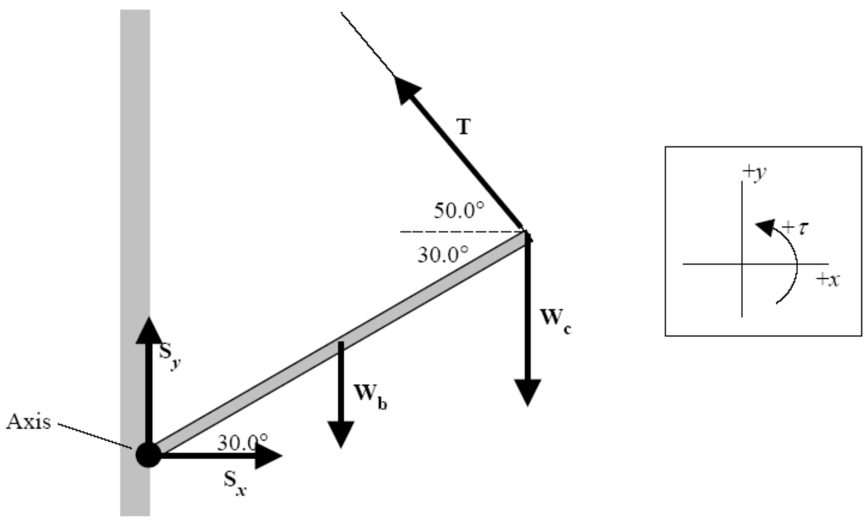
Workshop week 2

20. ***REASONING*** The drawing shows the beam and the five forces that act on it: the horizontal and vertical components $\mathbf{S}_x$ and $\mathbf{S}_y$ that the wall exerts on the left end of the beam, the weight $\mathbf{W}_b$ of the beam, the force due to the weight $\mathbf{W}_c$ of the crate, and the tension **T** in the cable. The beam is uniform, so its center of gravity is at the center of the beam, which is where its weight can be assumed to act. Since the beam is in equilibrium, the sum of the torques about any axis of rotation must be zero $(\Sigma\tau = 0)$, and the sum of the forces in the horizontal and vertical directions must be zero $\left(\Sigma F_x = 0,\ \Sigma F_y = 0\right)$. These three conditions will allow us to determine the magnitudes of $\mathbf{S}_x$, $\mathbf{S}_y$, and **T**.



SOLUTION

a. We will begin by taking the axis of rotation to be at the left end of the beam. Then the torques produced by $\mathbf{S}_x$ and $\mathbf{S}_y$ are zero, since their lever arms are zero. When we set the sum of the torques equal to zero, the resulting equation will have only one unknown, T, in it. Setting the sum of the torques produced by the three forces equal to zero gives (with L equal to the length of the beam)

$$\Sigma\tau = -W_b\left(\tfrac{1}{2}L\cos 30.0°\right) - W_c\left(L\cos 30.0°\right) + T\left(L\sin 80.0°\right) = 0$$

Algebraically eliminating L from this equation and solving for T gives

$$T = \frac{W_\text{b}\left(\frac{1}{2}\cos 30.0°\right) + W_\text{c}\left(\cos 30.0°\right)}{\sin 80.0°}$$

$$= \frac{(1220 \text{ N})\left(\frac{1}{2}\cos 30.0°\right) + (1960 \text{ N})\left(\cos 30.0°\right)}{\sin 80.0°} = \boxed{2260 \text{ N}}$$

b. Since the beam is in equilibrium, the sum of the forces in the vertical direction must be zero:

$$\Sigma F_y = +S_y - W_\text{b} - W_\text{c} + T\sin 50.0° = 0$$

Solving for S_y gives

$$S_y = W_\text{b} + W_\text{c} - T\sin 50.0° = 1220 \text{ N} + 1960 \text{ N} - (2260 \text{ N})\sin 50.0° = \boxed{1450 \text{ N}}$$

