Phys of Astro - wk 1 HW due wk 2 Zita

Phys.B: Giancoli 1-4, review Classical Mech. **Ch.1** # 23, 38, 56 (p.14); **Ch.2** # 16, 22, 28, 54, 68, *69*, 74 (p.39); **Ch.3** # 12-15, 24, *28*, 56, 62 (p.70); ~~**Ch.4** # 5, 22, 34, 40, 72 (p.98)~~ → next week

Ch.1 done in Phys A HW

Math.B: Boas 3.4, Vectors (practice problems with answers)

Q2

16. (II) An airplane travels 2100 km at a speed of 800 km/h, and then encounters a tailwind that boosts its speed to 1000 km/h for the next 1800 km. What was the total time for the trip? What was the average speed of the plane for this trip? [*Hint*: Think carefully before using Eq. 2–12d.]

$x_1 =$ $v_1 =$ $v_2 =$ x_2

Solve algebraically first

Speed = $\frac{\text{distance}}{\text{time}}$ $\quad v = \frac{x}{t} \rightarrow t = \frac{x\ (m)}{v\ (m/s)} = (s)$ ✓

$t_1 = \frac{x_1}{v_1} = \frac{2100\ km}{800\ km/hr} = \frac{21}{8}\ hr = \underline{2.63}\ hr$

$t_2 = \frac{x_2}{v_2} = \frac{1800\ km}{1000\ km/hr} = 1.8\ hr$

$t_{tot} = t_1 + t_2 = 2.63 + 1.8 = \underline{4.43}\ hr$ ✓

$V_{average} = \frac{x_{tot}}{t_{tot}} = \frac{2100 + 1800\ km}{4.43\ hr} = \underline{880}\ \frac{km}{hr}$ ✓

22. (I) Figure 2–32 shows the velocity of a train as a function of time. (*a*) At what time was its velocity greatest? (*b*) During what periods, if any, was the velocity constant? (*c*) During what periods, if any, was the acceleration constant? (*d*) When was the magnitude of the acceleration greatest?

a = acceleration

v = velocity

$a = \frac{dv}{dt}$

a = slope of $v(t)$

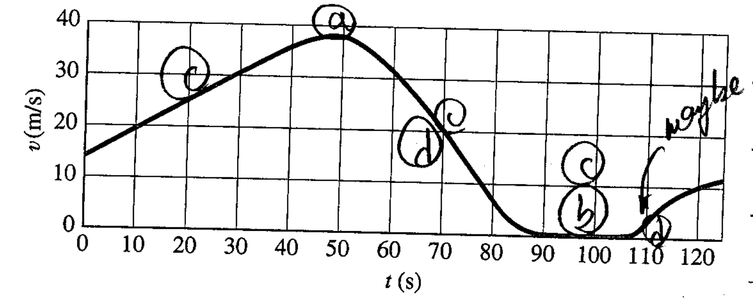

FIGURE 2–32 Question 18 and Problem 22.

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$V = \frac{dx}{dt}$, $a = \frac{dv}{dt}$

(28) (II) The position of a body is given by $x = At + 6Bt^3$, where x is in meters and t is in seconds. (*a*) What are the units of A and B? (*b*) What is the acceleration as a function of time? (*c*) What is the velocity and acceleration at $t = 5.0$ s? (*d*) What is the velocity as a function of time if $x = At + Bt^{-3}$?

(a) $x = At + 6Bt^3$

$(m) = [A](s) + [B](s^3)$

units of $A = [A] = \left(\frac{m}{s}\right)$

units of $B = [B] = \underline{\left(m/s^3\right)}$

(b) velocity $v = \frac{dx}{dt} = \frac{d}{dt}(At + 6Bt^3)$

$= \frac{d}{dt}(At) + \frac{d}{dt}(6Bt^3)$

$= A\frac{d}{dt}(t) + 6B\left(\frac{d}{dt}t^3\right)$

Differentiate: $V = A(1) + 6B(3t^2) = \underline{A + 18Bt^2}$

acceleration $a = \frac{dv}{dt} = \frac{d}{dt}(A + 18Bt^2)$

$= 0 + 18(2Bt)$ $\quad \underline{a = 36Bt}$

(c) $v(t = 5s) = A + 18B(5^2) = A + 18 \cdot 25B = A + \underline{450}B(m)$

$a(t = 5s) = 36B(5) = 18 \cdot 10B = \underline{180B\ (m/s^2)}$

(d) $x = At + Bt^{-3}$ $\quad V = \frac{dx}{dt} = \frac{d}{dt}(At) + \frac{d}{dt}(Bt^{-3})$

