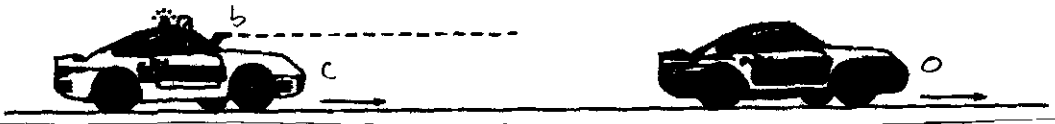
Physical Systems – spring 2007

EM HW 8a Tues. 22 May 2007 Griffiths **Ch.12** # 4, 13, 15, 20, 26, 34 (optional: #11)

EM HW 8b Thus. 24 May 2007 Griffiths **Ch.12** # 41, 43, 44

Problem 12.4 As the outlaws escape in their getaway car, which goes $\frac{3}{4}c$, the police officer fires a bullet from the pursuit car, which only goes $\frac{1}{2}c$ (Fig. 12.3). The muzzle velocity of the bullet (relative to the gun) is $\frac{1}{3}c$. Does the bullet reach its target (a) according to Galileo, (b) according to Einstein?

V_{cg} $= V_{og}$ V_{bc}



Mistic

(a) Galilean V_{bullet} (wrt ground) $= \frac{1}{2}c + \frac{1}{3}c = \frac{5}{6}c = \frac{10}{12}c = V_{bg}$

$V_{outlaws} = \frac{3}{4}c = \frac{9}{12}c$ ∴ outlaws get hit

(b) Relativistic V_{bullet} wrt ground $= \dfrac{V_{cg} + V_{bc}}{1 + V_{cg}V_{bc}/c^2} = \dfrac{\frac{3}{6} + \frac{2}{6}}{1 + \frac{1}{3}\cdot\frac{1}{2}/c^2} = \dfrac{\frac{5}{6}c}{\frac{6}{6} + \frac{1}{6}} = \frac{5}{7}c = \frac{20}{28}c$

$V_{outlaws} = \frac{3}{4}c = \frac{21}{28}c$ ∴ outlaws escape

Problem 12.15 You probably did Prob. 12.4 from the point of view of an observer on the *ground*. Now do it from the point of view of the police car, the outlaws, and the bullet. That is, fill in the gaps in the following table:

speed of → / relative to ↓	Ground	Police	Outlaws	Bullet	Do they escape?
Ground	0	$V_{cg} = \frac{1}{2}c$	$V_{og} = \frac{3}{4}c$	$V_{bg} = 5/7$	YES
Police	$V_{gc} = -\frac{1}{2}c$	$V_{cc} = 0$	$V_{oc} = \frac{2}{5}c$	$V_{bc} = \frac{1}{3}c$	Y
Outlaws	$V_{go} = -\frac{3}{4}c$	$V_{co} = -\frac{2}{5}c$	0	$V_{bo} = -\frac{1}{13}c$	Y
Bullet	$-5/7\,c$	$-\frac{1}{3}c$	$\frac{1}{13}c$	0	Y

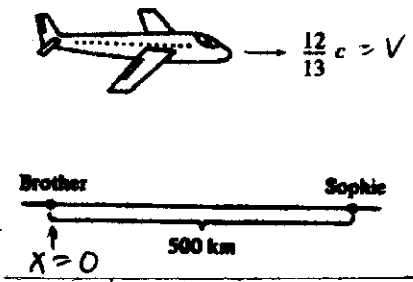
$V_{outlaws\ wrt\ cops} = V_{oc} = \dfrac{V_{og} - V_{cg}}{1 - V_{og}V_{cg}/c^2} = \dfrac{\frac{3}{4}c - \frac{2}{4}c}{1 - \frac{3}{4}\cdot\frac{1}{2}} = \dfrac{\frac{1}{4}c}{1 - \frac{3}{8}} = \frac{8}{5}\cdot\frac{1}{4}c = \frac{2}{5}c$

