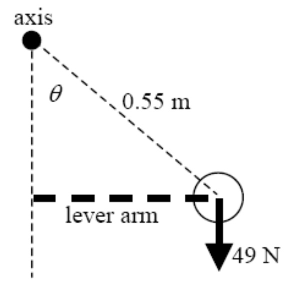
WORKSHOP WEEK 1

4. ***REASONING*** In both parts of the problem, the magnitude of the torque is given by Equation 9.1 as the magnitude F of the force times the lever arm ℓ. In part (a), the lever arm is just the distance of 0.55 m given in the drawing. However, in part (b), the lever arm is less than the given distance and must be expressed using trigonometry as $\ell = (0.55 \text{ m})\sin\theta$. See the drawing at the right.



SOLUTION

a. Using Equation 9.1, we find that

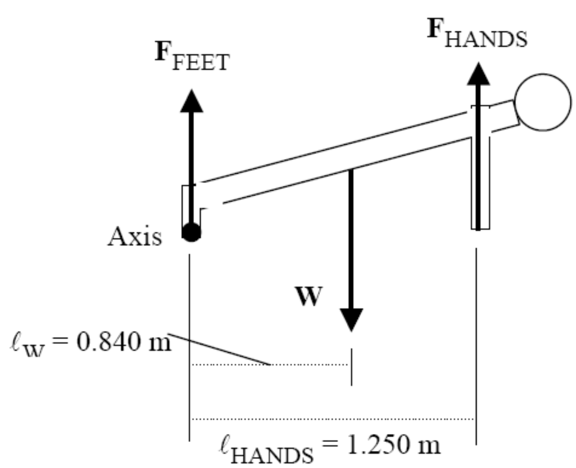
$$\text{Magnitude of torque} = F\ell = (49 \text{ N})(0.55 \text{ m}) = \boxed{27 \text{ N}\cdot\text{m}}$$

b. Again using Equation 9.1, this time with a lever arm of $\ell = (0.55 \text{ m})\sin\theta$, we obtain

$$\text{Magnitude of torque} = 15 \text{ N}\cdot\text{m} = F\ell = (49 \text{ N})(0.55 \text{ m})\sin\theta$$

$$\sin\theta = \frac{15 \text{ N}\cdot\text{m}}{(49 \text{ N})(0.55 \text{ m})} \quad \text{or} \quad \theta = \sin^{-1}\left[\frac{15 \text{ N}\cdot\text{m}}{(49 \text{ N})(0.55 \text{ m})}\right] = \boxed{34^\circ}$$

11. ***REASONING*** The drawing shows the forces acting on the person. It also shows the lever arms for a rotational axis perpendicular to the plane of the paper at the place where the person's toes touch the floor. Since the person is in equilibrium, the sum of the forces must be zero. Likewise, we know that the sum of the torques must be zero.



SOLUTION Taking upward to be the positive direction, we have

$$F_{\text{FEET}} + F_{\text{HANDS}} - W = 0$$

Remembering that counterclockwise torques are positive and using the axis and the lever arms shown in the drawing, we find

$$W\ell_{\text{W}} - F_{\text{HANDS}}\ell_{\text{HANDS}} = 0$$

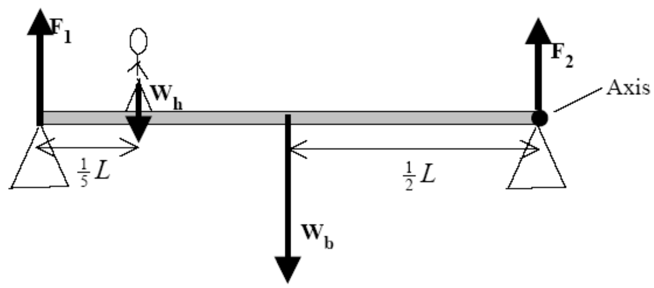
$$F_{\text{HANDS}} = \frac{W\ell_{\text{W}}}{\ell_{\text{HANDS}}} = \frac{(584\ \text{N})(0.840\ \text{m})}{1.250\ \text{m}} = 392\ \text{N}$$

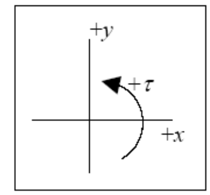
Substituting this value into the balance-of-forces equation, we find

$$F_{\text{FEET}} = W - F_{\text{HANDS}} = 584\ \text{N} - 392\ \text{N} = 192\ \text{N}$$

The force on each hand is half the value calculated above, or $\boxed{196\ \text{N}}$. Likewise, the force on each foot is half the value calculated above, or $\boxed{96\ \text{N}}$.

13. $\boxed{\text{SSM}}$ ***REASONING*** The drawing shows the bridge and the four forces that act on it: the upward force $\mathbf{F}_1$ exerted on the left end by the support, the force due to the weight $\mathbf{W}_\text{h}$ of the hiker, the weight $\mathbf{W}_\text{b}$ of the bridge, and the upward force $\mathbf{F}_2$ exerted on the right side by the support. Since the bridge is in equilibrium, the sum of the torques about any axis of rotation must be zero $(\Sigma\tau = 0)$, and the sum of the forces in the vertical direction must be zero $\left(\Sigma F_y = 0\right)$. These two conditions will allow us to determine the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$.





