PS - spring week 3, Thus 19 Apr 07 - QM Ch 6

Derive Ehrenfest's theorem

From Schrödinger eqn, we know that

$$H\psi = i\hbar \frac{\partial \psi}{\partial t} .$$ Solve for

$|\dot{\psi}\rangle = \frac{\partial \psi}{\partial t} =$

$\langle \dot{\psi} | =$

Time evolution of expectation value for Ω is

$$\frac{d}{dt}\langle \Omega \rangle = \frac{d}{dt}\langle \psi | \Omega | \psi \rangle = \langle \dot{\psi} | \Omega | \psi \rangle + \langle \psi | \Omega | \dot{\psi} \rangle + \langle \psi | \dot{\Omega} | \psi \rangle$$

write in terms of H: assume $\frac{d\Omega}{dt} = 0$

$\langle \dot{\psi} | \Omega | \psi \rangle =$

$\langle \psi | \Omega | \dot{\psi} \rangle =$

$\frac{d}{dt}\langle \Omega \rangle =$

$$= \left(-\frac{i}{\hbar}\right)\langle \psi | [\Omega, H] | \psi \rangle = \left(-\frac{i}{\hbar}\right)\langle [\Omega, H] \rangle$$

Classical Mechanics - Hamiltonian & Poisson Brackets

Lagrangian $\mathcal{L} = T - V$ = Kinetic energy - Potential energy

(2.5.8) Hamiltonian $\mathcal{H} = \sum_{i=1}^{n} p_i \dot{q}_i - \mathcal{L} \underset{\text{usually}}{=} T + V$

where q_i = coordinate and

(2.5.1) $p_i = \dfrac{\partial \mathcal{L}}{\partial \dot{q}_i}$ = momentum corresponding to q_i

Equations of motion: Hamilton's canonical equations

(2.5.12) $$\frac{\partial \mathcal{H}}{\partial p_i} = \dot{q}_i = \frac{dq_i}{dt} \qquad -\frac{\partial \mathcal{H}}{\partial q_i} = \dot{p}_i = \frac{dp_i}{dt}$$

(See Exercise 2.5.1 for SHO)

Notice that if q_i does not appear in $\mathcal{H}$ (e.g. $V \neq V(q)$), then p_i is CONSERVED.

Time dependence of some function $\omega(p,q)$ of the state variables p & q

(2.7.2) $$\frac{d\omega(q,p)}{dt} = \{\omega, \mathcal{H}\} \equiv \sum_i \left(\frac{\partial \omega}{\partial q_i} \frac{\partial \mathcal{H}}{\partial p_i} - \frac{\partial \omega}{\partial p_i} \frac{\partial \mathcal{H}}{\partial q_i} \right)$$

Poisson Brackets

Consequences of Ehrenfest's Theorem - Ch 6

(6.2) $\frac{d}{dt}\langle \Omega \rangle = -\frac{i}{\hbar}\langle [\Omega, H] \rangle$

(6.5) $\langle P \rangle = \langle \dot{X} \rangle m$ | classical: $p = m\frac{dx}{dt}$

$\langle \dot{X} \rangle = \langle \frac{\partial H}{\partial P} \rangle$ | classical: $\dot{x} = \frac{\partial \mathcal{H}}{\partial p}$

(6.8) $\langle \dot{P} \rangle = \langle -\frac{\partial H}{\partial X} \rangle$ | classical: $\dot{p} = -\frac{\partial \mathcal{H}}{\partial x}$

QM $\longrightarrow$ classical limit

for macroscopic systems, or whenever the mean is a good approximation.

($\mathcal{H} = f(x^n, p^m)$ where $n \leq 2$, $m \leq 2$)

Derive (6.5):

$$\frac{d}{dt}\langle X \rangle = -\frac{i}{\hbar}\langle [X, H] \rangle, \quad H = \frac{P^2}{2m} + V(X)$$

$[X, H] =$

$= [X, P^2/2m]$

$[X, P^2] = P[X, P] + [X, P]P =$

$= 2i\hbar P$