$V_{cops\ wrt\ outlaws} = V_{co} = -V_{oc} = -\frac{2}{5}c$

$V_{bullet\ wrt\ outlaws} = V_{bo} = \dfrac{V_{bg} - V_{og}}{1 - V_{bg}V_{og}/c^2} = \dfrac{\frac{5}{7}c - \frac{3}{4}c}{1 - \frac{5}{7}\cdot\frac{3}{4}} = \dfrac{\left(\frac{20}{28} - \frac{21}{28}\right)c}{\frac{28}{28} - \frac{15}{28}} = \frac{-1}{13}c$

$V_{cops\ wrt\ bullet} = V_{cb} = -V_{bc} = -\frac{1}{3}c$, $V_{outlaws\ wrt\ bullet} = V_{ob} = -V_{bo} = +\frac{1}{13}c$

Problem 12.13 Sophie Zabar, clairvoyante, cried out in pain at precisely the instant her twin brother, 500 km away, hit his thumb with a hammer. A skeptical scientist observed both events (brother's accident, Sophie's cry) from an airplane traveling at $\frac{12}{13}c$ to the right (see Fig. 12.19). Which event occurred first, according to the scientist? How *much* earlier was it, in seconds?



Let brother's accident occur at $x=0$ in both system S (Sophie's system - at rest) and system S' (scientist's system - moving at V).

In S, let accident occur at $t=0$ (to Sophie).

In S', it happens at $\bar{t} = \gamma(t - \frac{V}{c^2}x)$

$\bar{t} = \gamma(0 - \frac{V}{c^2})x = -\gamma\frac{Vx}{c^2} < 0$: Scientist observes brother's

(a) accident BEFORE Sophie's cry. How long before?

(b) $1-\frac{V^2}{c^2} = 1-(\frac{12}{13})^2 = \frac{169-144}{169} = \frac{25}{169}$ so $\gamma = \sqrt{\frac{1}{1-\frac{V^2}{c^2}}} = \sqrt{\frac{169}{25}} = \frac{13}{5}$

$\frac{x}{c} = 5\times10^5 m / 3\times10^8 \frac{m}{s} = \frac{5}{3}\times10^{-3} s$

So $\bar{t} = -\gamma\frac{V}{c}\frac{x}{c} = -\frac{13}{5}\frac{12}{13}\frac{5}{3}\times10^{-13} s = -\frac{12}{3}\times10^{-13} s =$ 4 ms earlier

Problem 12.20

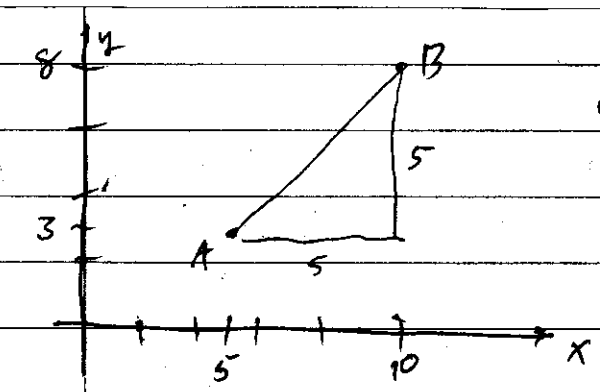
(a) Event A happens at point ($x_A = 5$, $y_A = 3$, $z_A = 0$) and at time t_A given by $ct_A = 15$; event B occurs at (10, 8, 0) and $ct_B = 5$, both in system S.

(i) What is the invariant interval between A and B?

(ii) Is there an inertial system in which they occur *simultaneously*? If so, find its velocity (magnitude and direction) relative to S.

(iii) Is there an inertial system in which they occur at the same point? If so, find its velocity relative to S

(b) Repeat part (a) for $A = (2, 0, 0)$, $ct = 1$; and $B = (5, 0, 0)$, $ct = 3$.



