Modern Physics

FOURTH EDITION

Paul A. Tipler
Formerly of Oakland University

Ralph A. Llewellyn
University of Central Florida

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The Greek Alphabet

Alpha	A	α	Iota	I	ι	Rho	P	ρ
Beta	B	β	Kappa	K	κ	Sigma	Σ	σ
Gamma	Γ	γ	Lambda	Λ	λ	Tau	T	τ
Delta	Δ	δ	Mu	M	μ	Upsilon	Y	υ
Epsilon	E	ϵ	Nu	N	ν	Phi	Φ	ϕ
Zeta	Z	ζ	Xi	Ξ	ξ	Chi	X	χ
Eta	H	η	Omicron	O	o	Psi	Ψ	ψ
Theta	Θ	θ	Pi	Π	π	Omega	Ω	ω

Prefixes for Powers of 10

Multiple	Prefix	Abbreviation
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Mathematical Symbols

$=$	is equal to	Δx	change in x
$\neq$	is not equal to	$\lvert x \rvert$	absolute value of x
$\approx$	is approximately equal to	$n!$	$n(n-1)(n-2)\cdots 1$
$\sim$	is of the order of	Σ	sum
$\propto$	is proportional to	lim	limit
$>$	is greater than	$\Delta t \rightarrow 0$	Δt approaches zero
$\geq$	is greater than or equal to	$\frac{dx}{dt}$	derivative of x with respect to t
$\gg$	is much greater than		
$<$	is less than	$\frac{\partial x}{\partial t}$	partial derivative of x with respect to t
$\leq$	is less than or equal to		
$\ll$	is much less than	$\int$	integral

Abbreviations for Units

A	ampere	keV	kilo-electron volts
Å	angstrom (10^{-10} m)	L	liter
atm	atmosphere	m	meter
Btu	British thermal unit	MeV	mega-electron volts
Bq	becquerel	min	minute
C	coulomb	mm	millimeter
°C	degree Celsius	ms	millisecond
cal	calorie	N	newton
Ci	curie	nm	nanometer (10^{-9} m)
cm	centimeter	rev	revolution
eV	electron volt	R	roentgen
°F	degree Fahrenheit	Sv	seivert
fm	femtometer, fermi (10^{-15} m)	s	second
G	gauss	T	tesla
Gy	gray	u	unified mass unit
g	gram	V	volt
H	henry	W	watt
h	hour	Wb	weber
Hz	hertz	y	year
J	joule	μm	micrometer (10^{-6} m)
K	kelvin	μs	microsecond
kg	kilogram	μC	microcoulomb
km	kilometer	Ω	ohm

Some Useful Combinations

$hc = 1.9864 \times 10^{-25}\ \text{J}\cdot\text{m} = 1239.8\ \text{eV}\cdot\text{nm}$

$\hbar c = 3.1615 \times 10^{-26}\ \text{J}\cdot\text{m} = 197.33\ \text{eV}\cdot\text{nm}$

Bohr radius $a_0 = \dfrac{4\pi\varepsilon_0\hbar^2}{m_e e^2} = 5.2918 \times 10^{-11}\ \text{m}$

$ke^2 = 1.440\ \text{eV}\cdot\text{nm}$

Fine structure constant $\alpha = \dfrac{e^2}{4\pi\varepsilon_0\hbar c} = 0.0072974 \approx \dfrac{1}{137}$

$kT = 2.5249 \times 10^{-2}\ \text{eV} \approx \frac{1}{40}\ \text{eV}$ at $T = 293$ K

Some Physical Constants

(See Appendix D for a complete list of fundamental constants.)

Avogadro's number	N_A	6.022142×10^{23} particles/mol
Boltzmann's constant	k	1.380650×10^{-23} J/K
Bohr magneton	$m_B = e\hbar$	$9.2740090 \times 10^{-24}$ J/T
Coulomb constant	$k = 1/4\pi\varepsilon_0$	$8.987551788 \times 10^{9}$ N·m²/C²
Compton wavelength	$\lambda_c = h/m_e c$	$2.42631022 \times 10^{-12}$ m
Fundamental charge	e	1.602176×10^{-19} C
Gas constant	$R = N_A k$	8.31447 J/mol·K = 1.98722 cal/mol·K
		$= 8.20578 \times 10^{-2}$ L·atm/mol·K
Gravitational constant	G	6.6731×10^{-11} N·m²/kg
Mass, of electron	m_e	9.109382×10^{-31} kg
		= 510.9989 keV/c^2
of proton	m_p	1.672622×10^{-27} kg
		= 938.2722 MeV/c^2
of neutron	m_n	1.674927×10^{-27} kg
		= 939.5653 MeV/c^2
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A²
Planck's constant	h	6.626069×10^{-34} J·s
		$= 4.135667 \times 10^{-15}$ eV·s
	$\hbar$	1.054572×10^{-34} J·s
		$= 6.582119 \times 10^{-16}$ eV·s
Speed of light	c	2.99792458×10^{8} m/s
Unified mass unit	u	1.660539×10^{-27} kg
		= 931.49401 MeV/c^2

Some Conversion Factors

1 yr = 3.156×10^{7} s	1 T = 10^4 G
1 light-year = 9.461×10^{15} m	1 Ci = 3.7×10^{10} Bq
1 cal = 4.186 J	1 barn = 10^{-28} m²
1 MeV/c = 5.344×10^{-22} kg·m/s	1 u = 1.66054×10^{-27} kg
1 eV = 1.6022×10^{-19} J	1 parsec = 3.26 light-years
1 kW·h = 3.6 MJ	1 rad = 57.30°

