Phys 1B - Giancoli Ch 3, 4, 6 - week 2
(due week 3)

Ch 3 - Finish from last week + 24, 56, 62

Ch 4 - Newton's Laws - 5, 21, 34, 40, 72, 55
(not 22)

next week

~~Ch 6 - Candidate Q: 4, 6, 8, 11, 13, 14, 18~~

~~p.152: 22, 29, 28, 36, 38~~

~~p.153: 42, 47, 53, 54, 57, 59~~

~~p.154: 60, 62~~

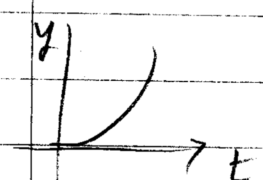
Ch 3

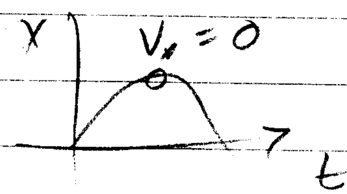
24. (II) A particle starts from the origin at $t = 0$ with an initial velocity of 5.0 m/s along the positive x axis. If the acceleration is $(-3.0\mathbf{i} + 4.5\mathbf{j})$ m/s², determine the velocity and position of the particle at the moment it reaches its maximum x coordinate.

$V_0 = V_{x_0} = 5 \frac{m}{s}$

$a_x = -3 \, m/s^2$

$a_y = 4.5 \, m/s^2 = 4\frac{1}{2} = \frac{8}{2} + \frac{1}{2} = \frac{9}{2} \, m/s^2$





$V_{y_0} = 0$

$y(t) = \frac{1}{2} a_y t^2$

$V_y(t) = \frac{dy}{dt} = \frac{1}{2}(2 a_y t)$

$V_y(t) = a_y t$

$x(t) = V_0 t + \frac{1}{2} a_x t^2$

$V_x(t) = \frac{dx}{dt} = V_0 + a_x t$

At peak, $V_x(t) = 0 = V_0 + a_x t$

At peak, $t = -\frac{V_0}{a_x} = \frac{-5 \, m/s}{-3 \, m/s^2} = \frac{5}{3} \, s$

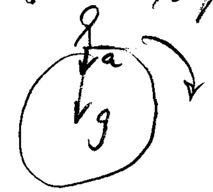
$V_y(t = \frac{5}{3} s) = a_y t = \left(\frac{9}{2} \frac{m}{s^2}\right)\left(\frac{5}{3} s\right) = 3 \cdot \frac{5}{2} = 7.5 \, m/s = V_{peak}$

Position at peak: $x = 5 \frac{m}{s}\left(\frac{5}{3} s\right) + \frac{1}{2}\left(-3 \frac{m}{s^2}\right)\left(\frac{5}{3} s\right)^2 = \underline{4.2} \; m$

$y = \frac{1}{2} a_y t^2 = \frac{1}{2}(4.5 \, m/s^2)(5/3 \, s)^2 = \underline{6.3} \; m$

16

Looking down on N pole



(56) (II) Because the Earth rotates once per day, the effective acceleration of gravity at the equator is slightly less than it would be if the Earth didn't rotate. Estimate the magnitude of this effect. What fraction of g is this?

$a = \frac{v^2}{R}$ $\quad v = \frac{2\pi R}{T}$ $T = 1\text{day}$ $\quad a = \frac{\left(\frac{2\pi R}{T}\right)^2}{R} = \frac{4\pi^2 R}{T^2}$

If Not spinning: Weight $= mg$ $\quad T = 1d \left| \frac{24hr}{d} \right| \frac{3600s}{hr} \Big|$

$T = 8.64 \times 10^4$ s

If spinning: part of g contributes to a: $W' = mg'$

$g' = g - a$

$a = \frac{4\pi^2 R}{T^2} \approx \frac{40\,(6.4 \times 10^6\,m)}{(8.64 \times 10^4\ s)^2} = 0.038\ \frac{m}{s^2}$

At the equator, effective gravity is less:

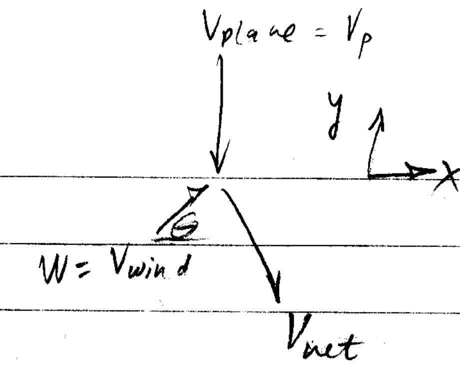
$g' = 9.8\,\frac{m}{s^2} - 0.038\,\frac{m}{s^2} \approx g$

