Physical Systems – spring 2007

QM HW 2a Tues.10 April 2007 - Shankar 4.2.1 (p.135, ed.1), 4.2.2 (p.146), 4.2.3

Exercise 4.2.1 (*Very Important*). Consider the following operators on a Hilbert space $\mathcal{V}^3(C)$:

$$L_x = \frac{1}{2^{1/2}}\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}, \qquad L_y = \frac{1}{2^{1/2}}\begin{bmatrix}0 & -i & 0\\ i & 0 & -i\\ 0 & i & 0\end{bmatrix}, \qquad L_z = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{bmatrix}$$

(a) What are the possible values one can obtain if L_z is measured?

(b) Take the state in which $L_z = 1$. In this state what are $\langle L_x\rangle$, $\langle L_x^2\rangle$, and ΔL_x?

(c) Find the normalized eigenstates and the eigenvalues of L_x in L_z basis.

(d) If the particle is in the state with $L_z = -1$, and L_x is measured, what are the possible outcomes and their probabilities?

(e) Consider the state

$$|\psi\rangle = \begin{bmatrix}1/2\\ 1/2\\ 1/2^{1/2}\end{bmatrix}$$

in the L_z basis. If L_z^2 is measured in this state and a result $+1$ is obtained, what is the state after the measurement? How probable was this result? If L_z is measured, what are the outcomes and respective probabilities?

(f) A particle is in a state for which the probabilities are $P(L_z = 1) = 1/4$, $P(L_z = 0) = 1/2$, and $P(L_z = -1) = 1/4$. Convince yourself that the most general, normalized state with this property is

$$|\psi\rangle = \frac{e^{i\delta_1}}{2}|L_z = 1\rangle + \frac{e^{i\delta_2}}{2^{1/2}}|L_z = 0\rangle + \frac{e^{i\delta_3}}{2}|L_z = -1\rangle$$

It was stated earlier on that if $|\psi\rangle$ is a normalized state then the state $e^{i\theta}|\psi\rangle$ is a physically equivalent normalized state. Does this mean that the factors $e^{i\delta_i}$ multiplying the L_z eigenstates are irrelevant? [Calculate for example $P(L_x = 0)$.]

*Exercise 4.2.2.** Show that for a real wave function $\psi(x)$, the expectation value of momentum $\langle P\rangle = 0$. (Hint: Show that the probabilities for the momenta $\pm p$ are equal). Generalize this result to the case $\psi = c\psi_r$, where ψ_r is real and c an arbitrary (real or complex) constant. (Recall that $|\psi\rangle$ and $\alpha|\psi\rangle$ are physically equivalent.)

*Exercise 4.2.3.** Show that if $\psi(x)$ has mean momentum $\langle P\rangle$, $e^{ip_0x/\hbar}\psi(x)$ has mean momentum $\langle P\rangle + p_0$.

QM HW 2b Thus.12 April 2007 - **Shankar 5.1.2** (p.161, ed.1), **5.2.1** (p.163), **5.3.2** (p.176. Why? See 4.2.2)

*Exercise 5.1.2.** By solving the eigenvalue equation (5.1.3) in the X basis, regain Eq. (5.1.8), i.e., show that the general solution of energy E is

$$\psi_E(x) = \beta \frac{\exp[i(2mE)^{1/2}x/\hbar]}{(2\pi\hbar)^{1/2}} + \gamma \frac{\exp[-i(2mE)^{1/2}x/\hbar]}{(2\pi\hbar)^{1/2}}$$

[The factor $(2\pi\hbar)^{-1/2}$ is arbitrary and may be absorbed into β and γ.] Though $\psi_E(x)$ will satisfy the equation even if $E < 0$, are these functions in the Hilbert space?

$$H\,|\,E\rangle = \frac{P^2}{2m}\,|\,E\rangle = E\,|\,E\rangle \quad (5.1.3)$$

eigenvalue eqn

$$|\,E\rangle = \beta\,|\,p = (2mE)^{1/2}\rangle + \gamma\,|\,p = -(2mE)^{1/2}\rangle \qquad (5.1.8)$$

*Exercise 5.2.1.** A particle is in the ground state of a box of length L. Suddently the box expands (symmetrically) to twice its size, leaving the wave function undisturbed. Show that the probability of finding the particle in the ground state of the new box is $(8/3\pi)^2$.

Exercise 5.3.2. Convince yourself that if $\psi = c\tilde{\psi}$, where c is constant (real or complex) and $\tilde{\psi}$ is real, the corresponding j vanishes.