Spring Physical Systems - Astrophysics - wk 3 14 Apr 03

E/Z

Ch 9 - Stellar Atmospheres p. 255

9.1 THE RADIATION FIELD

Intensity = $\frac{\text{power}}{\text{area}} = \langle I_\lambda \rangle = \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} I_\lambda \sin\theta \, d\theta \, d\phi$ (Fig 9.1 256)

Prob 9.1 Energy density $u = u_\lambda d\lambda = \frac{4\pi}{c}\langle I_\lambda \rangle d\lambda \rightarrow \frac{4\pi}{c} B_\lambda d\lambda = \frac{4\sigma T^4}{c}$ for a blackbody

Radiative flux $F = F_\lambda d\lambda = \int I_\lambda d\lambda \cos\theta \, d\Omega$ (also units of intensity)

p. 261 Radiation pressure $P_{rad\,\lambda} d\lambda = P_{rad} = \frac{4\pi}{3c} I_\lambda d\lambda \rightarrow \frac{1}{3}u$

ISOTROPIC blackbody

9.2 261 STELLAR OPACITY ≈ optical thickness of gas, ability to absorb & scatter photons

THERMODYNAMIC EQUILIBRIUM: Σ(Energy in + Energy Out) = 0 (not stars!)

Local LTE : $\frac{T}{\nabla T} > \lambda_{mfp}$: temperature is locally constant

photon travels many mfp before T changes

Temperature scale height $H_T = \frac{T}{\nabla T}$

Photon mean free path $l = \frac{1}{n\sigma}$ where σ = collision cross section

Ex 9.1 263: PHOTOSPHERE : $H_T = 6.66 \times 10^7$ cm $\gg l_{atoms} = 1.9 \times 10^{-2}$ cm $\therefore$ LTE for atoms

Prob 9.6 Ex 9.2 p. 266 $H_T \gtrsim l_{photons} = 1.5 \times 10^7$ cm : barely LTE for photons

Prob 9.7 **OPACITY** (absorption coeff) $K_\lambda = \frac{1}{l\rho} = \frac{n\sigma_\lambda}{\rho}$ $\left(\frac{cm^2}{g}\right)$ reduces intensity:

for uniform gas, $I_\lambda = I_{\lambda 0} e^{-K_\lambda \rho s}$ (s = distance traveled)

266 OPTICAL DEPTH $\tau_\lambda = 0$ where light travels unimpeded [SURFACE]

looking back along photons path $\quad$ $s=0 \longrightarrow s = \text{distance}$

$\longleftarrow \tau = 0$, τ = optical depth

Prob 9.12 $\quad \tau = \int_0^s \kappa_\lambda \rho \, ds$

$I = I_{\lambda_0} e^{-\tau_\lambda}$

optical depth = # of mfp from original position (of photon) to surface

p. 269 Interactions with electrons scatter & absorb photons.
(mainly excitation, ionization, and free-free absorption. ~~Electron scattering is~~
Electron scattering (Thomson, Compton, Rayleigh) is NEGLIGIBLE unless VERY HOT.

Average all sources of opacity: $\bar{\kappa} = \dfrac{\kappa_0 \rho}{T^{3.5}}$ (See Fig $\frac{9.10}{275}$)

9.3 RADIATIVE TRANSFER : absorbing & emitting radiation
276

Prob 9.11 Random walk: displacement $d = l\sqrt{N}$ $= \tau_\lambda l$, N = # of steps $= \tau_\lambda^2$ $(\tau_\lambda \gg 1)$

We can see to $\tau_\lambda = \frac{2}{3}$ at any angle $\rightarrow$ LIMB DARKENING Fig $\frac{9.12}{279}$

Radiation pressure $\rightarrow$ photon "breeze" out to surface of star

$$\nabla P_{rad} = -\frac{\bar{\kappa}\rho}{c} F_{rad} \qquad \frac{(9.25)}{280}$$