Too small to notice an effect: $\frac{\Delta g}{g} = \frac{0.038}{9.8} \approx 0.3\%$

17

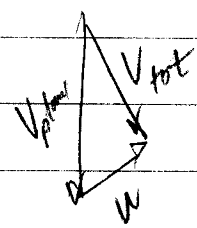
$V_p =$ south

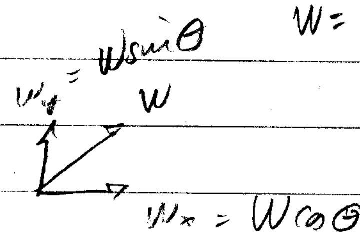
(62) (II) An airplane is heading due south at a speed of 550 km/h. If a wind begins blowing from the southwest at a speed of $W =$ 90.0 km/h (average), calculate: (*a*) the velocity (magnitude and direction) of the plane, relative to the ground, and (*b*) how far off course it will be after 12.0 min if the pilot takes no corrective action. [*Hint*: First draw a diagram.]



$\vec{V}_{net} = \vec{V}_{plane} + \vec{W}$

(a)





$\cos\theta = \cos 45^\circ = \frac{\sqrt{2}}{2} = \sin\theta$

$W_x = W_y = \frac{\sqrt{2}}{2} W = \frac{\sqrt{2}}{2}\left(90.0 \frac{km}{hr}\right) = \underline{63.6 \frac{km}{hr}}$

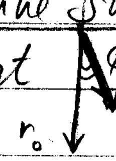
$V_{net_x} = 0 + W_x = \underline{63.6} \frac{km}{hr}$ EAST

$V_{net_y} = -V_{plane} + W_y = -550 \frac{km}{hr} + \underline{63.6} \frac{km}{hr}$

$= + \underline{486} \frac{km}{hr}$ SOUTH

$V_{net} = \sqrt{V_{net_x}^2 + V_{net_y}^2} = \underline{490} \ km/hr$ (SLOWER)

(b) After 12 min, plane should have been at $r_0 = V_p t$
but it's actually at $\phi = \tan^{-1}\left(\frac{V_{net\,y}}{V_{net\,x}}\right) = 8^\circ$



$\vec{r}_{net} = \vec{V}_{net}\, t$

$t = 12 \text{ min} \left|\frac{hr}{60 \text{ min}}\right| = \frac{1}{5} hr$

$r_0 = 550 \frac{km}{hr} \cdot \frac{1}{5} hr = 110$ km due SOUTH

TOO FAR

$\vec{r}_{net} = \vec{V}_{x_{net}}\, t + \vec{V}_{y_{net}}\, t$ where $V_{x_{net}} t = W_x t = \underline{63.6} \frac{km}{hr} \frac{1}{5} hr = \underline{12.7 \text{ km}}$ EAST

and $V_{y_{net}} t = \left(486 \frac{km}{hr}\right)\frac{1}{5} hr = \underline{97}$ km SOUTH NOT FAR ENOUGH

18

Giancoli

Answers to Ch 4: SEE WORK IN HWTS!

5 (a) 570 N

(b) 99 N

(c) 210 N

21 (a) $a = 2.2\ m/s^2$

(b) $v = 18\ m/s$

(c) $t = 93\ s$

34 $F_{lift} = 9 \times 10^4\ N$

$T = 1.2 \times 10^4\ N$

40 (a) $a = 3.67\ m/s^2$

(b) $v = 9.4\ m/s$

55 - See solutions worked out

72 - (a) $c = 41\ kg/s$

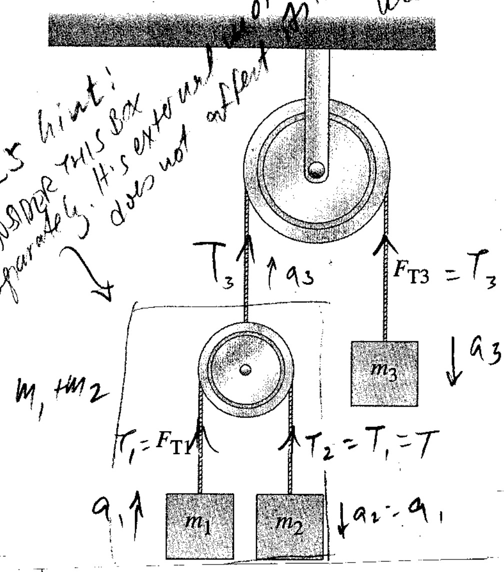
(b) $F_{applied} = 160\ N$

24

Let arrows indicate directions. Avoid $\ominus$ signs.