(a)(i) $I = -c^2\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$

$= -(5-15)^2 + (10-5)^2 - (8-3)^2 + 0^2$

$= -100 + 25 + 25 = -50$

(ii) NO - if so, $\Delta\bar{t} = 0$ and $I > 0$ but it's not

(iii) YES - $\bar{S}$ travels from B to A in time $\frac{10}{c}$ $\therefore V = \frac{-5\hat{x} - 5\hat{y}}{10/c} = -\frac{c}{2}\hat{x} - \frac{c}{2}\hat{y}$

(b)(i) $I = (-3-1)^2 + (5-2)^2 + 0 + 0) = -4 + 9 = 5$

(ii) YES - Lorentz trans: $\Delta(ct) = \gamma[\Delta(ct) - \beta(\Delta x)]$

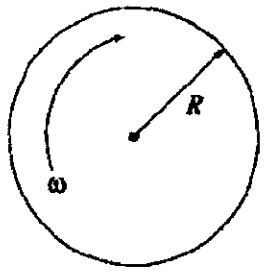
We want $\Delta \bar{t} = 0$, so $\Delta(ct) = \beta(\Delta x)$

$$\frac{v}{c} = \frac{\Delta(ct)}{\Delta x} = \frac{(3-1)}{(5-2)} = \frac{2}{3} \Rightarrow v = \frac{2}{3}c$$

in x dir

(ii) No, otherwise $\Delta x = \Delta y = \Delta z = 0$ and $I < 0$, but this not

Problem 12.11 A record turntable of radius R rotates at angular velocity ω (Fig. 12.15). The circumference is presumably Lorentz-contracted, but the radius (being perpendicular to the velocity) is *not*. What's the ratio of the circumference to the diameter, in terms of ω and R? According to the rules of ordinary geometry, that has to be π. What's going on here? [This is known as **Ehrenfest's paradox**; for discussion and references see H. Arzelies, *Relativistic Kinematics*, Chap. IX (Elmsford, NY: Pergamon Press, 1966) and T. A. Weber, *Am. J. Phys.* **65**, 486 (1997).]



$S = R\theta$

$v = R\omega$

How would the circumference contract? $C = 2\pi R$

$C' = \frac{2\pi R}{\gamma}$ where $\gamma = \frac{1}{\sqrt{1-\left(\frac{R\omega}{c}\right)^2}}$

Then $\frac{C'}{2R} = \pi\sqrt{1-\left(\frac{R\omega}{c}\right)^2}$

That does not make sense. If a rigid object could have its circumference contract, that would also DECREASE the RADIUS proportionally - BUT SR tells us the radius cannot decrease.

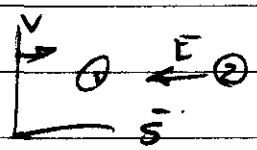
p.509 **12.26** $\eta^\mu \eta_\mu = -(\eta^0)^2 + \eta^2 = \left(\frac{1}{1-\frac{u^2}{c^2}}\right)(-c^2+u^2)$

$$= \frac{-c^2\left(1-\frac{u^2}{c^2}\right)}{\left(1-\frac{u^2}{c^2}\right)} = -c^2$$

p.575 **Problem 12.34** In the past, most experiments in particle physics involved stationary targets: one particle (usually a proton or an electron) was accelerated to a high energy E, and collided with a target particle at rest (Fig. 12.29a). Far higher *relative* energies are obtainable (with the same accelerator) if you accelerate *both* particles to energy E, and fire them at each other (Fig. 12.29b). *Classically*, the energy $\bar{E}$ of one particle, relative to the other, is just 4E (why?)—not much of a gain (only a factor of 4). But *relativistically* the gain can be *enormous*. Assuming the two particles have the same mass, m, show that

$$\bar{E} = \frac{2E^2}{mc^2} - mc^2. \tag{12.59}$$

Classically $E = \frac{1}{2}mv^2$, In COLLIDING BEAMS, $V_{rel} = 2V \rightarrow E_{tot} = 4E_1$, $E_1 \sim V_1^2$



In $\bar{S}$ let ① be at rest.
Relative to S, its speed is V, in S.