Some Particle Masses and Rest Energies

	kg	MeV/c^2	u
Electron	9.1094×10^{-31}	0.51100	5.4858×10^{-4}
Muon	1.8835×10^{-28}	105.66	0.11343
Proton	1.6726×10^{-27}	938.27	1.00728
Neutron	1.6749×10^{-27}	939.57	1.00866
Deuteron	3.3436×10^{-27}	1875.61	2.01355
α particle	6.6447×10^{-27}	3727.38	4.00151
W	1.43×10^{-25}	80×10^{3}	85.9
Z°	1.63×10^{-25}	91.2×10^{3}	97.9

Periodic Table

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 **H** 1.00797																	2 **He** 4.003
3 **Li** 6.941	4 **Be** 9.012											5 **B** 10.81	6 **C** 12.011	7 **N** 14.007	8 **O** 15.9994	9 **F** 19.00	10 **Ne** 20.179
11 **Na** 22.990	12 **Mg** 24.31											13 **Al** 26.98	14 **Si** 28.09	15 **P** 30.974	16 **S** 32.064	17 **Cl** 35.453	18 **Ar** 39.948
19 **K** 39.102	20 **Ca** 40.08	21 **Sc** 44.96	22 **Ti** 47.88	23 **V** 50.94	24 **Cr** 52.00	25 **Mn** 54.94	26 **Fe** 55.85	27 **Co** 58.93	28 **Ni** 58.69	29 **Cu** 63.55	30 **Zn** 65.38	31 **Ga** 69.72	32 **Ge** 72.59	33 **As** 74.92	34 **Se** 78.96	35 **Br** 79.90	36 **Kr** 83.80
37 **Rb** 85.47	38 **Sr** 87.62	39 **Y** 88.906	40 **Zr** 91.22	41 **Nb** 92.91	42 **Mo** 95.94	43 **Tc** (98)	44 **Ru** 101.1	45 **Rh** 102.905	46 **Pd** 106.4	47 **Ag** 107.870	48 **Cd** 112.41	49 **In** 114.82	50 **Sn** 118.69	51 **Sb** 121.75	52 **Te** 127.60	53 **I** 126.90	54 **Xe** 131.29
55 **Cs** 132.905	56 **Ba** 137.33	57-71 **Rare Earths**	72 **Hf** 178.49	73 **Ta** 180.95	74 **W** 183.85	75 **Re** 186.2	76 **Os** 190.2	77 **Ir** 192.2	78 **Pt** 195.09	79 **Au** 196.97	80 **Hg** 200.59	81 **Tl** 204.37	82 **Pb** 207.19	83 **Bi** 208.98	84 **Po** (210)	85 **At** (210)	86 **Rn** (222)
87 **Fr** (223)	88 **Ra** (226)	89-103 **Actinides**	104 **Rf** (261)	105 **Du** (260)	106 **Sg** (263)	107 **Bh** (262)	108 **Hs** (265)	109 **Mt** (266)	110 — (273)	111 — (?)	112 — (277)		114 — (?)				

Rare Earths (Lanthanides)	57 **La** 138.91	58 **Ce** 140.12	59 **Pr** 140.91	60 **Nd** 144.24	61 **Pm** (147)	62 **Sm** 150.36	63 **Eu** 152.0	64 **Gd** 157.25	65 **Tb** 158.92	66 **Dy** 162.50	67 **Ho** 164.93	68 **Er** 167.26	69 **Tm** 168.93	70 **Yb** 173.04	71 **Lu** 174.97
Actinides	89 **Ac** 227.03	90 **Th** 232.04	91 **Pa** 231.04	92 **U** 238.03	93 **Np** 237.05	94 **Pu** (244)	95 **Am** (243)	96 **Cm** (247)	97 **Bk** (247)	98 **Cf** (251)	99 **Es** (252)	100 **Fm** (257)	101 **Md** (258)	102 **No** (259)	103 **Lr** (260)

The 1–18 group designation has been recommended by the International Union of Pure and Applied Chemistry (IUPAC).

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Part 1

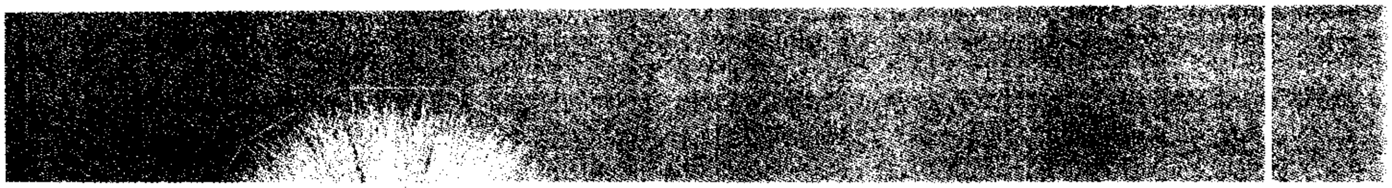
Relativity and Quantum Mechanics: The Foundations of Modern Physics

