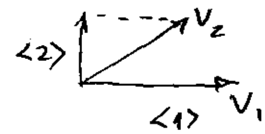
MATRIX IDENTITIES

$$[\Omega,\Lambda] = -[\Lambda,\Omega]$$
$$[\Omega,\Lambda\Theta] = \Lambda[\Omega,\Theta] + [\Omega,\Lambda]\Theta$$
$$[\Omega\Lambda,\Theta] = \Omega[\Lambda,\Theta] + [\Omega,\Theta]\Lambda$$
$$[\Omega,\Lambda+\Theta] = [\Omega,\Lambda] + [\Omega,\Theta]$$
$$[\Omega+\Lambda,\Theta] = [\Omega,\Theta] + [\Lambda,\Theta]$$

GRAM-SCHMIDT TH: constructing orthonormal $|1\rangle, |2\rangle, |3\rangle$ from vectors $|V1\rangle, |V2\rangle, |V3\rangle$

$$|1\rangle = |V1\rangle$$
$$|2\rangle = |V_2\rangle - \frac{|1\rangle\langle 1|V_2\rangle}{\langle 1|1\rangle}$$
$$|3\rangle = |V_3\rangle - \frac{|1\rangle\langle 1|V_3\rangle}{\langle 1|1\rangle} - \frac{|2\rangle\langle 2|V_3\rangle}{\langle 2|2\rangle}$$

etc.



MATRIX PROPERTIES

Hermitian $\Omega = \Omega^\dagger$

Commute $[\Omega,\Theta] = \Omega\Theta - \Theta\Omega = 0 = -[\Omega,\Theta] = [\Theta,\Omega]$

Unitary $UU^\dagger = I$

FINDING E-VALUES & E-VECTORS of MATRIX Ω:

e-values ω satisfy $0 = \det(\Omega - \omega I)$

e-vectors $|\omega = i\rangle$ satisfy $(\Omega - \omega I)|\omega = i\rangle = 0$

REVIEW - EARLY CHAPTERS

inner product satisfies $\langle V_i | V_i \rangle \geq 0$

$$\langle V_i | V_j \rangle = \langle V_j | V_i \rangle^*$$

linear (2nd) $\langle V_i | \alpha V_j + \beta V_k \rangle = \alpha \langle V_i | V_j \rangle + \beta \langle V_i | V_k \rangle$

antilinear (1st) $\langle \alpha V_i + \beta V_j | V_k \rangle = \alpha^* \langle V_i | V_k \rangle + \beta^* \langle V_j | V_k \rangle$

orthonormal: $\langle e_i | e_j \rangle = \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$

$$\langle V | V' \rangle = \sum_{i=1}^{3} \sum_{j=1}^{3} v_i^* v_j' \delta_{ij} = \sum_{i=1}^{3} v_i^* v_j' = \int_{(V(x))} v_i^* v_j \, dx$$

$\langle j | i' \rangle = \langle j | \Omega | i \rangle = \Omega_{ji}$ $\quad | i' \rangle = \Omega | i \rangle$ $\quad \Omega_{ji} \leftrightarrow \begin{array}{cc} & i \\ j & \left(\Omega \right) \end{array}$

$\Omega^\dagger_{ji} = \Omega^*_{ij}$ dagger = transpose conjugate

unitary $UU^\dagger = I \to U^\dagger U = I$

basis of operator is its eigenfxns, or it eigenvalues diagonalized by the matrix of eigenvalues.

commuting Hermitian operators have common diagonalizing basis

p.50 $\ddot{x}_1 = -\frac{2k}{m} x_1 + \frac{k}{m} x_2$ $\quad \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\quad \Omega_{11} = \Omega_{22} = \frac{-2k}{m}$

$\ddot{x}_2 = \frac{k}{m} x_1 - \frac{2k}{m} x_2$ $\quad \Omega_{12} = \Omega_{21} = \frac{k}{m}$

$$| \ddot{x}(t) \rangle = \Omega | x(t) \rangle$$

modes $|1\rangle \leftrightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ m_1 displaced, m_2 still $\quad |2\rangle \leftrightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix} \leftrightarrow$ m_1 still, m_2 displaced

arbitrary state: $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x_2$

$$| x \rangle = |1\rangle x_1 + |2\rangle x_2$$

In $|1\rangle, |2\rangle$ basis, $\Omega \longleftrightarrow \begin{bmatrix} -\frac{2k}{m} & \frac{k}{m} \\ \frac{k}{m} & -\frac{2k}{m} \end{bmatrix}$ but components x_1 & x_2 obey coupled eqns. So Diagonalize → uncouple