Thus 10 May 07
week 6

QM 66 - Finish Ch 12.6 *12.6.1, 12.6.2
Ch 13.2,3,4 H Atom *13.1.1, 13.1.3, 13.3.3

#1 **Exercise 12.6.1.*** A particle is described by the wave function

$$\psi_E(r, \theta, \phi) = Ae^{-r/a_0} \qquad (a_0 = \text{const})$$

(i) What is the angular momentum content of the state?

(ii) Assuming $\psi_E$ is an eigenstate in a potential that vanishes as $r \to \infty$, find $E$. (Match leading terms in Schrödinger's equation.) $E = -\hbar^2/2\mu a_0^2$

(iii) Having found $E$, consider finite $r$ and find $V(r)$. $V = \hbar^2/\mu a_0 r$

(a) This wave function does not depend on $\theta$ or $\varphi$. The only $Y_l^m$ the doesn't is $Y_0^0 = \sqrt{\frac{1}{4\pi}}$, so $l=0$ and $m=0$.
$L^2|\psi_E\rangle = l(l+1)|\psi_E\rangle = 0|\psi_E\rangle$ ; $L=0$

(b) This radial wave function must satisfy the radial Sch. eqn for some spherically symmetric potential $V(r)$:

(12.6.3) 349 $$\left\{\frac{-\hbar^2}{2\mu}\left[\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} - \frac{l(l+1)}{r^2}\right] + V(r)\right\}R_{El} = ER_{El}$$

We know $l=0$: $$\frac{-\hbar^2}{2\mu}\left[\frac{1}{r^2}\underbrace{\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}} + V(r)\right]\psi_E = E\psi_E$$

Do the $\frac{\partial}{\partial r}$ parts: $\frac{\partial}{\partial r}\psi_E =$

$$\frac{\partial}{\partial r}r^2\frac{\partial\psi_E}{\partial r} =$$

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}r^2\frac{\partial\psi_E}{\partial r}\right] =$$

(12.6½...) So Sch. Eqn. becomes (sub in & simplify)

Let's look at the limits for infinite and finite r:

lim ($r \to \infty$):

Sch. eqn. also has to work for finite r, and that will tell us V(r). Use the E we just found:

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Shankar

#12.6.2 - Derive (12.6.5) from (12.6.3)

(12.6.3) p.349 $$\left\{ \frac{-\hbar^2}{2\mu} \left[ \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} - \frac{\ell(\ell+1)}{r^2} \right] + V(r) \right\} R_{E\ell} = E R_{E\ell}$$

Substitute for the radial wavefunction $R_{E\ell} = \frac{U_{E\ell}}{r}$, or $R = \frac{U}{r}$

$$\frac{\partial R}{\partial r} = \frac{\partial}{\partial r}\left(\frac{U}{r}\right) =$$

$$r^2 \frac{\partial R}{\partial r} =$$

$$\frac{\partial}{\partial r} r^2 \frac{\partial R}{\partial r} =$$

$$ER = E\frac{U}{r} = \left\{ \frac{-\hbar^2}{2\mu} \left[ \qquad \right] + V(r) \right\} \frac{U}{r}$$

Multiply both sides by $-2\mu r/\hbar^2$ to get

$$\left\{ \frac{d^2}{dr^2} + \frac{2\mu}{\hbar^2} \left[ E - V(r) - \frac{\ell(\ell+1)\hbar^2}{2\mu r^2} \right] \right\} U_{E\ell} = 0 \quad (12.6.5)$$