The earliest recorded systematic efforts to assemble knowledge about motion as a key to understanding natural phenomena were those of the ancient Greeks. Set forth in sophisticated form by Aristotle, theirs was a natural philosophy (i.e., physics) of explanations deduced from assumptions rather than experimentation. For example, it was a fundamental assumption that every substance had a "natural place" in the universe. Motion then resulted when a substance was trying to reach its natural place. Time was given a similar absolute meaning, as moving from some instant in the past (the creation of the universe) toward some end goal in the future, its natural place. The remarkable agreement between the deductions of Aristotelian physics and motions observed throughout the physical universe, together with a nearly total absence of accurate instruments to make contradictory measurements, enabled acceptance of the Greek view for nearly 2000 years. Toward the end of that time a few scholars had begun to deliberately test some of the predictions of theory, but it was the Italian scientist Galileo Galilei who, with his brilliant experiments on motion, established for all time the absolute necessity of experimentation in physics and, coincidentally, initiated the disintegration of Aristotelian physics. Within 100 years Isaac Newton had generalized the results of Galileo's experiments into his three spectacularly successful laws of motion, and the natural philosophy of Aristotle was gone.

With the burgeoning of experimentation, the following 200 years saw a multitude of major discoveries and a concomitant development of physical theories to explain them. Most of the latter, then as now, failed to survive increasingly sophisticated experimental tests, but by the dawn of the twentieth century Newton's theoretical explanation of the motion of mechanical systems had been joined by equally impressive laws of electromagnetism and thermodynamics as expressed by Maxwell, Carnot, and others. The remarkable success of these laws led many scientists to believe that description of the physical universe was complete. Indeed, A. A. Michelson, speaking to scientists near the end of the nineteenth century, said, "The grand underlying principles have been firmly established . . . the future truths of physics are to be looked for in the sixth place of decimals."

Such optimism (or pessimism, depending on your point of view) turned out to be premature, as there were already vexing cracks in the foundation of what we now refer to as classical physics. Two of these were described by Lord Kelvin, in his famous Baltimore Lectures in 1900, as the "two clouds" on the horizon of twentieth-century physics: the failure of theory to account for the radiation spectrum emitted by a blackbody and the inexplicable results of the Michelson-Morley experiment. Indeed, the breakdown of classical physics occurred in many different areas: the Michelson-Morley null result contradicted Newtonian relativity; the blackbody radiation spectrum contradicted predictions of thermodynamics; the photoelectric effect and the spectra of atoms could not be explained by electromagnetic theory; and the exciting discoveries of x rays and radioactivity seemed to be outside the framework of classical physics entirely. The development of the theories of quantum mechanics and relativity in the early twentieth century not only dispelled Kelvin's "dark clouds," they provided answers to all of the puzzles listed here and many more. The applications of these theories to such microscopic systems as atoms, molecules, nuclei, and fundamental particles and to macroscopic systems of solids, liquids, gases, and plasmas have given us a deep understanding of the intricate workings of nature and have revolutionized our way of life.

In Part 1 we discuss the foundations of the physics of the modern era, relativity theory and quantum mechanics. Chapter 1 examines the apparent conflict between Einstein's principle of relativity and the observed constancy of the speed of light and shows how accepting the validity of both ideas led to the special theory of relativity. Chapter 2 concerns the relations connecting mass, energy, and momentum in special relativity and concludes with a brief discussion of general relativity and some experimental tests of its predictions. In Chapters 3, 4, and 5 the development of quantum theory is traced from the earliest evidences of quantization to de Broglie's hypothesis of electron waves. An elementary discussion of the Schrödinger equation is provided in Chapter 6, illustrated with applications to one-dimensional systems. Chapter 7 extends the application of quantum mechanics to many-particle systems and introduces the important new concepts of electron spin and the exclusion principle. Concluding the development, Chapter 8 discusses the wave mechanics of systems of large numbers of identical particles, underscoring the importance of the symmetry of wave functions. Beginning with Chapter 3, the chapters in Part 1 should be studied in sequence because each of Chapters 4 through 8 depends on the discussions, developments, and examples of the previous chapters.



Chapter 1 Relativity I

1-1 The Experimental Basis of Relativity
1-2 Einstein's Postulates
1-3 The Lorentz Transformation
1-4 Time Dilation and Length Contraction
1-5 The Doppler Effect
1-6 The Twin Paradox and Other Surprises

The relativistic character of the laws of physics began to be apparent very early in the evolution of classical physics. Even before the time of Galileo and Newton, Nicolaus Copernicus[1] had shown that the complicated and imprecise Aristotelian method of computing the motions of the planets, based on the assumption that Earth was located at the center of the universe, could be made much more simple and accurate if it were assumed that the planets move about the sun instead of Earth. Although Copernicus did not publish his work until very late in life, it became widely known through correspondence with his contemporaries and helped pave the way for acceptance a century later of the heliocentric theory of planetary motion. While the Copernican theory led to a dramatic revolution in human thought, the aspect that concerns us here is that it did not consider the location of Earth to be special or favored in any way. Thus, the laws of physics discovered on Earth could apply equally well with any point taken as the center—i.e., the same equations would be obtained regardless of the origin of coordinates. This invariance of the equations that express the laws of physics is what we mean by the term *relativity.*

We will begin this chapter by investigating briefly the relativity of Newton's laws and then concentrate on the theory of relativity as developed by Albert Einstein (1879–1955). The theory of relativity consists of two rather different theories, the special theory and the general theory. The special theory, developed by Einstein and others in 1905, concerns the comparison of measurements made in different frames of reference moving with constant velocity relative to each other. Contrary to popular opinion, the special theory is not difficult to understand. Its consequences, which can be derived with a minimum of mathematics, are applicable in a wide variety of situations in physics and engineering. On the other hand, the general theory, also developed by Einstein (around 1916), is concerned with accelerated reference frames and gravity. Although a thorough understanding of the general theory requires more sophisticated mathematics (e.g., tensor analysis), a number of its basic ideas and important predictions can be discussed at the level of this book. The general theory is of great importance in cosmology and in understanding events that occur in the vicinity of very large masses (e.g., stars), but is rarely encountered in other areas of physics and engineering. We will devote this chapter entirely to the special theory (often referred to as *special relativity*) and discuss the general theory in the final section of Chapter 2, following the sections concerned with special relativistic mechanics.