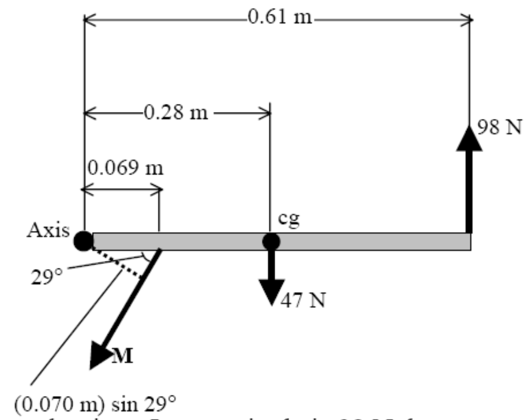
The sum of the forces in the horizontal direction must also be zero:

$$\Sigma F_x = +S_x - T\cos 50.0° = 0$$

so that

$$S_x = T\cos 50.0° = (2260 \text{ N})\cos 50.0° = \boxed{1450 \text{ N}}$$

22. ***REASONING*** The arm, being stationary, is in equilibrium, since it has no translational or angular acceleration. Therefore, the net external force and the net external torque acting on the arm are zero. Using the fact that the net external torque is zero will allow us to determine the magnitude of the force **M**. The drawing at the right shows three forces: **M**, the 47-N weight of the arm acting at the arm's center of gravity (cg), and the 98-N force that acts upward on the right end of the arm. The 98-N force is applied to the arm by the ring. It is the reaction force that arises in response to the arm pulling downward on the ring. Its magnitude is 98 N, because it supports the 98-N weight hanging from the pulley system. Other forces also act on the arm at the shoulder joint, but we can ignore them. The reason is that their lines of action pass directly through the axis at the shoulder joint, which is the axis that we will use to determine torques. Thus, these forces have zero lever arms and contribute no torque.



SOLUTION The magnitude of each individual torque is the magnitude of the force times the corresponding lever arm. The forces and their lever arms are as follows:

Force	Lever Arm
98 N	0.61 m
47 N	0.28
M	(0.069 m) sin 29°

Each torque is positive if it causes a counterclockwise rotation and negative if it causes a clockwise rotation about the axis. Thus, since the net torque must be zero, we see that

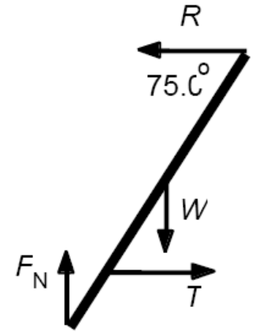
$$(98\text{ N})(0.61\text{ m})-(47\text{ N})(0.28\text{ m})-M\left[(0.069\text{ m})\sin 29°\right]=0$$

Solving for M gives

$$M=\frac{(98\text{ N})(0.61\text{ m})-(47\text{ N})(0.28\text{ m})}{(0.069\text{ m})\sin 29°}=\boxed{1400\text{ N}}$$

26. ***REASONING AND SOLUTION*** The weight W of the left side of the ladder, the normal force F_N of the floor on the left leg of the ladder, the tension T in the crossbar, and the reaction force R due to the right-hand side of the ladder, are shown in the following figure. In the vertical direction $-W+F_N=0$, so that

$$F_N = W = mg = (10.0 \text{ kg})(9.80 \text{ m/s}^2) = 98.0 \text{ N}$$



In the horizontal direction it is clear that $R = T$. The net torque about the base of the ladder is

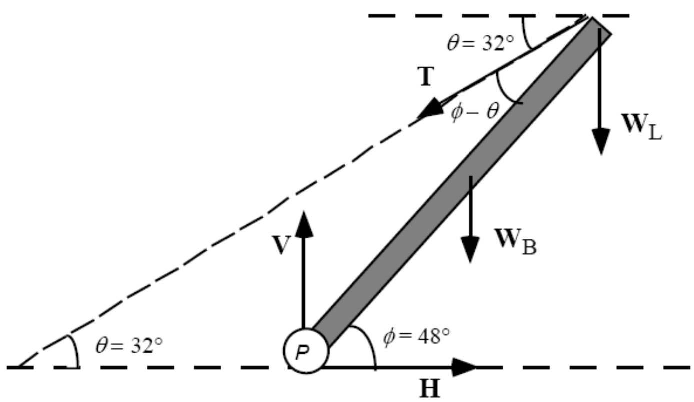
$$\Sigma\tau = -T[(1.00 \text{ m})\sin 75.0^\circ] - W[(2.00 \text{ m})\cos 75.0^\circ] + R[(4.00 \text{ m})\sin 75.0^\circ] = 0$$

