Non-Linear Dynamics Homework Solutions

Week 8

Chris Small

March 10, 2007

I apologize for how late this solution set is, as well as for it's incompleteness. Please email me at smachr09@evergreen.edu with any questions or concerns reguarding these solutions.

Credit and thanks go to Taiyo and Chris Davis for Problem 8.4.3, and again to Taiyo for Problem 8.5.1. I reviewed their homework solutions before typing these up.

8.4.1 Consider the system $\dot{r} = r(1 - r^2)$ and $\dot{\theta} = \mu - \sin\theta$ for μ slightly greater than 1. Sketch waveforms of $x(t)$ and $y(t)$.

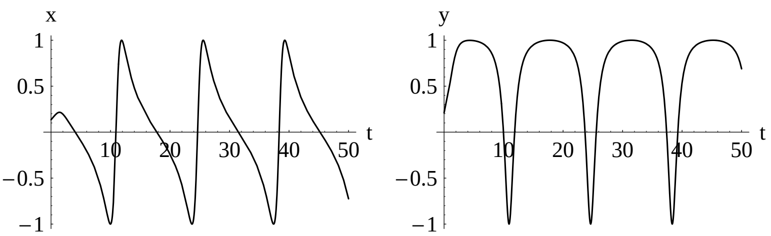

Figure 1: Plot of the waveforms of $x(t)$ and $y(t)$

8.4.2 Discuss the bifurcations of the system $\dot{r} = r(\mu - \sin r)$, $\dot{\theta} = 1$ as μ varies.

This system has no fixed points, but $\dot{r}$ can take on zero values when $|\mu| \leq 0$. When equality holds, an infinite number of fold bifurcations occur, each sperated from the next by 2π. When μ passes below or above either of these critical values, the bistable limit cycles which arise when equality holds, split apart into two limit cycles, one stable and one unstable. As these move apart they collide with those which arose from neighboring fold bifurcations and anhilate eachother as $|\mu|$ becomes greater than 1.

8.4.3 Using numerical integration, find the value of μ at which the system

$$\begin{aligned}\dot{x} &= \mu x + y - x^2 \\ \dot{y} &= -x + \mu y + 2x^2\end{aligned}$$

undergoes a homoclinic bifurcation. Sketch the phase portrait just above and below the bifurcation.

Numerical plots of the system using Mathematica show us that $.06 < \mu_c < .07$. Phase portraits are shown in Figure 2.

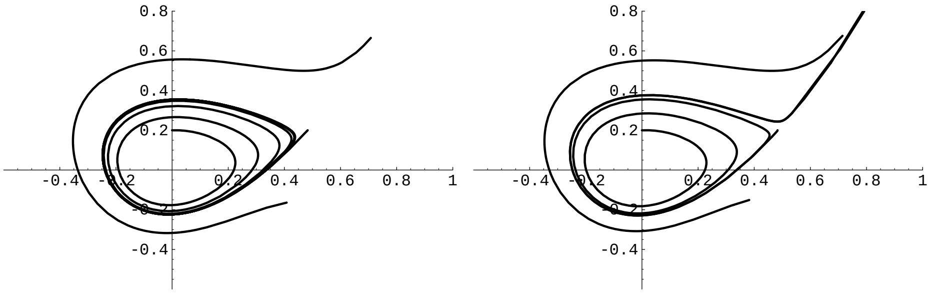

Figure 2: Numerical plots for $\mu = .06$ and $.07$ from left to right

8.5.1 Show that $[ln(I - I_c)]^{-1}$ has infinite derivatives of all orders at I_c.

We use the hint on the first couple of derivatives to get some intuition as to what the derivatives are doing. We find that

$$\frac{d}{dt}\frac{1}{\ln[I - I_c]} = \frac{-1}{(I - I_c)\ln[I - I_c]^2}$$

$$\frac{d^2}{dt^2}\frac{1}{\ln[I - I_c]} = \frac{1}{(I - I_c)^2\ln[I - I_c]} + \frac{2}{(I - I_c)^2\ln[I - I_c]^3}$$

If we continue in this fassion we see that we will continue to get terms of $(I - I_c)$ in the denomenator of the summands involved in each derivative. Thus, each derivative is undefined at $I = I_c$.