9.4 STRUCTURE OF SPECTRAL LINES : line width $W = \int \frac{F_{continuum} - F_\lambda}{F_c} \, d\lambda$
293
- Natural broadening due to HUP: $\Delta\lambda \approx \frac{\lambda^2}{2\pi c}\left(\frac{1}{\Delta t_i} + \frac{1}{\Delta t_f}\right)$ = SMALL (Fig 9.18 p. 294)
- Doppler broadening $\Delta\lambda = \frac{2\lambda}{c}\sqrt{\frac{2kT}{m}} \gg$ natural
- Pressure (coll) broadening $\Delta\lambda = \frac{\lambda^2}{c} \frac{1}{\pi \Delta t}$ $\approx \frac{\lambda^2}{c} \frac{n\sigma}{\pi} \sqrt{\frac{2kT}{m}}$ (turbulent broadening negligible) Ap-22

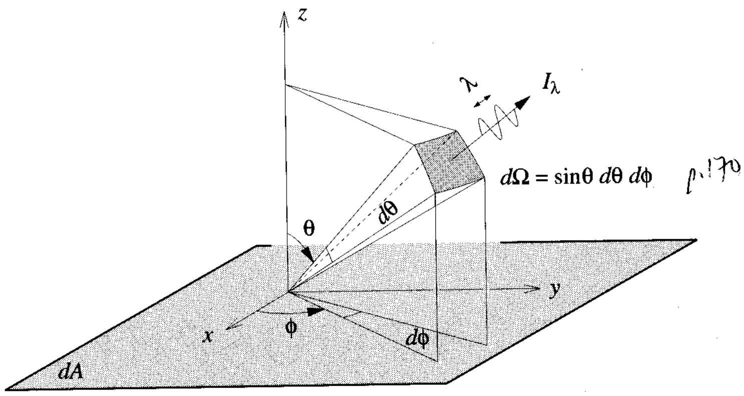


Figure 9.1 Intensity I_λ.

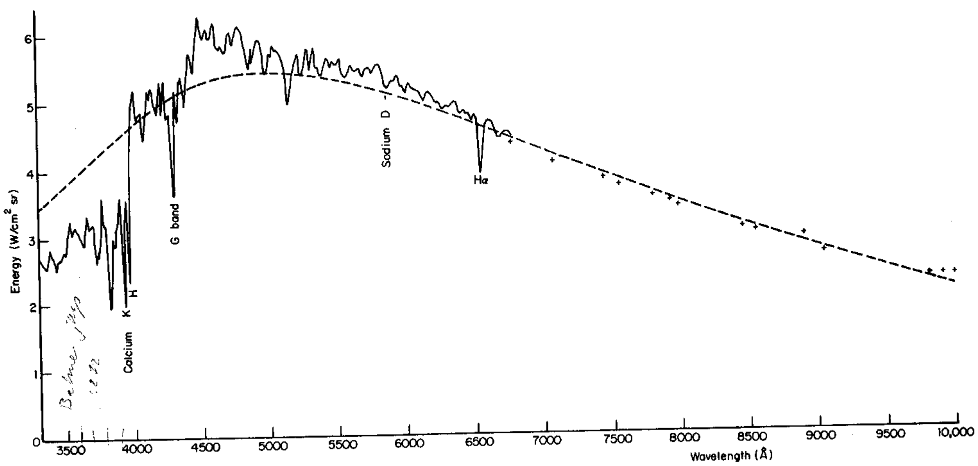


Figure 9.5 The spectrum of the Sun. The dashed line is the curve of an ideal blackbody having the Sun's effective temperature. (Figure from Aller, *Atoms, Stars, and Nebulae*, Revised Edition, Harvard University Press, Cambridge, MA, 1971.)

