Workshop 2

1. Find first integrals of the Euler-Lagrange equations corresponding to the optimization of following functionals.

   (a)
$$I[y] = \int_{x_1}^{x_2} x\sqrt{x^2 + (y')^2}\,\mathrm{d}x$$

   (b)
$$I[y] = \int_{x_1}^{x_2} \sqrt{1 + (yy')^2}\,\mathrm{d}x$$

2. Assuming that light travels in a medium where the refractive index varies like $n(x, y) = 1/y$, Use the methods of variational calculus and Fermat's Principle to show that the light rays travel in circular paths in this medium. Find the equation of the light ray which starts at (0,1) moving in the horizontal direction.

Homework 2

1. Find the solution to the brachistochrone problem for a bead moving between $A(0,0)$ and $B(2, 2+3\pi)$. Sketch the solution.

2. In a certain medium the refractive index is $n(x,y) = e^y$. Use Fermat's principle to explicitly determine the light ray path between the points (-1,1) and (1,1).

3. Find a first integral of the Euler-Lagrange equations corresponding to the optimization the following functional.
$$I[y] = \int_{x_1}^{x_2} \frac{1+(y')^2}{y}\,\mathrm{d}x$$
. Solve the resulting first order differential equation for y given that the curve passes through the points (0,0) and (1,0).

4. Solve the following problems from *Perfect Form* by Lemons: 2.5(b), 3.1 and 3.2