$\underline{V = A - 3Bt^{-4}}$

$a = \frac{d}{dt}(A) - 3B\frac{d}{dt}(t^{-4})$

$a = 0 - 3B(-4t^{-5})$ $\quad a = 12Bt^{-5}$

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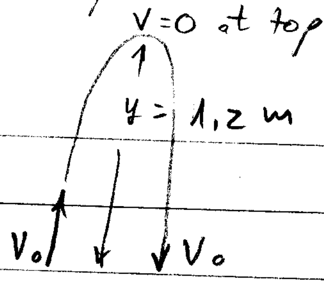
92.7 Follow the example 2-15 of ball thrown upward

(54) (II) The best rebounders in basketball have a vertical leap (that is, the vertical movement of a fixed point on their body) of about 120 cm. (*a*) What is their initial "launch" speed off the ground? (*b*) How long are they in the air?

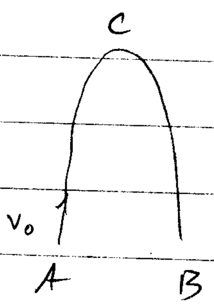


(a) $V^2 = V_0^2 + 2ay \qquad (a = -g = -9.8\,\frac{m}{s^2})$

$0 = V_0^2 - 2gy$

launch speed $V_0 = \sqrt{2gy} = \sqrt{2 \cdot 9.8\,\frac{m}{s^2} \cdot 1.2\,m} = \underline{4.85\,\frac{m}{s}}$

(b)



Total air time is time to go from A→B←C

$\Delta y = y_B - y_A = V_0 t + \frac{1}{2} a t^2$

$0 = V_0 t + \frac{1}{2} a t^2$

$(a = -g) \qquad (c = 0)$

Solve for t: $a t^2 + 2V_0 t = 0 = a t^2 + bt + c = 0$

$b = 2V_0$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2}}{2a} = \frac{-b \pm b}{2a} = 0, \frac{-2b}{2a}$

t = 0 is the time to go from A→A (nowhere)

$t = \frac{-2b}{2a} = \frac{-2b}{2g} = \frac{b}{g} = \frac{2V_0}{g}$ is the total air time

$t = \frac{2(4.85\,\frac{m}{s})}{9.8\,\frac{m}{s^2}} \cong \underline{1}\ s$

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*68. (III) The acceleration of a particle is given by $a = A\sqrt{t}$ where $A = 2.0\ \mathrm{m/s^{5/2}}$. At $t = 0$, $v_o = 10\ \mathrm{m/s}$ and $x_o = 0$. (*a*) What is the speed as a function of time? (*b*) What is the displacement as a function of time? (*c*) What are the acceleration, speed and displacement at $t = 5.0$ s?

$= At^{1/2}$

Recall that for a = const:

a = acceleration $= \frac{dv}{dt} \rightarrow \Delta v = \int a\,dt = at$

$v(t) - v_0 = at$

$v(t) = at + v_0$

v = velocity $= \frac{dx}{dt}$

$\rightarrow \Delta x = \int v\,dt = \int (at + v_0)\,dt$

$= \int at\,dt + \int v_0\,dt$

$x(t) - x_0 = \frac{at^2}{2} + v_0\,dt$

$x(t) = \frac{at^2}{2} + v_0 dt + x_0$

In this case, however $a \neq$ constant.
You can use the same method! however

$a = \frac{dv}{dt} \rightarrow \Delta v = v(t) - v_0 = \int a\,dt = A\int t^{1/2}\,dt$

Integrate and find $v(t) = v_0 + \frac{2}{3}At^{3/2} = 10 + \frac{4}{3}t^{3/2}$

$\left(\frac{m}{s}\right)$ ✓ $\quad \left(\frac{m}{s}\right)\ \left(\frac{m}{s^{5/2}}\right)s^{3/2}$

$v = \frac{dx}{dt} \rightarrow \Delta x = x(t) - x_0 = \int v(t)\,dt = \int \left(10 + \frac{4}{3}t^{3/2}\right)dt$