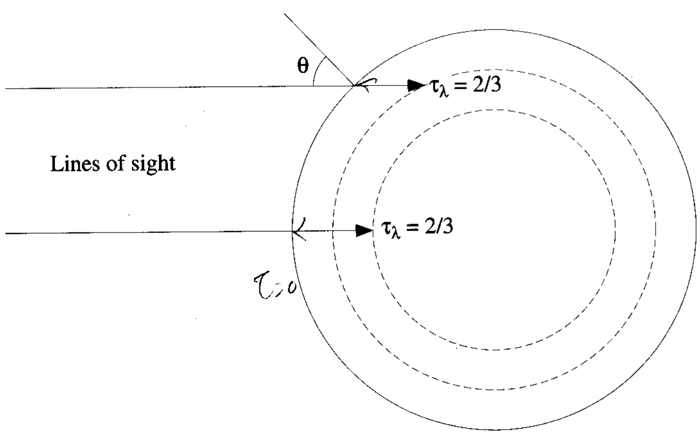
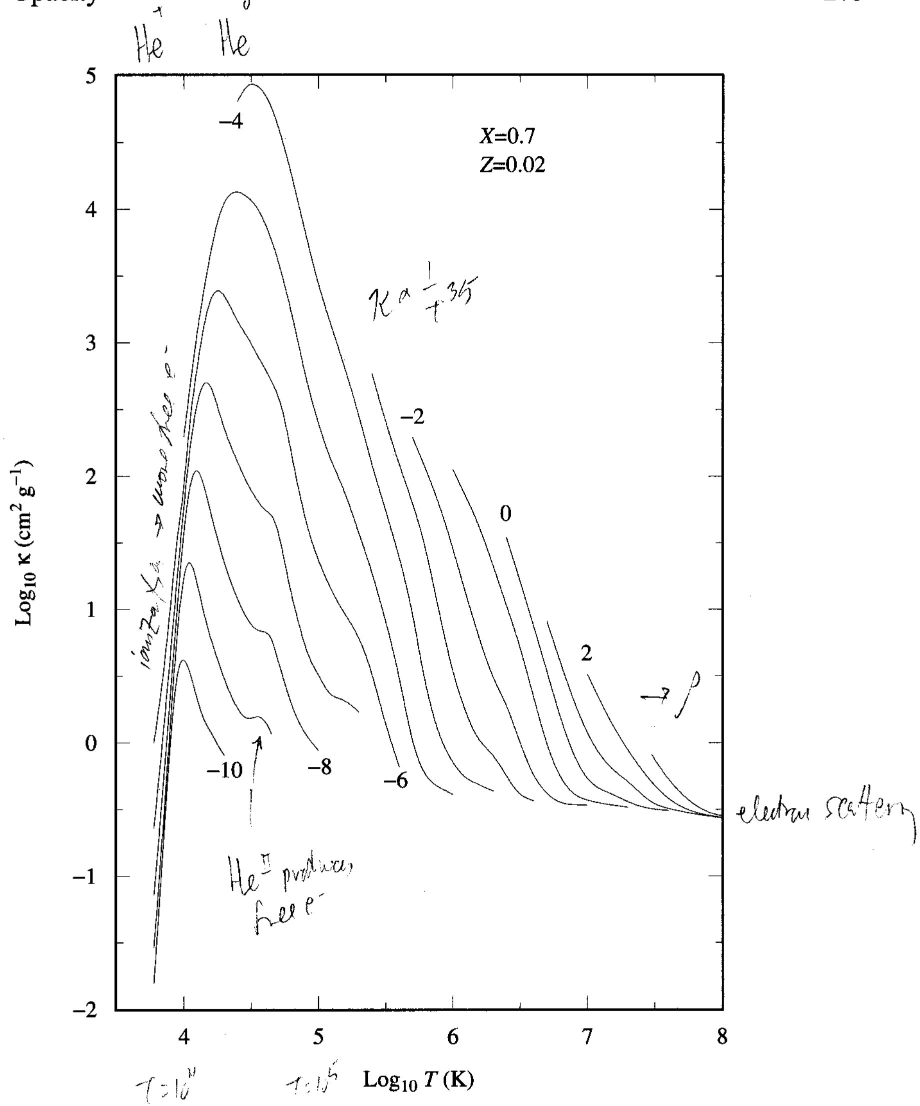


Figure 9.12 Limb darkening.



igure 9.10 Rosseland mean opacity. The curves are labeled by the value the density ($\log_{10}\rho$ in g cm^{-3}). (Data from Rogers and Iglesias, *Ap. J. uppl.*, *79*, 507, 1992.)