1-1 The Experimental Basis of Relativity

Classical Relativity

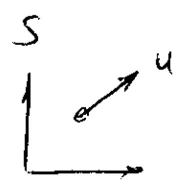
Galileo was the first to recognize the concept of acceleration when, in his studies of falling objects, he showed that the rate at which the velocity changed was always constant, indicating that the motion of the falling body was intimately related to its *changing* velocity. It was this observation, among others, that Newton generalized into his second law of motion:

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt} = m\mathbf{a} \qquad \textbf{1-1}$$

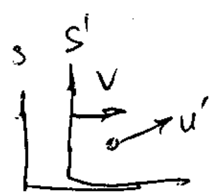
where $d\mathbf{v}/dt = \mathbf{a}$ is the acceleration of the mass m and $\mathbf{F}$ is the net force acting on it. (Recall that letters and symbols printed in boldface type are vectors.) Newton's first law of motion, the law of inertia, is also implied in Equation 1-1: the velocity of an object acted upon by no net force does not change; i.e., its acceleration is zero.

Frames of Reference An important question regarding the laws of motion, one that concerned Newton himself and one that you likely studied in first-year physics, is that of the reference frame in which they are valid. It turns out that they work correctly only in what is called an *inertial reference frame,* a reference frame in which the law of inertia holds.[2] Newton's laws of motion for mechanical systems are *not* valid in systems that accelerate relative to an inertial reference frame; i.e., an accelerated reference frame is not an inertial reference frame. Figures 1-1 and 1-2 illustrate inertial and noninertial reference frames.

Galilean Transformation Newton's laws brought with them an enormous advance in the relativity of the laws of physics. The laws are *invariant*, or unchanged, in reference systems that move at constant velocity with respect to an inertial frame. Thus, not only is there no special or favored position for measuring space and time, there is no special or favored velocity for inertial frames of reference. All such frames are equivalent. If an observer in an inertial frame S measures the velocity of an object to be $\mathbf{u}$ and an observer in a reference frame S' moving at constant velocity $\mathbf{v}$ in the $+x$ direction with respect to S measures the velocity of the object to be $\mathbf{u}'$, then $\mathbf{u}' = \mathbf{u} - \mathbf{v}$, or, in terms of the coordinate systems in Figure 1-3 (page 5),



$$u'_x = u_x - v \qquad u'_y = u_y \qquad u'_z = u_z \qquad \textbf{1-2}$$



If we recall that $u'_x = dx'/dt$, $u_x = dx/dt$, and so forth, then, integrating each of the Equations 1-2, the *velocity transformation* between S and S', yields Equations 1-3, the *Galilean transformation of coordinates:*

$$x' = x - vt \qquad y' = y \qquad z' = z \qquad \textbf{1-3}$$

assuming the origins of S and S' coincided at $t = 0$. Differentiating Equations 1-2 leads to