$\bar{p}^0 = \gamma(p^0 - \beta p^1) \rightarrow \frac{\bar{E}}{c} = \gamma\left(\frac{E}{c} - \beta p\right)$ where $p = p_2$ in S

$E = \gamma Mc^2 \rightarrow \gamma = \frac{E}{Mc^2}$, $p = -\gamma M v = -\gamma M \beta c$

$\bar{E} = \gamma\left(\frac{E}{c} + \beta\gamma M\beta c\right)c = \gamma(E + \gamma Mc^2\beta^2)$

$\gamma = \frac{1}{1-\beta^2} \rightarrow 1-\beta^2 = \frac{1}{\gamma^2} \rightarrow \beta^2 = 1 - \frac{1}{\gamma^2} = \frac{\gamma^2-1}{\gamma^2}$

$\bar{E} = \frac{E}{Mc^2}E + \left[\left(\frac{E}{Mc^2}\right)^2 - 1\right]Mc^2 = \frac{E^2}{Mc^2} + \frac{E^2}{Mc^2} - Mc^2 = \frac{2E^2}{Mc^2} - Mc^2$

For $E = 30\,GeV$, $Mc^2 = 1\,GeV \rightarrow \bar{E} = \frac{2 \cdot 900}{1} - 1 = 1800 - 1 = 1799\,GeV = \underline{60E}$

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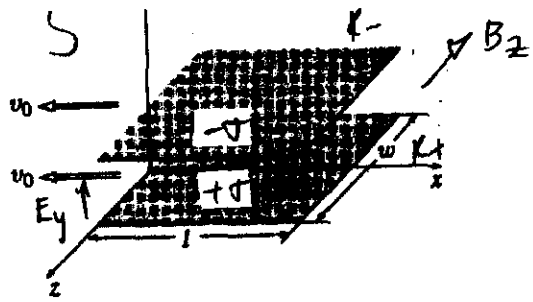
Problem 12.41 Why can't the electric field in Fig. 12.35b have a z component? After all, the *magnetic* field does. p.526



(10.29) 4/27 The general $E(r,t) = \frac{1}{4\pi\epsilon_0}\int\left[\frac{\rho(r',t_r)}{\mathscr{r}^2}\hat{\mathscr{r}} + \frac{\dot{\rho}(r',t_r)}{c\mathscr{r}}\hat{\mathscr{r}} - \frac{\dot{J}(r',t_r)}{c^2\mathscr{r}}\right]d\tau'$

These uniform planes of charge moving in the x-z plane have $\dot{J}=0$ and $\dot{\rho}=0$, and ρ (that is, σ), is time-independent, so retardation does nothing. So the field is the same as for a plane at rest, except for the amplification of σ due to length contraction.

Another way to think of it - recall from Ch. 1 Prob 1.10 p.12

VECTORS $E \to -E$ under coordinate inversion, e.g. reflection

PSEUDOVECTORS (e.g. $B = \nabla\times A$) $\to$ unchanged under reflection
or $L = r\times p$

What if we reflect the configuration above in the x-y plane? The configuration would be UNCHANGED, which says E_z should not change. But since E is a vector, E_z should REVERSE under reflection. Since it can't do both, $E_z=0$ must be.

(If you reflect the configuration $\frac{+\sigma}{-\sigma}$ the E_y reverses - that's fine)

Problem 12.43

(a) Check that Gauss's law, $\int \mathbf{E} \cdot d\mathbf{a} = (1/\epsilon_0) Q_{enc}$, is obeyed by the field of a point charge in uniform motion, by integrating over a sphere of radius R centered on the charge.