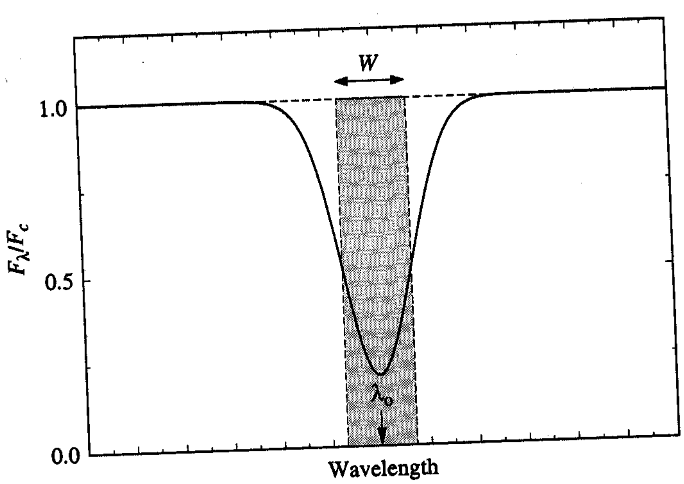


Figure 9.18 The shape of a typical spectral line.

Ap Cid HW #1

Evaluate the energy of the blackbody photons inside your eye. Compare this with the visible energy inside your eye while looking at a 100-W (10^9 erg s^{-1}) light bulb that is 100 cm away. (You can assume that the light bulb is 100% efficient, although in reality it converts only a few percent of its 100 watts into visible photons. Take your eye to be a hollow sphere of radius 1.5 cm at a temperature of 37°C. The area of the eye's pupil is about 0.1 cm^2.) Why is it dark when you close your eyes?

$T = 37+273 = 310\,K$

BB $\frac{(9.5)}{258}$ energy density $u = \frac{4\sigma T^4}{c} = aT^4$ where $a = 7.566 \times 10^{-15} \frac{erg}{cm^3 K^4}$

Energy = $u \cdot$ volume

Light bulb: Flux $= \frac{L}{4\pi d^2}$, Power entering eye = Flux · area = $\frac{\text{Energy}}{\text{time}}$

(d = distance from eye to bulb)

Time for light to cross eye? speed = $\frac{\text{distance}}{\text{time}}$

(radius of eye)

time = $\frac{\text{distance}}{\text{speed}} = \frac{2r}{c}$ Combine these to find

$E_{\text{from light bulb to eye}} =$

Dark: When you close your eyes, the blackbody photons are still hitting retina. What is their wavelength?

Wien's law (3.15)

9.2

(a) **Find an expression for $n_\lambda \, d\lambda$, the number density of blackbody photons (the number of blackbody photons per cm^3) with a wavelength between λ and $\lambda + d\lambda$.**

(b) **Find the total number of photons inside a kitchen oven set at 400°F, assuming a volume of 1 m^3.**

(9.3) 257

Specific energy density $du = \frac{\partial u}{\partial \lambda} d\lambda = u_\lambda \, d\lambda = \frac{8\pi hc/\lambda^5}{e^{hc/\lambda kT} - 1} d\lambda$

Specific number density $n_\lambda \, d\lambda = \frac{u_\lambda d\lambda}{hc} =$

(b) $n = \int_0^\infty n_\lambda \, d\lambda$

Hint: $\int_0^\infty \frac{x^2}{e^x - 1} dx = 2.404$

$n =$ ____________

Kitchen oven: Number of photons = density · volume

$N = n V$

9.3

(a) Use the results of Problem 9.2 to find the total number density, n, of blackbody photons of all wavelengths. Also show that the average energy per photon, u/n, is

$$\frac{u}{n} = \frac{\pi^4 kT}{15(2.404)} = 2.70kT. \qquad (9.60)$$

(b) Find the average energy per blackbody photon at the center of the Sun, where $T = 1.58 \times 10^7$ K, and in the solar photosphere, where $T = 5770$ K. Express your answers in units of electron volts (eV).