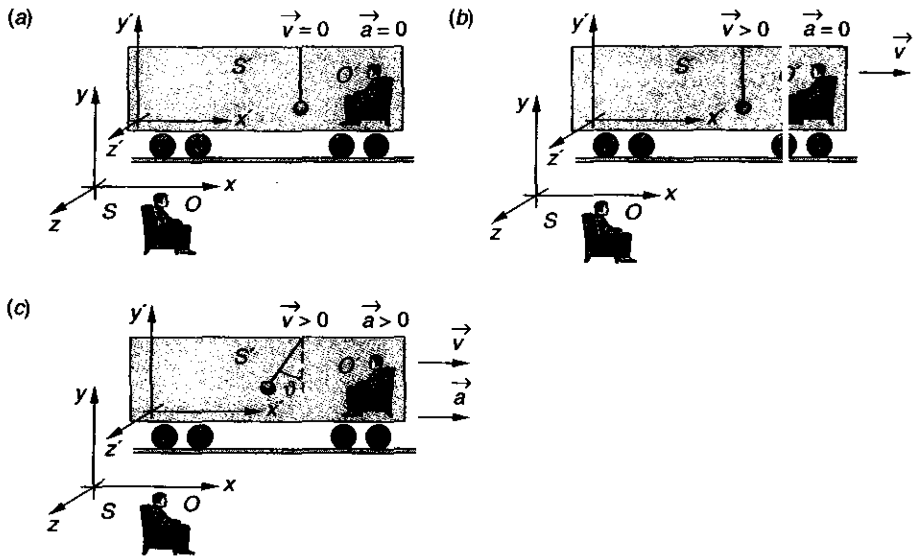


Fig. 1-1 A mass suspended by a cord from the roof of a railroad boxcar illustrates the relativity of Newton's second law $\mathbf{F} = m\mathbf{a}$. The only forces acting on the mass are its weight $m\mathbf{g}$ and the tension $\mathbf{T}$ in the cord. (*a*) The boxcar sits at rest in *S*. Since the velocity $\mathbf{v}$ and the acceleration $\mathbf{a}$ of the boxcar (i.e., the system S') are both zero, both observers see the mass hanging vertically at rest with $\mathbf{F} = \mathbf{F}' = 0$. (*b*) As S' moves in the $+x$ direction with $\mathbf{v}$ constant, both observers see the mass hanging vertically, but moving at $\mathbf{v}$ with respect to *O* in *S* and at rest with respect to the S' observer. Thus, $\mathbf{F} = \mathbf{F}' = 0$. (*c*) As S' moves in the $+x$ direction with $\mathbf{a} > 0$ with respect to *S*, the mass hangs at an angle $\theta > 0$ with respect to the vertical. However, it is still at rest (i.e., in equilibrium) with respect to the observer in S', who now "explains" the angle θ by adding a pseudoforce F_p in the $-x'$ direction to Newton's second law.

$$a'_x = \frac{du'_x}{dt} = \frac{du_x}{dt} = a_x \qquad a'_y = \frac{du'_y}{dt} = \frac{du_y}{dt} = a_y \qquad a'_z = \frac{du'_z}{dt} = \frac{du_z}{dt} = a_z \tag{1-4}$$

and the conclusion that $\mathbf{a}' = \mathbf{a}$. Thus, we see that $\mathbf{F} = m\mathbf{a} = m\mathbf{a}' = \mathbf{F}'$ in Figure 1-3 and Figure 1-1*b* and, indeed, in every situation where the relative velocity $\mathbf{v}$ of the reference

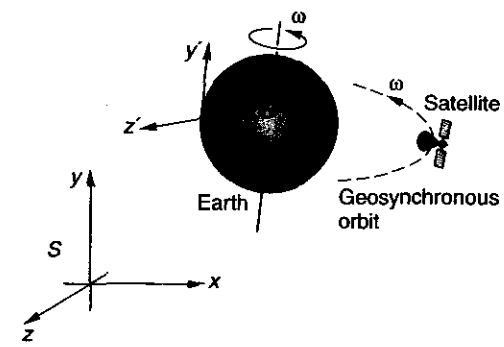


Fig. 1-2 A geosynchronous satellite has an orbital angular velocity equal to that of Earth and, therefore, is always located above a particular point on Earth; i.e., it is at rest with respect to the surface of Earth. An observer in *S* accounts for the radial, or centripetal, acceleration $\mathbf{a}$ of the satellite as the result of the net force $\mathbf{F}_G$. For an observer O' at rest on Earth (in S'), however, $\mathbf{a}' = 0$ and $\mathbf{F}'_G \neq m\mathbf{a}'$. To explain the acceleration being zero, observer O' must add a pseudoforce $\mathbf{F}_p = -\mathbf{F}_G$.

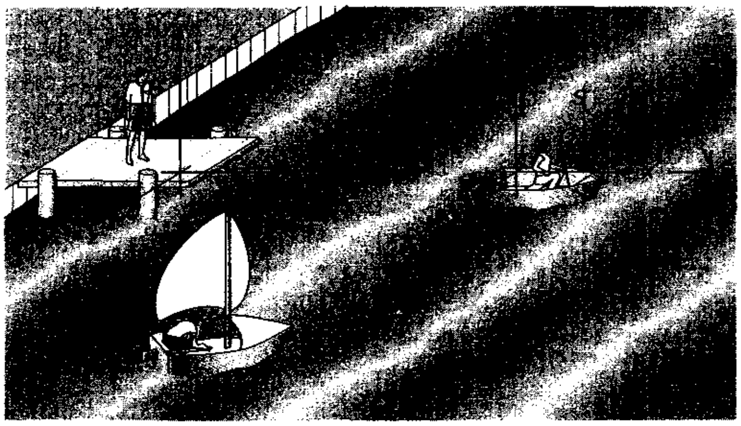


Fig. 1-3 The observer in S on the dock measures $\mathbf{u}$ for the sailboat's velocity. The observer in S' (in the motorboat) moving at constant velocity $\mathbf{v}$ with respect to S measures $\mathbf{u}'$ for the sailboat. The invariance of Newton's equations between these two systems means that $\mathbf{u}' = \mathbf{u} - \mathbf{v}$.

frames is constant. Constant relative velocity $\mathbf{v}$ of the frames means that $d\mathbf{v}/dt = 0$; hence the observers measure identical accelerations for moving objects and agree on the results when applying $\mathbf{F} = m\mathbf{a}$. Note that S' is thus also an inertial frame and neither frame is preferred or special in any way. This result can be generalized as follows:

> Any reference frame which moves at constant velocity with respect to an inertial frame is also an inertial frame. Newton's laws of mechanics are invariant in all reference systems connected by a Galilean transformation.

The second of the preceding statements is the *Newtonian principle of relativity*. Note the tacit assumption in the foregoing that the clocks of both observers keep the same time, i.e., $t' = t$.