(12.92) $E = \frac{1}{4\pi\epsilon_0} \frac{q(1-\frac{v^2}{c^2})}{(1-\frac{v^2}{c^2}\sin^2\theta)^{3/2}} \frac{\hat{R}}{R^2}$ $\qquad \int E \cdot da = \int E \cdot R^2 \sin\theta \, d\theta \, d\phi$

$= 2\pi \int_0^\pi E \sin\theta \, d\theta$

$\int E \cdot da = \frac{2\pi}{4\pi\epsilon_0} \frac{R^2}{R^2} q(1-\frac{v^2}{c^2}) \int_0^\pi \frac{\sin\theta \, d\theta}{(1-\frac{v^2}{c^2}\sin^2\theta)^{3/2}}$

let $u = \cos\theta$

$du = -\sin\theta \, d\theta$

$\sin^2\theta = 1-\cos^2\theta = 1-u^2$

$1-\frac{v^2}{c^2}\sin^2\theta = 1-\frac{v^2}{c^2}(1-u^2) = 1-\frac{v^2}{c^2}+\frac{v^2}{c^2}u^2$

$= \frac{v^2}{c^2}(\frac{c^2}{v^2}-1+u^2)$

$\int E \cdot da = \frac{q(1-\frac{v^2}{c^2})}{2\epsilon_0} \int_{-1}^{1} \frac{-du}{(\frac{v^2}{c^2})^{3/2}(\frac{c^2}{v^2}-1+u^2)^{3/2}} = \frac{-q(1-\frac{v^2}{c^2})}{2\epsilon_0}\left(\frac{c}{v}\right)^3 \int_{-1}^{1} \frac{du}{(a^2+u^2)^{3/2}}$

where $a^2 = \frac{c^2}{v^2}-1$. $\int_{-1}^{1} \frac{du}{(a^2+u^2)^{3/2}} =$

12.43

(b) Find the Poynting vector for a point charge in uniform motion. (Say the charge is going in the z direction at speed v, and calculate $\mathbf{S}$ at the instant q passes the origin.)

(12.110) Example 12.14 p. 532

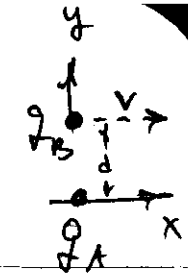
$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{qv\left(1-\frac{v^2}{c^2}\right)\sin\theta}{\left[1-\frac{v^2}{c^2}\sin^2\theta\right]^{3/2} R^2}\,\hat{\phi}$$

(12.92) Example 12.13 p. 527-8

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{q\left(1-\frac{v_0^2}{c^2}\right)}{\left[1-\frac{v_0^2}{c^2}\sin^2\theta\right]^{3/2} R^2}\,\hat{R}$$

$$\mathbf{S} = \frac{\mathbf{E}\times\mathbf{B}}{\mu_0}, \quad \hat{R}\times\hat{\phi} = -\hat{\theta}$$

Problem 12.44



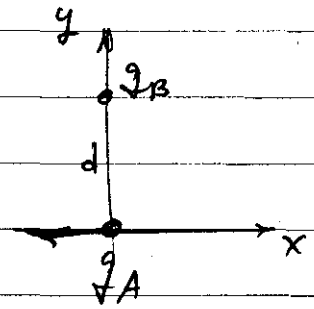
(a) Charge q_A is at rest at the origin in system $\mathcal{S}$; charge q_B flies by at speed v on a trajectory parallel to the x axis, but at $y = d$. What is the electromagnetic force on q_B as it crosses the y axis?

(b) Now study the same problem from system $\bar{\mathcal{S}}$, which moves to the right with speed v. What is the force on q_B when q_A passes the $\bar{y}$ axis? [Do it two ways: (i) by using your answer to (a) and transforming the force; (ii) by computing the fields in $\bar{\mathcal{S}}$ and using the Lorentz force law.]

(a) Field of q_A at $y=d$ is $\mathbf{E}_A = \frac{1}{4\pi\epsilon_0}\frac{q_A}{d^2}\hat{y}$, $\mathbf{B}=0$

Force on q_B is $\mathbf{F}_B = q_B \mathbf{E}_A =$

(b)



Now that q_A is moving, its field at $y=d$ is given by (12.92) 527-8

$$\mathbf{E}_A = \frac{q_A}{4\pi\epsilon_0}\frac{\left(1-\frac{v^2}{c^2}\right)}{\left[1-\frac{v^2}{c^2}\sin^2\theta\right]^{3/2} d^2}\hat{y}$$

$\theta = 90°$

Simplify

$\mathbf{E}_A =$

What is $\mathbf{B}_A$? Is it zero? If not, what will be the magnetic force on q_B?

