Homework week 2

44. ***REASONING*** Let us assume that the skater is moving horizontally along the $+x$ axis. The time t it takes for the skater to reduce her velocity to $v_x = +2.8$ m/s from $v_{0x} = +6.3$ m/s can be obtained from one of the equations of kinematics:

$$v_x = v_{0x} + a_x t \tag{3.3a}$$

The initial and final velocities are known, but the acceleration is not. We can obtain the acceleration from Newton's second law $\left(\Sigma F_x = ma_x, \text{ Equation 4.2a}\right)$ in the following manner. The kinetic frictional force is the only horizontal force that acts on the skater, and, since it is a resistive force, it acts opposite to the direction of the motion. Thus, the net force in the x direction is $\Sigma F_x = -f_{\mathrm{k}}$, where f_{k} is the magnitude of the kinetic frictional force. Therefore, the acceleration of the skater is $a_x = \Sigma F_x/m = -f_{\mathrm{k}}/m$.

The magnitude of the frictional force is $f_{\mathrm{k}} = \mu_{\mathrm{k}} F_{\mathrm{N}}$ (Equation 4.8), where μ_{k} is the coefficient of kinetic friction between the ice and the skate blades and F_{N} is the magnitude of the normal force. There are two vertical forces acting on the skater: the upward-acting normal force $\mathbf{F_N}$ and the downward pull of gravity (her weight) $m\mathbf{g}$. Since the skater has no vertical acceleration, Newton's second law in the vertical direction gives (taking upward as the positive direction) $\Sigma F_y = F_{\mathrm{N}} - mg = 0$. Therefore, the magnitude of the normal force is $F_{\mathrm{N}} = mg$ and the magnitude of the acceleration is

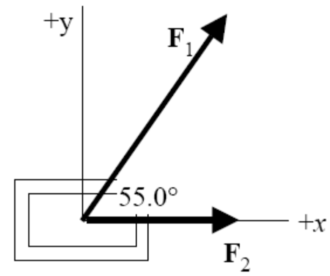
$$a_x = \frac{-f_{\mathrm{k}}}{m} = \frac{-\mu_{\mathrm{k}} F_{\mathrm{N}}}{m} = \frac{-\mu_{\mathrm{k}} \cancel{m} g}{\cancel{m}} = -\mu_{\mathrm{k}} g$$

SOLUTION

Solving the equation $v_x = v_{0x} + a_x t$ for the time and substituting the expression above for the acceleration yields

$$t = \frac{v_x - v_{0x}}{a_x} = \frac{v_x - v_{0x}}{-\mu_{\mathrm{k}} g} = \frac{2.8 \text{ m/s} - 6.3 \text{ m/s}}{-(0.081)\left(9.80 \text{ m/s}^2\right)} = \boxed{4.4 \text{ s}}$$

45. ***REASONING*** The diagram shows the two applied forces that act on the crate. These two forces, plus the kinetic frictional force $\mathbf{f}_k$ constitute the net force that acts on the crate. Once the net force has been determined, Newtons' second law, $\Sigma\mathbf{F} = m\mathbf{a}$ (Equation 4.1) can be used to find the acceleration of the crate.



SOLUTION The sum of the two applied forces is $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$. The x-component of this sum is $F_x = F_1 \cos 55.0° + F_2 = (88.0\text{ N}) \cos 55.0° + 54.0\text{ N} = 104\text{ N}$. The y-component of $\mathbf{F}$ is $F_y = F_1 \sin 55.0° = (88.0\text{ N}) \sin 55.0° = 72.1\text{ N}$. The magnitude of $\mathbf{F}$ is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(104\text{ N})^2 + (72.1\text{ N})^2} = 127\text{ N}$$

Since the crate starts from rest, it moves along the direction of $\mathbf{F}$. The kinetic frictional force $\mathbf{f}_k$ opposes the motion, so it points opposite to $\mathbf{F}$. The net force acting on the crate is the sum of $\mathbf{F}$ and $\mathbf{f}_k$. The magnitude a of the crate's acceleration is equal to the magnitude ΣF of the net force divided by the mass m of the crate

$$a = \frac{\Sigma F}{m} = \frac{-f_k + F}{m} \qquad (4.1)$$

According to Equation 4.8, the magnitude f_k of the kinetic frictional force is given by $f_k = \mu_k F_N$, where F_N is the magnitude of the normal force. In this situation, F_N is equal to the magnitude of the crate's weight, so $F_N = mg$. Thus, the x-component of the acceleration is

$$a = \frac{-\mu_k mg + F}{m} = \frac{-(0.350)(25.0\text{ kg})(9.80\text{ m/s}^2) + 127\text{ N}}{25.0\text{ kg}} = \boxed{1.65\text{ m/s}^2}$$

The crate moves along the direction of $\mathbf{F}$, whose x and y components have been determined previously. Therefore, the acceleration is also along $\mathbf{F}$. The angle ϕ that $\mathbf{F}$ makes with the x-axis can be found using the inverse tangent function:

$$\phi = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{F_1 \sin 55.0°}{F_1 \cos 55.0° + F_2}\right)$$

$$= \tan^{-1}\left[\frac{(88.0\text{ N}) \sin 55.0°}{(88.0\text{ N})\cos 55.0° + 54.0\text{ N}}\right] = \boxed{34.6° \text{ above the } x \text{ axis}}$$

109. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION***

a. The left mass (mass 1) has a tension T_1 pulling it up. Newton's second law gives

$$T_1 - m_1 g = m_1 a \qquad (1)$$

The right mass (mass 3) has a different tension, T_3, trying to pull it up. Newton's second for it is

$$T_3 - m_3 g = -m_3 a \qquad (2)$$

The middle mass (mass 2) has both tensions acting on it along with friction. Newton's second law for its horizontal motion is

$$T_3 - T_1 - \mu_k m_2 g = m_2 a \qquad (3)$$

Solving Equation (1) and Equation (2) for T_1 and T_3, respectively, and substituting into Equation (3) gives

$$a = \frac{\left(m_3 - m_1 - \mu_k m_2\right) g}{m_1 + m_2 + m_3}$$

Hence,

$$a = \frac{\left[25.0\text{ kg} - 10.0\text{ kg} - (0.100)(80.0\text{ kg})\right]\left(9.80\text{ m/s}^2\right)}{10.0\text{ kg} + 80.0\text{ kg} + 25.0\text{ kg}} = \boxed{0.60\text{ m/s}^2}$$

b. From part a:

$$T_1 = m_1(g + a) = (10.0\text{ kg})\left(9.80\text{ m/s}^2 + 0.60\text{ m/s}^2\right) = \boxed{104\text{ N}}$$

$$T_3 = m_3(g - a) = (25.0\text{ kg})\left(9.80\text{ m/s}^2 - 0.60\text{ m/s}^2\right) = \boxed{230\text{ N}}$$