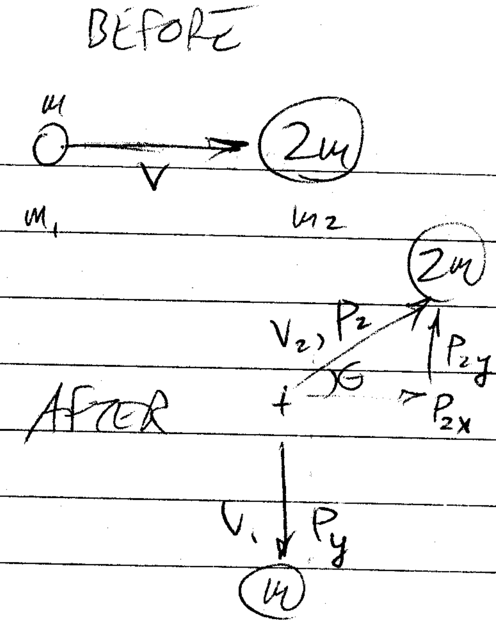
We ignored the details of the gravitational interaction and considered endpoints before & after the interaction.

Saturn can be modeled as a moving wall, off which the spaceship bounces.

$$V_2 = V_1 + 2V_S = 10.4\,\frac{km}{s} + 2\left(9.6\,\frac{km}{s}\right) = 29.6\,\text{km/s}$$

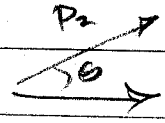
53. (II) An atomic nucleus of mass m traveling with speed v collides elastically with a target particle of mass $2m$ (initially at rest) and is scattered at 90°. (*a*) At what angle does the target particle move after the collision? (*b*) What are the final speeds of the two particles? (*c*) What fraction of the initial kinetic energy is transferred to the target particle?

Find θ
Find v_1, v_2



Solve for θ, V_1, V_2
in terms of V, m

Initially (BEFORE) $p_1 = m_1 V$ in the x-dir



AFTER: in the x-direction $p_{2x} =$

$$= m_2 V_{2x} = p_2 \cos\theta$$

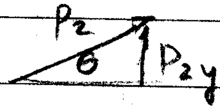
$$p_{2x} = 2m\, V_2 \cos\theta$$

Since $p_1 = p_{1x} = p_{2x}$

$$m_1 V = 2m V_2 \cos\theta$$

$$\boxed{V = 2V_2 \cos\theta} \quad (1)$$

BEFORE: $P_y = 0$



AFTER: $p_{2y} = 2m V_2 \sin\theta$

P_y before $= P_y$ after $\quad P_{1y} = -V_1 m$

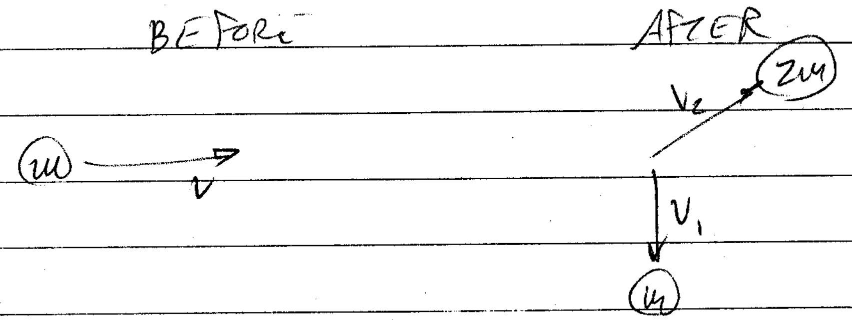
$$0 = -m V_1 + 2m V_2 \sin\theta$$

$$\boxed{0 = -V_1 + 2V_2 \sin\theta} \quad (2)$$

We have 2 eqns in 3 unknowns: (1) $v = 2v_2 \cos\theta$

(2) $v_1 = 2v_2 \sin\theta$

We need a third equation: ENERGY CONSERVATION holds for elastic collisions



$$\tfrac{1}{2} m v^2 = \tfrac{1}{2} m v_1^2 + \tfrac{1}{2}(2m) v_2^2$$

$$\boxed{v^2 = v_1^2 + 2v_2^2} \quad (3)$$

Now solve 3 equations in 3 unknowns

Hint - Square and add (1) and (2) first: That eliminates θ

$$v^2 = 4v_2^2 \cos^2\theta \qquad v_1^2 = 4v_2^2 \sin^2\theta$$

$$v^2 + v_1^2 = 4v_2^2 (\cos^2\theta + \sin^2\theta) = 4v_2^2 \quad (4)$$

Now Solve (3) & (4) for v_1 & v_2. I'll eliminate v_1^2 first:

$$v_1^2 = \underbrace{4v_2^2 - v^2}_{(4)} = \underbrace{v^2 - 2v_2^2}_{(3)}$$

$$6v_2^2 = 2v^2 \rightarrow v_2^2 = \tfrac{1}{3} v^2$$

$$v_2 = \frac{v}{\sqrt{3}} \quad (5)$$

We can find v_1 by eliminating v_2 from ③ & ⑤

③ $v_2^2 = \frac{1}{2}(v^2 - v_1^2) = \frac{v^2}{3}$

$$3(v^2 - v_1^2) = 2v^2$$

$$(3-2)v^2 = v^2 = 3v_1^2 \rightarrow v_1 = v/\sqrt{3}$$

Now can find θ from ②; $\frac{v_1}{v_2} = 2\sin\theta = \frac{v/\sqrt{3}}{v/\sqrt{3}} = 1$

$$\sin\theta = \frac{1}{2} \rightarrow \underline{\theta = 30^\circ}$$

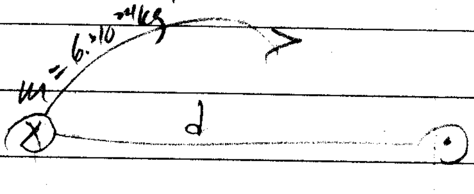
ⓒ What fraction of the initial $K = \frac{1}{2}mv^2$ is transferred to the target particle: $K_2 = \frac{1}{2}(2m)v_2^2$

$$K_2 = \frac{mv^2}{3}$$

$$\frac{K_2}{K} = \frac{mv^2/3}{\frac{1}{2}mv^2} = \frac{2}{3}$$

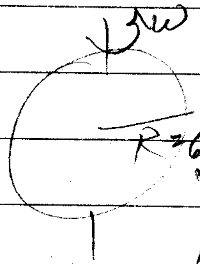
Giancoli Ch 10 - #6, 17, 53, 57, 86

6. (II) Calculate the angular velocity of the Earth (*a*) in its orbit around the Sun and (*b*) about its axis.



(a) $L = mvd = md\left(\frac{2\pi d}{T}\right)$

$L_{orbit} = \frac{2\pi m d^2}{T} = \frac{2\pi(6\times10^{24}\text{ kg})(1.5\times10^{11}\text{ m})^2}{1\text{ yr} = \pi\times10^7\text{ s}} = \underline{2.7\times10^{40}\ \text{kg m}^2/\text{s}}$



$L_{spin} = I\omega$ where $\omega = \frac{2\pi\text{ rad}}{1\text{ day}}\left|\frac{d}{24h}\right|\frac{h}{3.6\times10^3\text{ s}}$

$\omega_{spin} = \underline{7.27\times10^{-5}\text{ rad/s}}$

Moment of inertia of sphere $I = \frac{2}{5}MR^2$

Assume Earth has uniform density:

$I = \frac{2}{5}(6\times10^{24}\text{ kg})(6.4\times10^6\text{ m})^2 = \underline{9.8\times10^{37}\ \text{kg m}^2}$

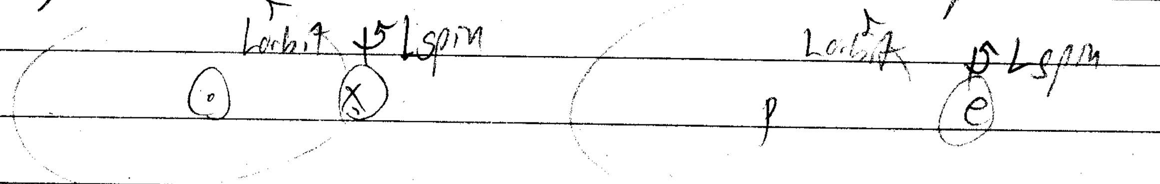
$I\omega = L_{spin} = \left(7.27\times10^{-5}\frac{\text{rad}}{\text{s}}\right)(9.8\times10^{37}\text{ kg m}^2) = \underline{7.1\times10^{33}\ \frac{\text{kg m}^2}{\text{s}}}$

57. (II) Determine the angular momentum of the Earth (*a*) about its rotation axis (assume the Earth is a uniform sphere), and (*b*) in its orbit around the Sun (treat the Earth as a particle orbiting the Sun). The Earth has mass = 6.0×10^{24} kg, radius = 6.4×10^6 m, and is 1.5×10^8 km from the Sun.

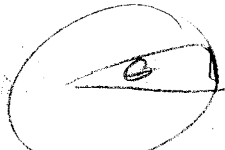
Oops - I answered this above, Skilled

(a) $\omega_{orbit} = \frac{2\pi\text{ rad}}{1\text{ yr} = \pi\times10^7\text{ s}} \sim 2\times10^{-7}\text{ rad/s}$

In Quantum Mechanics, we will similarly consider the spin and orbital angular momentum of particles e.g. of an electron in orbit around a proton



17. (II) The angular acceleration of a wheel, as a function of time, is $\alpha = 5.0t^2 - 3.5t$, where α is in rad/s^2 and t in seconds. If the wheel starts from rest ($\theta = \omega = 0$ at $t = 0$), determine a formula for (*a*) the angular velocity ω and (*b*) the angular position θ, both as a function of time. (*c*) Evaluate ω and θ at $t = 2.0$ s.