(55.) (III) The double Atwood machine shown in Fig. 4–52 has frictionless, massless pulleys and cords. Determine (*a*) the acceleration of masses m_1, m_2, and m_3, and (*b*) the tensions F_{T1} and F_{T3} in the cords.

Paul L's hint! CONSIDER THIS BOX Separately. It's external motion does not affect internal motion



$$\sum F_3 = m_3 a_3$$

$$\downarrow \sum F_3 = m_3 g - T_3$$

$$\underline{m_3 a_3 = m_3 g - T_3} \quad (1)$$

T ↑, m_3, a_3 ↓, $m_3 g$

T_3 ↑, a_3 ↑, $(m_1 + m_2) g$

$$\uparrow \sum F = ma$$

$$\underline{T_3 - (m_1 + m_2) g = (m_1 + m_2) a_3} \quad (2)$$

$T_1 = T$ (INSIDE THE BOX)

m_1: ↑ $a_1 = a$, $m_1 g$

m_2: $T_2 = T_1 = T$, ↓ $a_2 = a_1 = a$, $m_2 g$

$$\sum F_1 = m_1 a_1 \qquad \sum F_2 = m_2 a_2$$

$$\underline{T - m_1 g = m_1 a} \qquad \underline{m_2 g - T = m_2 a}$$

We can solve these last two eqns for T and a:

Eliminate $T = m_1 (a + g) = m_2 (g - a)$

Find a: $m_1 a + m_2 a = a(m_1 + m_2) = g(m_2 - m_1) \rightarrow a = g\left(\frac{m_2 - m_1}{m_2 + m_1}\right)$, $a_1 = a_2 =$

Sub in: So $T = T_1 = T_2 = m_1 (a + g) = m_1 g \left(\frac{m_2 - m_1}{m_2 + m_1}\right) + m_1 g \left(\frac{m_2 + m_1}{m_2 + m_1}\right)$

$$T = \frac{m_1 g}{m_2 + m_1} (m_2 - m_1 + m_2 + m_1 = 2m_2) = \frac{2 m_1 m_2 g}{m_1 + m_2}$$

Returning to the top two eqns, we can solve for T_3, a_3

Eliminate: $T_3 = m_3 g - m_3 a_3 = (m_1 + m_2) a_3 + (m_1 + m_2) g$

Find a_3: $a_3 (m_1 + m_2 + m_3) = g(m_3 - m_1 - m_2) \rightarrow a_3 = g\left(\frac{m_3 - m_1 - m_2}{m_1 + m_2 + m_3}\right)$

Sub in: $T_3 = m_3 g \left(\frac{m_1 + m_2 + m_3}{m_1 + m_2 + m_3}\right) - m_3 g \left(\frac{m_3 - m_1 - m_2}{m_1 + m_2 + m_3}\right) = \frac{m_3 g}{(m_1 + m_2 + m_3)} (2m_1 + 2m_2)$

Note that the total accelerations of m_1 and m_2 DIFFER (a_1', a_2')

$a_3 \uparrow$ $\boxed{m_1}$ $\uparrow a$

$$\uparrow a_1' = a + a_3 = g\left(\frac{m_2-m_1}{m_1+m_2}\right) + g\left(\frac{m_3-m_1-m_2}{m_1+m_2+m_3}\right)$$

$$\frac{a_1'}{g} = \frac{(m_2-m_1)(m_1+m_2+m_3) + (m_3-m_1-m_2)(m_1+m_2)}{(m_1+m_2)(m_1+m_2+m_3)}$$

$$(m_1+m_2)(m_1+m_2+m_3)\frac{a_1'}{g} = (\cancel{m_1m_2} - m_1^2 + \cancel{m_2^2} - m_1m_2 + m_3m_2 - m_1m_3) + (m_1m_3 - m_1^2 - m_1m_2 + m_2m_3 - m_1m_2 - m_2^2)$$

$$= (-m_1^2 + m_3m_2) + (-m_1^2 - 2m_1m_2 + m_2m_3)$$

$$= 2(-m_1^2 + m_2m_3 + m_1m_2)$$

$$\frac{a_1'}{g} = \frac{2(-m_1^2 + m_1m_2 + m_2m_3)}{(m_1+m_2)(m_1+m_2+m_3)}$$

$\uparrow a_3$ $\boxed{m_2}$ $a \downarrow$

$$\downarrow a_2' = a - a_3 = g\left(\frac{m_2-m_1}{m_1+m_2}\right) + g\left(\frac{-m_3+m_1+m_2}{m_1+m_2+m_3}\right)$$