Substituting for W and using $R = T$, we obtain

$$T = \frac{(9.80 \text{ N})(2.00 \text{ m})\cos 75.0^\circ}{(3.00 \text{ m})\sin 75.0^\circ} = \boxed{17.5 \text{ N}}$$

68. ***REASONING*** If we assume that the system is in equilibrium, we know that the vector sum of all the forces, as well as the vector sum of all the torques, that act on the system must be zero.

The figure below shows a free body diagram for the boom. Since the boom is assumed to be uniform, its weight $\mathbf{W}_B$ is located at its center of gravity, which coincides with its geometrical center. There is a tension $\mathbf{T}$ in the cable that acts at an angle θ to the horizontal, as shown. At the hinge pin P, there are two forces acting. The vertical force $\mathbf{V}$ that acts on the end of the boom prevents the boom from falling down. The horizontal force $\mathbf{H}$ that also acts at the hinge pin prevents the boom from sliding to the left. The weight $\mathbf{W}_L$ of the wrecking ball (the "load") acts at the end of the boom.



By applying the equilibrium conditions to the boom, we can determine the desired forces.

SOLUTION The directions upward and to the right will be taken as the positive directions. In the x direction we have

$$\sum F_x = H - T\cos\theta = 0 \tag{1}$$

while in the y direction we have

$$\sum F_y = V - T\sin\theta - W_\mathrm{L} - W_\mathrm{B} = 0 \tag{2}$$

Equations (1) and (2) give us two equations in three unknown. We must, therefore, find a third equation that can be used to determine one of the unknowns. We can get the third equation from the torque equation.

In order to write the torque equation, we must first pick an axis of rotation and determine the lever arms for the forces involved. Since both V and H are unknown, we can eliminate them from the torque equation by picking the rotation axis through the point P (then both V and H have zero lever arms). If we let the boom have a length L, then the lever arm for $\mathbf{W}_\mathrm{L}$ is $L\cos\phi$, while the lever arm for $\mathbf{W}_\mathrm{B}$ is $(L/2)\cos\phi$. From the figure, we see that the lever arm for $\mathbf{T}$ is $L\sin(\phi-\theta)$. If we take counterclockwise torques as positive, then the torque equation is

$$\sum \tau = -W_\mathrm{B}\left(\frac{L\cos\phi}{2}\right) - W_\mathrm{L} L\cos\phi + TL\sin\left(\phi-\theta\right) = 0$$

Solving for T, we have

$$T = \frac{\frac{1}{2}W_\mathrm{B} + W_\mathrm{L}}{\sin(\phi-\theta)}\cos\phi \tag{3}$$

a. From Equation (3) the tension in the support cable is

$$T = \frac{\frac{1}{2}(3600\ \mathrm{N}) + 4800\ \mathrm{N}}{\sin(48° - 32°)}\cos 48° = \boxed{1.6\times10^4\ \mathrm{N}}$$

b. The force exerted on the lower end of the hinge at the point P is the vector sum of the forces $\mathbf{H}$ and $\mathbf{V}$. According to Equation (1),

$$H = T\cos\theta = \left(1.6\times10^4\ \mathrm{N}\right)\cos 32° \ = \ 1.4\times10^4\ \mathrm{N}$$

and, from Equation (2)

$$V = W_L + W_B + T\sin\theta = 4800\ \mathrm{N} + 3600\ \mathrm{N} + \left(1.6\times10^4\ \mathrm{N}\right)\sin 32° = 1.7\times10^4\ \mathrm{N}$$

Since the forces $\mathbf{H}$ and $\mathbf{V}$ are at right angles to each other, the magnitude of their vector sum can be found from the Pythagorean theorem:

$$F_\mathrm{P} = \sqrt{H^2 + V^2} = \sqrt{(1.4\times10^4\ \mathrm{N})^2 + (1.7\times10^4\ \mathrm{N})^2} = \boxed{2.2\times10^4\ \mathrm{N}}$$