$s = r\theta$

$v = r\omega \qquad \omega = \frac{d\theta}{dt}$

$a = r\alpha \qquad \alpha = \frac{d\omega}{dt}$

Since $\alpha = \frac{d\omega}{dt}$, then $\omega = \int \alpha\, dt$

(a) $\omega = \int \left(5t^2 - \frac{7}{2}t\right) dt = \frac{5}{3}t^3 - \frac{7}{4}t^2 + \overset{0}{\omega_0}$

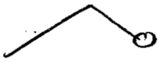
(b) $\theta = \int \omega\, dt = \int \left(\frac{5}{3}t^3 - \frac{7}{4}t^2\right) dt = \frac{5}{12}t^4 - \frac{7}{12}t^3 + \overset{0}{\theta_0}$

(c) $\omega(2s) = \frac{5}{3}\cdot 8 - \frac{7}{4}\cdot 4 = \frac{40}{3} - 7 = \frac{40}{3} - \frac{21}{3} = \underline{\frac{19}{3}}\ \frac{rad}{s} \quad (6,33)$

$\theta(2s) = \frac{5}{12}16 - \frac{7}{12}8 = 5\cdot\frac{4}{3} - 7\cdot\frac{2}{3} = \frac{20}{3} - \frac{14}{3} = \frac{6}{3} = \underline{2\ rad}$

53. (I) A diver (such as the one in Fig. 10–30) can reduce her moment of inertia by a factor of about 3.5 when changing from the straight position to the tuck position. If she makes two rotations in 1.5 s when in the tuck position, what is her angular speed (rev/s) when in the straight position?

STRAIGHT



TUCK

BIGGER moment of inertia $I_0 = 3.5\, I = \frac{7}{2} I$ $\qquad I = \frac{2}{7} I_0$ (TIGHTE

SLOWER spin frequency $f_0 = \frac{2}{7} f$ $\qquad f = \frac{2\,rev}{1.5\,s} = \frac{2}{3/2} = \frac{4}{3}\ \frac{re}{s}$

CONSERVATION OF ANGULAR MOMENTUM: (FASTER

$L_0 = L$

$I_0\omega_0 = I\omega \quad$ where $\omega = 2\pi f$

$I_0\, 2\pi f_0 = I\, 2\pi f$

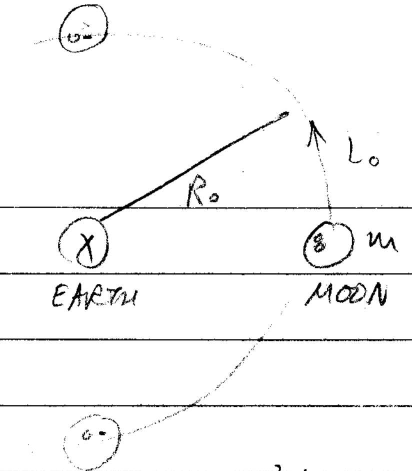
$I_0 f_0 = I f$

Spin frequency in straight dive:

$f_0 = f\,\frac{I}{I_0} = \frac{2}{7} f = \frac{2}{7}\,\frac{4}{3}\left(\frac{rev}{sec}\right) = \frac{8}{21} = 0.38\ \underline{\frac{rev}{sec}}$

86. The Moon orbits the Earth so that the same side always faces the Earth. Determine the ratio of its spin angular momentum (about its own axis) to its orbital angular momentum. (In the latter case, treat the Moon as a particle orbiting the Earth.)

Looking down on N pole:



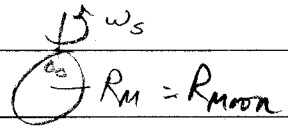
Orbital angular momentum

$$L_o = mVR_o = mR_o\left(\frac{2\pi R_o}{T_o}\right)$$

$T_o \sim 28\,d \left|\frac{3.6 \cdot 10^3\,s}{d}\right| = \_\_\_\_\_\ s$

$$L_o = \frac{2\pi m R_o^2}{T_o}$$

orbital period ~ 4 weeks



Spin angular momentum

$p = mv$

$$L_s = I\omega_s$$

Assume moon is a constant-density sphere; $I = \frac{2}{5}MR_M^2$

Phase locked: we always see the same face of the moon because its spin period $T_s = T_o$ orbit period, $\omega_s = \frac{2\pi}{T_s}$

$$L_s = \frac{2}{5}MR_M^2\,\frac{2\pi}{T_o}$$

So $$\frac{L_{spin}}{L_{orbit}} = \frac{\frac{2}{5}2\pi MR_M^2/T_o}{2\pi MR_o^2/T_o} = \frac{2}{5}\left(\frac{R_{moon}}{R_{orbit}}\right)^2 = \frac{2}{5}\left(\frac{1.74 \cdot 10^6\,m}{3.84 \cdot 10^8\,m}\right)^2$$

$$\frac{L_s}{L_o} = 8.21 \cdot 10^{-6}$$

ORBIT has much more angular momentum than SPIN