Integrate and find $x(t) = x_0 + 10t + \frac{4}{3}\left(\frac{2}{5}\right)t^{5/2}$

$= 0 + 10t + \frac{8}{15}t^{5/2}$

✓ (m) $\quad \left(\frac{m}{s}\right)(s) \quad \left(\frac{m}{s^{5/2}}\right)(s^{5/2})$

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c) at $t = 5\,s$, $x = 10t + \frac{8}{15}t^{5/2}$

$$x(5) = 10 \cdot 5 + \frac{8}{15}(5)^{5/2} = \underline{80} \; m$$

$$v = 10 + \frac{4}{3}t^{3/2}$$

$$v(5) = 10 + \frac{4}{3}(5)^{3/2} = \underline{25} \; m/s$$

$$a = A\sqrt{t} = 2\left(\frac{m}{s^{5/2}}\right)(5s)^{1/2} = \underline{4.5} \; m/s^2$$

$$\frac{m \cdot s^{1/2}}{s^{5/2}} = \frac{m}{s^{4/2}} = \frac{m}{s^2} \checkmark$$

Q2

(69.) The acceleration due to gravity on the Moon is about one sixth what it is on Earth. If an object is thrown vertically upward on the Moon, how many times higher will it go than it would on Earth, assuming the same initial velocity?

(Notice you don't need to know V_0 !)

From Ex 2-15, $y = \dfrac{V_0^2}{2g}$ where V_0 = initial speed, g = accl of gravity

$$\frac{y_{\text{on moon}}}{y_{\text{on Earth}}} = \frac{V_0^2/2g_{moon}}{V_0^2/2g_{earth}} = \frac{g_{earth}}{g_{moon}} = \frac{g}{g/6} = \frac{6 \text{ times}}{\text{higher}}$$

(FIRST simplify algebraically; then plug in $g_{moon} = \frac{1}{6} g_{earth}$ and evaluate.)

74.) Figure 2–40 is a position versus time graph ... of an object along the x axis. As the object moves from A to B: (a) Is the object moving in the positive or negative direction? (b) Is the object speeding up or slowing down? (c) Is the acceleration of the object positive or negative? Next, for the time interval from D to E: (d) Is the object moving in the positive or negative direction? (e) Is the object speeding up or slowing down? (f) Is the acceleration of the object positive or negative? (g) Finally, answer these same three questions for the time interval from C to D.

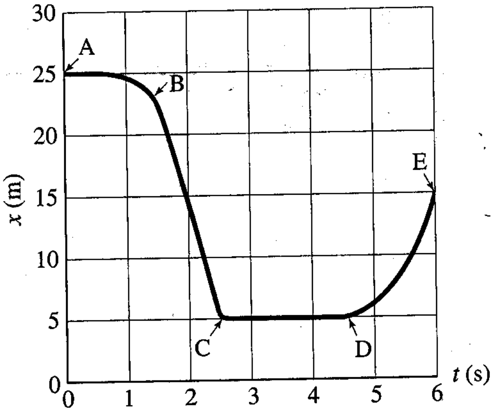

FIGURE 2–40 Problem 74.

(a) A→B going backward ($-\Delta x$)

(b) speeding up (steeper slope)

(c) $a = \dfrac{d^2x}{dt^2} < 0$: concave down

(d) D→E : going forwards ($+\Delta x$)

(e) Speeding up (steeper slope)

(f) $a = \dfrac{d^2x}{dt^2} > 0$: concave up

(g) C→D: not moving ($\Delta x = 0$)

constant speed ($v = 0$)

no acceleration ($\frac{dv}{dt} = 0$)

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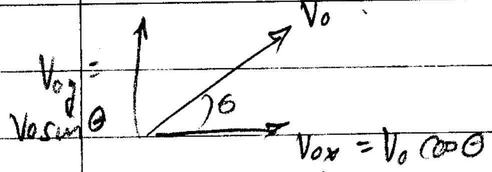
Ch 3

(28) (I) Determine how much farther a person can jump on the Moon as compared to the Earth if the takeoff speed and angle are the same. The acceleration due to gravity on the Moon is one-sixth what it is on Earth.