$\mathbf{F}_B =$

Problem 12.46

(a) Show that $(\mathbf{E} \cdot \mathbf{B})$ is relativistically invariant.

(b) Show that $(E^2 - c^2 B^2)$ is relativistically invariant.

(c) Suppose that in one inertial system $\mathbf{B} = 0$ but $\mathbf{E} \neq 0$ (at some point P). Is it possible to find another system in which the *electric* field is zero at P?

For motion along x with speed v,

(12.108) 531

$\bar{E}_x = E_x \qquad \bar{E}_y = \gamma(E_y - vB_z) \qquad \bar{E}_z = \gamma(E_z + vB_y)$

$\bar{B}_x = B_x \qquad \bar{B}_y = \gamma(B_y + \frac{v}{c^2}E_z) \qquad \bar{B}_z = \gamma(B_z - \frac{v}{c^2}E_y)$

(a) $(\mathbb{E} \cdot \mathbb{B}) = E_x B_x + E_y B_y + E_z B_z$. (In the frame at rest)

In the moving frame,

$\mathbb{E} \cdot \mathbb{B} =$

Problem 12.47 An electromagnetic plane wave of (angular) frequency ω is traveling in the x direction through the vacuum. It is polarized in the y direction, and the amplitude of the electric field is E_0.

(a) Write down the electric and magnetic fields, $\mathbf{E}(x, y, z, t)$ and $\mathbf{B}(x, y, z, t)$. [Be sure to define any auxiliary quantities you introduce, in terms of ω, E_0, and the constants of nature.]

(b) This same wave is observed from an inertial system $\bar{S}$ moving in the x direction with speed v relative to the original system S. Find the electric and magnetic fields in $\bar{S}$, and express them in terms of the $\bar{S}$ coordinates: $\bar{\mathbf{E}}(\bar{x}, \bar{y}, \bar{z}, \bar{t})$ and $\bar{\mathbf{B}}(\bar{x}, \bar{y}, \bar{z}, \bar{t})$. [Again, be sure to define any auxiliary quantities you introduce.]

(c) What is the frequency $\bar{\omega}$ of the wave in $\bar{S}$? Interpret this result. What is the wavelength $\bar{\lambda}$ of the wave in $\bar{S}$? From $\bar{\omega}$ and $\bar{\lambda}$, determine the speed of the waves in $\bar{S}$. Is it what you expected?

For motion in the z direction

(9.48) p 370 $\quad \mathbb{E}(z,t) = E_0 \cos(kz - \omega t)\,\hat{x} \qquad \mathbb{B}(z,t) = \frac{E_0}{c}\cos(kz - \omega t)\,\hat{y} \qquad k = \frac{\omega}{c}$

For a wave traveling in the x direction:

$\mathbb{E}(x,t) = \qquad\qquad \mathbb{B}(x,t) =$

(b) Transform the fields with (12.108) as usual; let $\alpha = \gamma(1 - \frac{v}{c}) = \sqrt{\frac{1-v/c}{1+v/c}}$

$\bar{E}_x = \qquad \bar{E}_y =$

$\bar{B}_x = \qquad \bar{B}_z =$

Inverse Lorentz transformation (12.19) $x = \gamma(\bar{x} + v\bar{t})$, $t = \gamma(\bar{t} + \frac{v}{c^2}\bar{x})$

$kx - \omega t =$

$\bar{\mathbb{E}}(\bar{x}, \bar{y}, \bar{z}, \bar{t}) =$

$\bar{\mathbb{B}}(\bar{x}, \bar{y}, \bar{z}, \bar{t}) =$