

Examine time dependence of Particle in a Box ψ

$$\begin{aligned} \text{Ex: } |\psi(0)\rangle &= \sqrt{\frac{1}{2}}|1\rangle + \sqrt{\frac{1}{2}}|2\rangle \\ &= \sqrt{\frac{1}{2}}\psi_1(x) + \sqrt{\frac{1}{2}}\psi_2(x) = \sqrt{\frac{1}{2}}\left[\cos\left(\frac{\pi x}{L}\right) + \sin\left(\frac{2\pi x}{L}\right)\right] \end{aligned}$$

$$\psi(x,t) = \sqrt{\frac{1}{2}} \left[\cos\left(\frac{\pi x}{L}\right) e^{-iEt} + \sin\left(\frac{2\pi x}{L}\right) e^{-i4Et} \right] \quad \text{where } E = \frac{E_1}{4}, \quad E_n = n^2 E_1$$

QUALITATIVELY:

$$\psi(x,0) = \underbrace{\cos\left(\frac{\pi x}{L}\right)}_{x=0} + \sin\left(\frac{2\pi x}{L}\right) = \text{graph} \rightarrow |\psi(x,0)|^2 = \text{graph}$$

$$\psi\left(x, \frac{\pi}{E}\right) = \underbrace{\cos\left(\frac{\pi x}{L}\right)}_{\text{graph}} + \underbrace{\sin\left(\frac{2\pi x}{L}\right)}_{\text{graph}} = \text{graph} \rightarrow \left|\psi\left(x, \frac{\pi}{E}\right)\right|^2 = \text{graph}$$

MATHEMATICALLY:

$$|\psi(x,t)|^2 = \frac{1}{2} \left[\cos\left(\frac{\pi x}{L}\right) e^{iEt} + \sin\left(\frac{2\pi x}{L}\right) e^{i4Et} \right] \left[\cos\left(\frac{\pi x}{L}\right) e^{-iEt} + \sin\left(\frac{2\pi x}{L}\right) e^{-i4Et} \right]$$

$$= \frac{1}{2} \left[\cos^2\left(\frac{\pi x}{L}\right) + \sin^2\left(\frac{2\pi x}{L}\right) + \sin\left(\frac{2\pi x}{L}\right) \cos\left(\frac{\pi x}{L}\right) e^{i3Et} + \cos\left(\frac{\pi x}{L}\right) \sin\left(\frac{2\pi x}{L}\right) e^{-i3Et} \right]$$

$$= \frac{1}{2} \left[\cos^2\left(\frac{\pi x}{L}\right) + \sin^2\left(\frac{2\pi x}{L}\right) + \sin\left(\frac{2\pi x}{L}\right) \cos\left(\frac{\pi x}{L}\right) \left\{ e^{i3Et} + e^{-i3Et} \right\} \right]$$

use $2\cos\theta = e^{i\theta} + e^{-i\theta}$ and simplify

What is the oscillation frequency of the overall $|\psi(x,t)|^2$?