Peak height

Use the results of Ch 2 # 69 : $y_{moon} = 6\, y_{earth}$

and the method of Ch 2 # 57



$y(t) = V_{oy} t + \frac{1}{2} a t^2$ where $a = -\frac{1}{6} g$

$x(t) = V_{ox} t$ and $g = 9.8 \frac{m}{s^2}$

(x_{max})

How far you can jump depends on how high you can jump (y_{max}) Total time = T. $x_{max} = V_{ox} T$

$\Delta y = 0 = V_{oy} T + \frac{1}{2} a T^2$

Equate $T = \frac{x_{max}}{V_{ox}}$ with

$\frac{1}{2}\frac{1}{6} g T^2 = V_{oy} T$

$T = 12 V_{oy}/g \leftarrow \frac{1}{12} g T = V_{oy}$

$x_{max} = V_{ox} \left(12 V_{oy}/g\right)$

We've seen that $x = x_{max}$ when $\theta = 45^\circ$ so $\sin\theta = \cos\theta = \frac{\sqrt{2}}{2}$

$x_{max} = \frac{12 V_0 \cos\theta \cdot V_0 \sin\theta}{g} = \frac{12 V_0^2}{g} \left[\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}\right]$

MOON: $x_{max} = \frac{6 V_0^2}{g}$ = 6 times further than on Earth

EARTH: $x_{max} = \frac{V_0^2}{g}$

96 | 9

do these NEXT week - not now

Ch 3 # 12-15 (24) 28, (56, 62)

(12) (II) Three vectors are shown in Fig. 3–41. Their magnitudes are given in arbitrary units. Determine the sum of the three vectors. Give the resultant in terms of (*a*) components, (*b*) magnitude and angle with *x* axis.

$A_x = 44.0 \cos 28° = \underline{38.8}$

$A_y = 44.0 \sin 28° = \underline{20.7}$

$B_x = -26.5 \cos 56° = \underline{-14.8}$

$B_y = 26.5 \sin 56° = \underline{22.0}$

$C_y = -31.0$

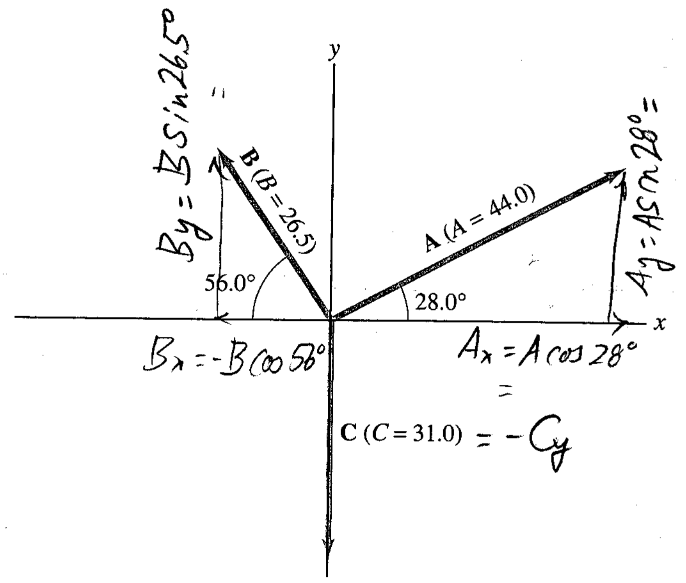


FIGURE 3–41 Problems 12, 13, 14, and 15. Vector magnitudes are given in arbitrary units.

(13) (II) (*a*) Given the vectors **A** and **B** shown in Fig. 3–41, determine **B** − **A**. (*b*) Determine **A** − **B** without using your answer in (*a*). Then compare your results and see if they are opposite.

(14) (II) For the vectors given in Fig. 3–41, determine (*a*) **A** − **B** + **C**, (*b*) **A** + **B** − **C**, and (*c*) **C** − **A** − **B**.

(15) (II) For the vectors shown in Fig. 3–41, determine (*a*) **B** − 2**A**, (*b*) 2**A** − 3**B** + 2**C**.