SOLUTION

a. We will begin by taking the axis of rotation about the right end of the bridge. The torque produced by $\mathbf{F}_2$ is zero, since its lever arm is zero. When we set the sum of the torques equal to zero, the resulting equation will have only one unknown, $\mathbf{F}_1$, in it. Setting the sum of the torques produced by the three forces equal to zero gives

$$\Sigma\tau = -F_1 L + W_\text{h}\left(\tfrac{4}{5}L\right) + W_\text{b}\left(\tfrac{1}{2}L\right) = 0$$

Algebraically eliminating the length L of the bridge from this equation and solving for F_1 gives

$$F_1 = \tfrac{4}{5}W_\text{h} + \tfrac{1}{2}W_\text{b} = \tfrac{4}{5}(985\text{ N}) + \tfrac{1}{2}(3610\text{ N}) = \boxed{2590\text{ N}}$$

b. Since the bridge is in equilibrium, the sum of the forces in the vertical direction must be zero:

$$\Sigma F_y = F_1 - W_\text{h} - W_\text{b} + F_2 = 0$$

Solving for F_2 gives

$$F_2 = -F_1 + W_\text{h} + W_\text{b} = -2590\text{ N} + 985\text{ N} + 3610\text{ N} = \boxed{2010\text{ N}}$$

47. ***REASONING*** The rotational kinetic energy of a solid sphere is given by Equation 9.9 as $\text{KE}_\text{R} = \tfrac{1}{2}I\omega^2$, where I is its moment of inertia and ω its angular speed. The sphere has translational motion in addition to rotational motion, and its translational kinetic energy is $\text{KE}_\text{T} = \tfrac{1}{2}mv^2$ (Equation 6.2), where m is the mass of the sphere and v is the speed of its center of mass. The fraction of the sphere's total kinetic energy that is in the form of rotational kinetic energy is $\text{KE}_\text{R}/(\text{KE}_\text{R} + \text{KE}_\text{T})$.

SOLUTION The moment of inertia of a solid sphere about its center of mass is $I = \tfrac{2}{5}mR^2$, where R is the radius of the sphere (see Table 9.1). The fraction of the sphere's total kinetic energy that is in the form of rotational kinetic energy is

$$\frac{\mathrm{KE_R}}{\mathrm{KE_R}+\mathrm{KE_T}}=\frac{\frac{1}{2}I\omega^2}{\frac{1}{2}I\omega^2+\frac{1}{2}mv^2}=\frac{\frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2}{\frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2+\frac{1}{2}mv^2}=\frac{\frac{2}{5}R^2\omega^2}{\frac{2}{5}R^2\omega^2+v^2}$$

Since the sphere is rolling without slipping on the surface, the translational speed v of the center of mass is related to the angular speed ω about the center of mass by $v = R\omega$ (see Equation 8.12). Substituting $v = R\omega$ into the equation above gives

$$\frac{\mathrm{KE_R}}{\mathrm{KE_R}+\mathrm{KE_T}}=\frac{\frac{2}{5}R^2\omega^2}{\frac{2}{5}R^2\omega^2+v^2}=\frac{\frac{2}{5}R^2\omega^2}{\frac{2}{5}R^2\omega^2+(R\omega)^2}=\boxed{\frac{2}{7}}$$

49. $\boxed{\textbf{SSM}}$ ***REASONING AND SOLUTION*** The only force that does work on the cylinders as they move up the incline is the conservative force of gravity; hence, the total mechanical energy is conserved as the cylinders ascend the incline. We will let $h = 0$ on the horizontal plane at the bottom of the incline. Applying the principle of conservation of mechanical energy to the solid cylinder, we have

$$mgh_\mathrm{s}=\tfrac{1}{2}mv_0^2+\tfrac{1}{2}I_\mathrm{s}\omega_0^2 \qquad (1)$$

where, from Table 9.1, $I_\mathrm{s}=\frac{1}{2}mr^2$. In this expression, v_0 and ω_0 are the initial translational and rotational speeds, and h_s is the final height attained by the solid cylinder. Since the cylinder rolls without slipping, the rotational speed ω_0 and the translational speed v_0 are related according to Equation 8.12, $\omega_0 = v_0/r$. Then, solving Equation (1) for h_s, we obtain

$$h_\mathrm{s}=\frac{3v_0^2}{4g}$$

Repeating the above for the hollow cylinder and using $I_\mathrm{h}=mr^2$ we have

$$h_\mathrm{h}=\frac{v_0^2}{g}$$

The height h attained by each cylinder is related to the distance s traveled along the incline and the angle θ of the incline by

$$s_\mathrm{s}=\frac{h_\mathrm{s}}{\sin\theta} \qquad \text{and} \qquad s_\mathrm{h}=\frac{h_\mathrm{h}}{\sin\theta}$$

Dividing these gives

$$\frac{s_\mathrm{s}}{s_\mathrm{h}}=\boxed{3/4}$$