EXAMPLE 1-1 Velocity of One Boat Relative to Another **What will a person in the motorboat in Figure 1-3 measure for the velocity of the sailboat? The motorboat is sailing due east at 3.0 m/s with respect to the dock. The person on the dock measures the velocity of the sailboat as 1.5 m/s toward the northeast. The coordinate system S is attached to the dock and S' is attached to the motorboat.**

Solution

1. The magnitude of the sailboat's velocity $\mathbf{u}'$ is given by:

$$u' = \sqrt{u_x'^2 + u_y'^2 + u_z'^2}$$

2. The components of $\mathbf{u}'$ are given by Equations 1-2 with $v = 3.0$ m/s, $u_x = 1.5 \cos 45°$, $u_y = 0$, and $u_z = -1.5 \sin 45°$:

$$u_x' = 1.5 \cos 45° - 3.0$$
$$u_y' = 0$$
$$u_z' = -1.5 \sin 45°$$

3. Substituting these into $\mathbf{u}'$ above yields:

$$u' = \sqrt{(3.76\ \text{m}^2/\text{s}^2 + 1.13\text{m}^2/\text{s}^2)}$$
$$= 2.2\ \text{m/s}$$

4. The direction of $\mathbf{u}'$ relative to north (the $-z$ axis) is given by:

$$\theta' = \tan^{-1}(u'_x/u'_z)$$

5. Substituting from above:

$$\theta' = \tan^{-1}(-1.94/-1.06)$$
$$= 61^\circ \text{ west of north}$$

***Remarks:** Note that the observers in S and S′ obtain different values for the speed and direction of the sailboat. It is the equations that are invariant between inertial systems, not necessarily the numbers calculated from them. Since neither reference frame is special or preferred, both results are correct!*

Speed of Light

In about 1860 James Clerk Maxwell discovered that the experimental laws of electricity and magnetism could be summarized in a consistent set of four concise mathematical statements, the Maxwell equations, one consequence of which was the prediction of the possibility of electromagnetic waves. It was recognized almost immediately, indeed by Maxwell himself, that the Maxwell equations did not obey the principle of Newtonian relativity, i.e., the equations were not invariant when transformed between inertial reference frames using the Galilean transformations. That this is the case can be seen by considering Figure 1-4, which shows an infinitely long wire with a uniform negative charge density λ per unit length and a point charge q located a distance y_1 above the wire. The wire and charge are at rest in the S frame. A second reference frame S' moves at constant speed v in the $+x$ direction with respect to S. An observer at rest in S' sees the wire and charge q moving in the $-x'$ direction at speed v. The observers in S and S' thus have *different forms* for the electromagnetic force acting on the point charge q near the wire, implying that Maxwell's equations are not invariant under a Galilean transformation.

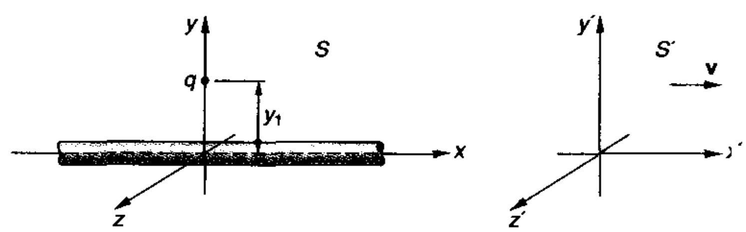


Fig. 1-4 The observers in S and S' see identical electric fields $2k\lambda/y_1$ at a distance $y_1 = y'_1$ from an infinitely long wire carrying uniform charge λ per unit length. Observers in both S and S' measure a force $2kq\lambda/y_1$ on q due to the line of charge; however, the S' observer measures an additional force $-\mu_0\lambda v^2 q/(2\pi y_1)$ due to the magnetic field at y'_1 arising from the motion of the wire in the $-x'$ direction. Thus, the electromagnetic force does not have the same form in different inertial systems, implying that Maxwell's equations are *not* invariant under a Galilean transformation.

A fair question at this point would be, Why does anyone care that Maxwell's electromagnetic laws are not invariant between inertial systems the way Newton's laws of mechanics are? Scientists of the time probably *wouldn't* have cared a great deal, except that Maxwell's equations predict the existence of electromagnetic waves whose speed would be a particular value $c = 1/(\mu_0\epsilon_0)^{1/2} = 3.00 \times 10^8$ m/s. The excellent agreement between this number and the measured value of the speed of light[3] and between the predicted polarization properties of electromagnetic waves and those observed for light provided strong confirmation of the assumption that light was an electromagnetic wave and, therefore, traveled at speed c.[4]

That being the case, it was postulated in the nineteenth century that electromagnetic waves, like all other waves, propagated in a suitable material medium. Called the *ether,* this medium filled the entire universe including the interior of matter. (The Greek philosopher Aristotle had first suggested that the universe was permeated with "ether" 2000 years earlier.) It had the inconsistent properties, among others, of being extremely rigid (in order to support the stress of the high electromagnetic wave speed) while offering no observable resistance to motion of the planets, which was fully accounted for by Newton's law of gravitation. The implication of this postulate is that a light wave, moving with velocity $\mathbf{c}$ with respect to the ether, would, according to the classical transformation (Equations 1-2), travel at velocity $\mathbf{c}' = \mathbf{c} + \mathbf{v}$ with respect to a frame of reference moving through the ether at $\mathbf{v}$. This would require that Maxwell's equations have a different form in the moving frame so as to predict the speed of light to be c', instead of $c = 1/(\mu_0\epsilon_0)^{1/2}$. That would in turn reserve for the ether the status of a favored or special frame for the laws of electromagnetic theory. It should then be possible to design an experiment that would detect the existence of the favored frame.