(13) $\vec{B} - \vec{A} = (B_x - A_x)\hat{i} + (B_y - A_y)\hat{j}$

$B_x - A_x = -14.8 - 38.8 = -53.6$

$B_y - A_y = 22 - 20.7 = 1.3$

$\vec{B} - \vec{A} = (-53.6)\hat{i} + (1.3)\hat{j}$

$\vec{A} - \vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}$

$A_x - B_x = 38.8 - (-14.8)$

$A_y - B_y = 20.7 - 22.0$

$\vec{A} - \vec{B} = (+53.6)\hat{i} + (-1.3)\hat{j}$

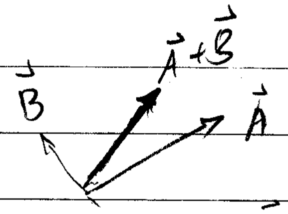
✓ $= -(\vec{B} - \vec{A})$

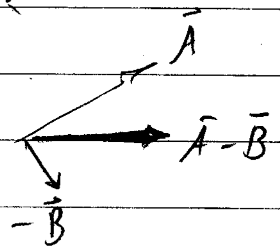
(14) $\vec{A} - \vec{B} + \vec{C} = (A_x - B_x + C_x)\hat{i} + (A_y - B_y + C_y)\hat{j}$

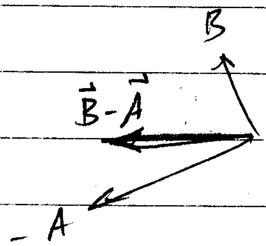
$= (38.8 + 14.8 + 0)\hat{i} + (20.7 - 22 - 31)\hat{j} = \underline{53.6\hat{i} - 32.3\hat{j}}$

(b) $\vec{A} + \vec{B} - \vec{C} = (A_x + B_x - C_x)\hat{i} + (A_y + B_y - C_y)\hat{j} =$

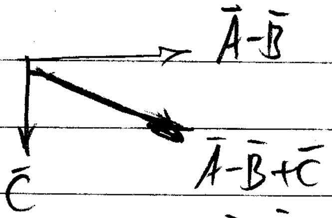
$= (38.8 - 14.8 - 0)\hat{i} + (20.7 + 22 + 31)\hat{j} = 24\hat{i} + 73.7\hat{j}$



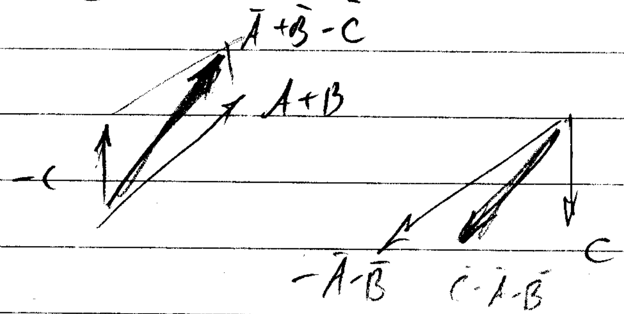




14ⓐ



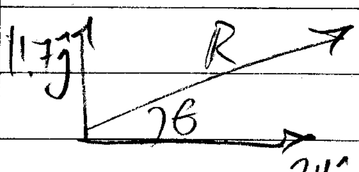
14ⓑ 14ⓒ



(12) $A + B + C = (A_x + B_x + C_x)\,\hat{i} + (A_y + B_y + C_y)\,\hat{j}$

$= (38.8 + 14.8 + 0)\,\hat{i} + (20.7 + 22.0 - 31.0)\,\hat{j}$

$R = 24\,\hat{i} + 11.7\,\hat{j}$



$|R| = R = \sqrt{24^2 + 11.7^2} = 26.7$

$\tan\theta = \frac{R_y}{R_x} = \frac{11.7}{24} \rightarrow \theta = 26^\circ$

14 ⓐ $\vec{C} - \vec{A} - \vec{B} = (C_x - A_x - B_x)\hat{i} + (C_y - A_y - B_y)\hat{j}$

$= (6 - \qquad)\hat{i} + (-31 \qquad)\hat{j}$

$= \qquad \hat{i} + \qquad \hat{j}$

15 ⓐ $B - 2A = (B_x - 2A_x)\hat{i} + (B_y - 2A_y)\hat{j}$

$= (\qquad)\hat{i} + (\qquad)\hat{j}$

$= \qquad \hat{i} + \qquad \hat{j}$

ⓑ $2A - 3B + 2C = (2A_x - 3B_x + 2C_x)\hat{i} + (2A_y - 3B_y + 2C_y)\hat{j}$

$= (\qquad)\hat{i} + (\qquad)\hat{j}$

$= \qquad \hat{i} + \qquad \hat{j}$