$$\frac{a^{2'}}{g} = \frac{(m_2-m_1)(m_1+m_2+m_3) + (m_1+m_2)(-m_3+m_1+m_2)}{(m_1+m_2)(m_1+m_2+m_3)}$$

$$(m_1+m_2)(m_1+m_2+m_3)\frac{a_2'}{g} = (\cancel{m_1m_2} - m_1^2 + m_2^2 - \cancel{m_1m_2} + \cancel{m_2m_3} - m_1m_3) + (-m_1m_3 - \cancel{m_2m_3} + m_1^2 + m_1m_2 + m_1m_2 + m_2^2)$$

$$= (-\cancel{m_1^2} + m_2^2 - m_1m_3) + (-m_1m_3 + \cancel{m_1^2} + m_2^2 + 2m_1m_2)$$

$$= 2(m_2^2 - m_1m_3 + m_1m_2)$$

$$\frac{a_2'}{g} = \frac{2(m_2^2 + m_1m_2 - m_1m_3)}{(m_1+m_2)(m_1+m_2+m_3)}$$

...these two equations, we get:

$a = g \tan\theta$, and $F_T = m_2g/\cos\theta$.

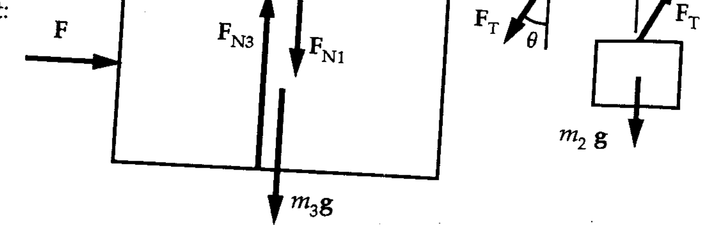
For the mass m_1, we have $\sum \mathbf{F} = m\mathbf{a}$:

x-component: $F_T = m_1a$;

y-component: $F_{N1} - m_1g = 0$.

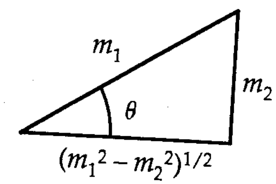
We can combine the two x-equations to get

$\sin\theta = m_2/m_1$.



If we put this in a triangle, as shown, we can find $\tan\theta$. Thus the acceleration is

$a = g\tan\theta = m_2g/(m_1^2 - m_2^2)^{1/2}$.



For the block m_3 (including the pulley), we have $\sum \mathbf{F} = m\mathbf{a}$:

x-component: $F - F_T\sin\theta - F_T = m_3a$;

y-component: $F_{N3} - F_{N1} - F_T\cos\theta - m_3g = 0$.

When we use the previous results in the x-equation, we get

$F = F_T\sin\theta + F_T + m_3a$

$= (m_2g/\cos\theta)\sin\theta + m_1a + m_3a = (m_2 + m_1 + m_3)a = \boxed{(m_1 + m_2 + m_3)m_2g/(m_1^2 - m_2^2)^{1/2}}$.

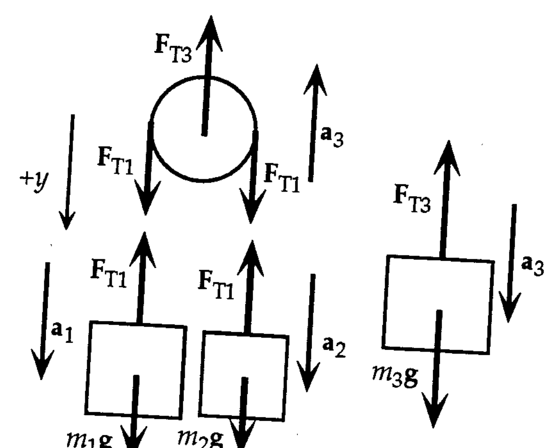
Note that we would obtain this result directly if we had chosen the three blocks and the pulley as the system. The only horizontal force would be F, which would produce the acceleration of the three blocks.

55. The force diagrams for each of the masses and the movable pulley are shown. Note that we take down as positive and the indicated accelerations are relative to the fixed pulley. A downward acceleration of m_3 means an upward acceleration of the movable pulley. If we call a_r the (downward) acceleration of m_1 with respect to the movable pulley, we have

$a_1 = a_r - a_3$ and $a_2 = -a_r - a_3$,

because the acceleration of m_2 with respect to the pulley must be the negative of m_1's acceleration with respect to the pulley. If the mass of the pulley is negligible, for the movable pulley we write $\sum F_y = ma_y$:

$2F_{T1} - F_{T3} = (0)(-a_3)$, so $2F_{T1} = F_{T3}$.



