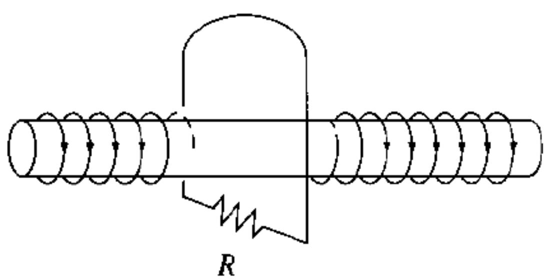
EM 1b Physical Systems HW Thus.5.April 2007
Griffiths Ch.7 # 12, 17, 22, 25, 26, 28

Problem 7.12 A long solenoid, of radius a, is driven by an alternating current, so that the field inside is sinusoidal: $\mathbf{B}(t) = B_0 \cos(\omega t)\,\hat{\mathbf{z}}$. A circular loop of wire, of radius $a/2$ and resistance R, is placed inside the solenoid, and coaxial with it. Find the current induced in the loop, as a function of time.

Problem 7.17 A long solenoid of radius a, carrying n turns per unit length, is looped by a wire with resistance R, as shown in Fig. 7.27.

(a) If the current in the solenoid is increasing at a constant rate ($dI/dt = k$), what current flows in the loop, and which way (left or right) does it pass through the resistor?

(b) If the current I in the solenoid is constant but the solenoid is pulled out of the loop, turned around, and reinserted, what total charge passes through the resistor?



Problem 7.22 Find the self-inductance per unit length of a long solenoid, of radius R, carrying n turns per unit length.

Problem 7.25 A capacitor C is charged up to a potential V and connected to an inductor L, as shown schematically in Fig. 7.38. At time $t = 0$ the switch S is closed. Find the current in the circuit as a function of time. How does your answer change if a resistor R is included in series with C and L?

Problem 7.26 Find the energy stored in a section of length l of a long solenoid (radius R, current I, n turns per unit length), (a) using Eq. 7.29 (you found L in Prob. 7.22); (b) using Eq. 7.30 (we worked out $\mathbf{A}$ in Ex. 5.12); (c) using Eq. 7.34; (d) using Eq. 7.33 (take as your volume the cylindrical tube from radius $a < R$ out to radius $b > R$).

old
See 7.41

Problem 7.28 A long cable carries current in one direction uniformly distributed over its (circular) cross section. The current returns along the surface (there is a very thin insulating sheath separating the currents). Find the self-inductance per unit length.

Giancoli:

MPLE 30–3 Solenoid inductance. (a) Determine a formula for the nductance L of a tightly wrapped solenoid (a long coil) containing N turns re in its length l and whose cross-sectional area is A. (b) Calculate the value if $N = 100$, $l = 5.0\,\text{cm}$, $A = 0.30\,\text{cm}^2$ and the solenoid is air filled. (c) Cal- L if the solenoid has an iron core with $\mu = 4000\,\mu_0$.

UTION (a) To determine the inductance L, it is usually simplest to start Eq. 30–4, so we need to first determine the flux. According to Eq. 28–4, the etic field inside a solenoid (ignoring end effects) is constant: $B = \mu_0 n I$ $n = N/l$. The flux is $\Phi_B = BA = \mu_0 NIA/l$, so

$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 A}{l}.$$

nce $\mu_0 = 4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}$

$$L = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(100)^2(3.0 \times 10^{-5}\,\text{m}^2)}{(5.0 \times 10^{-2}\,\text{m})} = 7.5\,\mu\text{H}.$$

ere we replace μ_0 by $\mu = 4000\,\mu_0$ so L will be 4000 times larger: .030 H = 30 mH.

Griffiths:

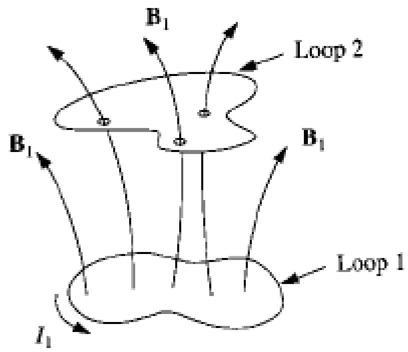


Figure 7.29

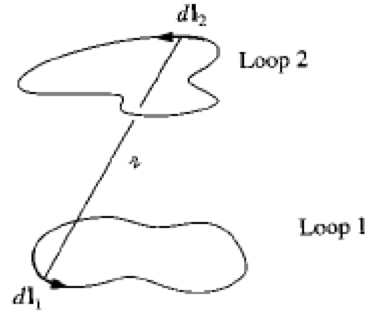


Figure 7.30

Thus

$$\Phi_2 = M_{21} I_1, \tag{7.21}$$

where M_{21} is the constant of proportionality; it is known as the **mutual inductance** of the two loops.

There is a cute formula for the mutual inductance, which you can derive by expressing the flux in terms of the vector potential and invoking Stokes' theorem:

$$\Phi_2 = \int \mathbf{B}_1 \cdot d\mathbf{a}_2 = \int (\nabla \times \mathbf{A}_1) \cdot d\mathbf{a}_2 = \oint \mathbf{A}_1 \cdot d\mathbf{l}_2.$$

Now, according to Eq. 5.63, p. 235

$$\mathbf{A}_1 = \frac{\mu_0 I_1}{4\pi} \oint \frac{d\mathbf{l}_1}{\mathscr{r}},$$

and hence

$$\Phi_2 = \frac{\mu_0 I_1}{4\pi} \oint \left(\oint \frac{d\mathbf{l}_1}{\mathscr{r}} \right) \cdot d\mathbf{l}_2.$$

Evidently

$$M_{21} = \frac{\mu_0}{4\pi} \oint \oint \frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{\mathscr{r}}. \tag{7.22}$$

This is the **Neumann formula**; it involves a double line integral—one integration around loop 1, the other around loop 2 (Fig. 7.30). It's not very useful for practical calculations, but it does reveal two important things about mutual inductance:

1. M_{21} is a purely geometrical quantity, having to do with the sizes, shapes, and relative positions of the two loops.

2. The integral in Eq. 7.22 is unchanged if we switch the roles of loops 1 and 2; it follows that

$$M_{21} = M_{12}. \tag{7.23}$$