The problem with the ether postulate at the time it was made was not that it became a favored frame of reference for Maxwell's equations (Newton had postulated a similar status for the "fixed stars" for the laws of mechanics), but that, unlike the media through which other kinds of waves moved (e.g., water, air, solids), it offered no other evidence of its existence. Many experiments were performed to establish the existence of the ether, but nearly all of them suffered from the same serious limitation.

Let's use Fizeau's classic measurement of the speed of light to illustrate that limitation (see Figure 1-5). The time t for the light beam to make a round trip (wheel to mirror back to wheel) is $2L/c$; therefore, the speed of light would be

$$c = \frac{2L}{t}$$

However, the motion of Earth relative to the ether at some speed v (unknown) would affect the time measured in an "out and back" terrestrial measurement of the light's speed, such as Fizeau's. If Earth moves toward the right in Figure 1-5 at speed v, then in the outbound leg the speed of light relative to the laboratory is $c' = c - v$ and in the return leg $c' = c + v$. The round-trip time t is then

$$t = \frac{L}{c - v} + \frac{L}{c + v} = \frac{2Lc}{c^2 - v^2} = \frac{2L}{c}\frac{1}{1 - \dfrac{v^2}{c^2}} \tag{1-5}$$

$$= \frac{2L}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1} \approx \frac{2L}{c}\left(1 + \frac{v^2}{c^2} + \dots\right)$$

slightly slower

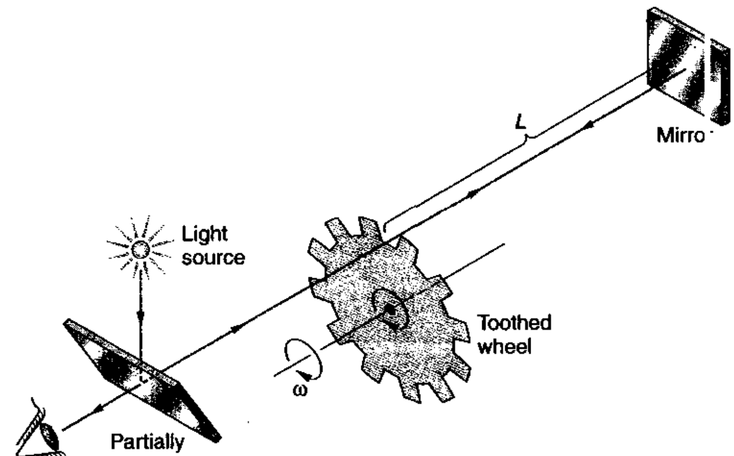


Fig. 1-5 Fizeau measured the speed of light in 1849 by aiming a beam of light at a distant mirror through the gap between two teeth in a wheel, in effect changing the light beam into pulses. A light pulse traveling at speed c would take $2L/c$ seconds to go from the wheel to the mirror and back to the wheel, If, during that time, rotation of the wheel moved a tooth into the light's path, the observer could not see the light. But if the angular velocity ω were such that the pulse arrived back at the wheel coincident with the arrival of the next gap, the observer saw the light.

where the term $(1 - v^2/c^2)^{-1}$ has been expanded using the binomial expansion in powers of the small quantity v^2/c^2 (see Appendix B4) and only the first two terms have been retained. Although the speed of Earth relative to the ether was unknown, one could reasonably expect that at some season of the year it should be at least equal to Earth's orbital speed around the sun, about 30 km/s. Thus, the maximum observable effect would only be of the order of $v^2/c^2 = (3 \times 10^4/3 \times 10^8)^2 = 10^{-8}$, or about 1 part in 10^8. The experimental accuracy of Fizeau's measurement was too poor by a factor of about 10^4 to detect this small an effect. A large number of experiments intended to detect the effect of Earth's motion on the propagation speed of light were proposed, but for all of them except one the accuracy possible with the apparatus available was, like Fizeau's, insufficient to detect the small effect. The one exception was the experiment of Michelson and Morley.[5]

EXAMPLE 1-2 Earth's Orbital Speed Determine Earth's average orbital speed with respect to an inertial frame of reference attached to the center of the sun. The mean value of Earth's orbit radius R is 1.496×10^8 km.

Solution

1. The average orbital speed is given in terms of the orbital circumference C and the time required to complete one orbit:

$$v = C/t$$

2. The circumference is given in terms of the orbit radius R. The mean value of R is a convenient unit of length used for distances within the solar system; it is called the *astronomical unit* (AU).

$$C = 2\pi R$$
$$= 2\pi(1.496 \times 10^8 \text{ km})$$
$$= 9.40 \times 10^8 \text{ km}$$

3. Earth travels a distance equal to C in $t = 1 \text{ y} = 3.16 \times 10^7$ s. The average speed is then given by:

$$v = \frac{9.40 \times 10^8 \text{ km}}{3.16 \times 10^7 \text{s}}$$
$$= 29.8 \text{ km/s}$$

QUESTIONS

1. What would the relative velocity of the inertial systems in Figure 1-4 need to be in order for the S' observer to measure no net electromagnetic force on the charge q?
2. Discuss why the very large value for the speed of the electromagnetic waves would imply that the ether be rigid, i.e., have a large bulk modulus. $v = \sqrt{\frac{B}{\rho}}$