We found total number density of blackbody photons

$n = 2.404 \dfrac{8\pi k^3 T^3}{h^3 c^3}$ and total energy density $u = \dfrac{4\sigma T^4}{c} = \dfrac{4T^4}{c} \dfrac{2\pi^5 k^4}{15 c^2 h^3}$

$\dfrac{(9.5)}{258}$ (Prob 3.12)

Average energy per photon $= \dfrac{u}{n} =$

This depends on the temperature T.

(b) For the center of Sun, $\dfrac{u}{n} =$

Solar photosphere : $\dfrac{u}{n} =$

9.4 Derive Eq. (9.9) for the blackbody radiation pressure,

$$P_{\text{rad}} = \frac{4\pi}{3c}\int_0^\infty B_\lambda(T)\,d\lambda = \frac{4\sigma T^4}{3c} = \frac{1}{3}aT^4 = \frac{1}{3}u.$$

(9.9), $\frac{(9.8)}{261}$ $P_{rad} = \frac{4\pi}{3c}\int_0^\infty B_\lambda \, d\lambda =$

Let $x = \frac{hc}{\lambda kT}$: $P_{rad} =$

Hint: $\int_0^\infty \frac{x^3}{e^x - 1}\,dx = \frac{\pi^4}{15}$

9.6 Using the root-mean-square speed, v_{rms}, estimate the mean free path of the nitrogen molecules in your classroom at room temperature (300 K). What is the average time between collisions? Take the radius of a nitrogen molecule to be 1 Å, and the density of air to be 1.2×10^{-3} g cm^{-3}. A nitrogen molecule contains 28 nucleons (protons and neutrons).

$\frac{(8.3)}{229}$ $v_{rms} = \sqrt{\frac{3kT}{28\, m_p}}$

$\frac{(9.10)}{264}$ mfp $= l = \frac{1}{n\sigma} = \frac{1}{n\pi(2r)^2}$

density of N_2 at room temp? pressure $= \frac{\rho}{n} = 3kT$

So number density $n =$

Finally, $l =$

Time between collisions time $= \frac{\text{distance}}{\text{speed}} =$

Ap-27

Prob 9.7

Calculate how far you could see through Earth's atmosphere if it had the opacity of the solar photosphere. Use the value for the Sun's opacity from Example 9.2 and 1.2×10^{-3} g cm^{-3} for the density of Earth's atmosphere.

$\kappa_{5000} = 0.264 \frac{cm^2}{g}$

$\rho = 1.2 \times 10^{-3} \frac{g}{cm^3}$

mean free path $l = \frac{1}{\rho \kappa_{5000}} =$

We could see in to $\tau_\lambda = \frac{2}{3}$ or $\frac{2}{3} l =$ ______

9.11 According to a "standard model" of the Sun, the central density is 162 g cm^{-3} and the Rosseland mean opacity at the center is 1.16 cm^2 g^{-1}.

(a) Calculate the mean free path of a photon at the center of the Sun.

(b) If this mean free path remained constant for the photon's journey to the surface, calculate the average time it would take for the photon to escape from the Sun.

(a) Average photon mfp $l = \frac{1}{\rho \bar{\kappa}} =$

(b) $N =$ Number of random-walk steps from (9.23) 277 $d = l\sqrt{N}$

If $d = R_\odot =$ ______ then $N =$ ______

Time for a photon to ~~go~~ random-walk out of the sun $= \frac{\text{distance}}{\text{speed}} = \frac{Nl}{c} =$

Ap-78

9.12 If the temperature of a star's atmosphere is *increasing* outward, what type of spectral lines would you expect to find in the star's spectrum at those wavelengths where the opacity is greatest?

226
312 Derive Eq. (9.55) for the uncertainty in the wavelength of a spectral line due to Heisenberg's uncertainty principle.

Heisenberg Uncertainty principle $\left(\frac{5.19}{146}\right)$ $\Delta E \Delta t \approx \hbar$

$E = \frac{hc}{\lambda} = E_{initial} - E_{final} \rightarrow \lambda =$ ______

$\frac{\partial \lambda}{\partial E_{initial}} =$ ______

$\frac{\partial \lambda}{\partial E_{final}} =$ ______

total uncertainty in λ is $\Delta\lambda$ = sum =

Ap-20