For each of the masses, for $\sum F_y = ma_y$ we get

mass m_1: $m_1g - F_{T1} = m_1a_1 = m_1(a_r - a_3)$,

mass m_2: $m_2g - F_{T1} = m_2a_2 = m_2(-a_r - a_3)$,

mass m_3: $m_3g - F_{T3} = m_3a_3$.

We have four equations for the four unknowns:

F_{T1}, F_{T3}, a_r, and a_3.

After some careful algebra, we get

$$\boxed{\begin{aligned} a_3 &= [(m_1m_3 + m_2m_3 - 4m_1m_2)/(m_1m_3 + m_2m_3 + 4m_1m_2)]g; \\ a_r &= [2(m_1m_3 - m_2m_3)/(m_1m_3 + m_2m_3 + 4m_1m_2)]g; \\ F_{T1} &= [4m_1m_2m_3/(m_1m_3 + m_2m_3 + 4m_1m_2)]g; \text{ and} \\ F_{T3} &= [8m_1m_2m_3/(m_1m_3 + m_2m_3 + 4m_1m_2)]g. \end{aligned}}$$

We can now find the other accelerations:

$$\boxed{\begin{aligned} a_1 &= [(m_1m_3 - 3m_2m_3 + 4m_1m_2)/(m_1m_3 + m_2m_3 + 4m_1m_2)]g; \\ a_2 &= [(-3m_1m_3 + m_2m_3 + 4m_1m_2)/(m_1m_3 + m_2m_3 + 4m_1m_2)]g. \end{aligned}}$$

$\int_0^x dx = \int_0$

57. From the result
$a = [m_2 + m$
To simplify the
Thus we have
$a = dv/dt =$
This contains th
because $v = dy/$
$dv/dt = (dv$
$[m_2 + m_C(y/$
When we integr
$\int_0^y [m_2 + (m_C$
$[m_2y + m_C(y^2$

Physics of Star Trek ROCKET PROBLEM

p.25 - "To accelerate to half the speed of light, The Enterprise must burn 81 times its entire mass in Hydrogen fuel" (1% of which streams out as He atoms after fusion)

$V_{final} = \frac{c}{2}$ $\qquad M_0$ - INITIALLY Enterprise Mass + fuel

$V_{exhaust\ gas} = \frac{c}{8}$ (relative to rocket)

Show that $M_0 = 81\, M_{final} = 81\, M_{ship}$

First try it NONRELATIVISTICALLY (too simple) as in Ex 9-18 on Giancoli p. 229

$V_{gas} \longleftarrow \bigcirc\, m(t) \longrightarrow V(t)$

We'll ignore initial acceleration to overcome gravity - assume we're out in space.

$dm \longrightarrow F_{thrust} = V_{gas} \frac{dm}{dt}$

($F = ma$ is, more generally, $F = \frac{d}{dt} mv = m\frac{dv}{dt} + v\frac{dm}{dt}$)

Momentum conservation: $-m\frac{dv}{dt} = V_{gas}\frac{dm}{dt}$

$$-m\,dv = V_{gas}\,dm$$

$$-\frac{dv}{V_{gas}} = \frac{dm}{m}$$

$$\frac{1}{V_{gas}}\int_{t=0}^{t} dv = \frac{1}{V_{gas}}(V(t) - \cancelto{0}{V_0}) = \frac{V_{final}}{V_{gas}} = -\int\frac{dm}{m} = -\ln m\Big|_{m_0}^{m_f}$$

$$= -(\ln m_f - \ln m_0) = \ln\left(\frac{m_0}{m_f}\right)$$

$$\frac{V_{final}}{V_{gas}} = \frac{c/2}{c/8} = 4 = \ln\left(\frac{m_0}{m_f}\right)$$

$$e^4 = \frac{m_0}{m_f} = \frac{m_0}{m_{ship}} \approx 3^4 = 9 \cdot 9 = 81$$

Relativistically (correctly) it would take even more fuel, because the mass gets harder and harder to accelerate as it goes faster.

Mass APPEARS to increase as $m' = \gamma m$ where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ so $m' = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}$

So the equation would instead become

$$-m' dv = V_{gas}\, dm = \frac{-m}{\sqrt{1-\frac{v^2}{c^2}}}\, dv$$

$$-V_{gas}\int_{m_0}^{m_f} \frac{dm}{m} = \int_{V_0=0}^{V_F=c/2} \frac{dv}{\sqrt{1-\frac{v^2}{c^2}}}$$

Try those integrals!