The Michelson-Morley Experiment

All waves that were known to nineteenth-century scientists required a medium in order to propagate. Surface waves moving across the ocean obviously require the water. Similarly, waves move along a plucked guitar string, across the surface of a struck drumhead, through Earth after an earthquake, and, indeed, in all materials acted upon by suitable forces. The speed of the waves depends on the properties of the medium and is derived *relative to the medium*. For example, the speed of sound waves in air, i.e., their absolute motion relative to still air, can be measured. The Doppler effect for sound in air depends not only on the relative motion of the source and listener, but also on the motion of each relative to still air. Thus, it was natural for scientists of that time to expect the existence of some material like the ether to support the propagation of light and other electromagnetic waves *and* to expect that the absolute motion of Earth through the ether should be detectable, despite the fact that the ether had not been observed previously. $C_s = \sqrt{\frac{\gamma P}{\rho}}$

Michelson realized that, although the effect of Earth's motion on the results of any "out and back" speed of light measurement, such as shown generically in Figure 1-6, would be too small to measure directly, it should be possible to measure v^2/c^2 by

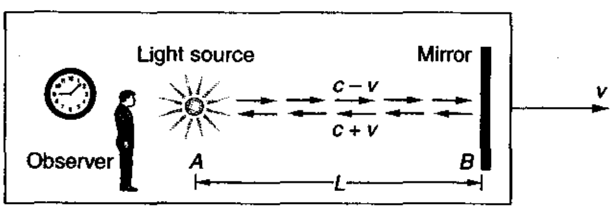


Fig. 1-6 Light source, mirror, and observer are moving with speed v relative to the ether. According to classical theory, the speed of light c, relative to the ether, would be $c - v$ relative to the observer for light moving from the source toward the mirror and $c + v$ for light reflecting from the mirror back toward the source.

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The numbers in column I of the preceding table are thus translated:

3 = good

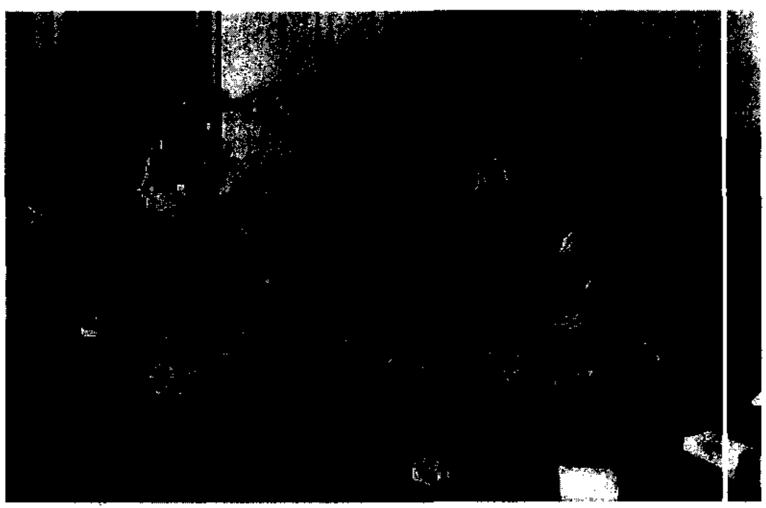
2 = fair

1 = poor

These numbers do not however represent the relative weights.

Mean result	299728
Correction for temp. (P 51)	+12
Velocity of Light in air	299740
* Correction for vacuo	+88
Velocity of Light in vacuo	299828 Kilometers per second.

* Should be	+80



Albert A. Michelson made the first accurate measurement of the speed of light. Above, in his own handwriting, is the value as recorded on page 107 of his laboratory records of the 1878 experiment. (*Below*) Michelson in his laboratory. [*Courtesy of American Institute of Physics, Niels Bohr Library.*]

a difference measurement, using the interference property of the light waves as a sensitive "clock." The apparatus that he designed to make the measurement is called the *Michelson interferometer*. The purpose of the Michelson-Morley experiment was to measure the speed of light relative to the interferometer (i.e., relative to Earth), thereby detecting Earth's motion through the ether and, thus, verifying the latter's existence. To illustrate how the interferometer works and the reasoning behind the experiment, let us first describe an analogous situation set in more familiar surroundings.

EXAMPLE 1-3 A Boat Race Two equally matched rowers race each other over courses as shown in Figure 1-7*a*. Each oarsman rows at speed c in still water; the current in the river moves at speed v. Boat 1 goes from A to B, a distance L, and back. Boat 2 goes from A to C, also a distance L, and back. A, B, and C are marks on the riverbank. Which boat wins the race, or is it a tie? (Assume $c > v$.)

PREDICT

Solution

The winner is, of course, the boat that makes the round trip in the shortest time, so to discover which boat wins we compute the time for each. Using the classical velocity transformation (Equations 1-2), the speed of 1 relative to the ground is $(c^2 - v^2)^{1/2}$, as shown in Figure 1-7*b;* thus the round-trip time t_1 for boat 1 is

$$t_1 = t_{A\to B} + t_{B\to A} = \frac{L}{\sqrt{c^2 - v^2}} + \frac{L}{\sqrt{c^2 - v^2}} = \frac{2L}{\sqrt{c^2 - v^2}}$$

$$= \frac{2L}{c\sqrt{1 - \dfrac{v^2}{c^2}}} = \frac{2L}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \approx \frac{2L}{c}\left(1 + \frac{1v^2}{2c^2} + \ldots\right) \qquad \textbf{1-6}$$

(*a*)

(*b*)

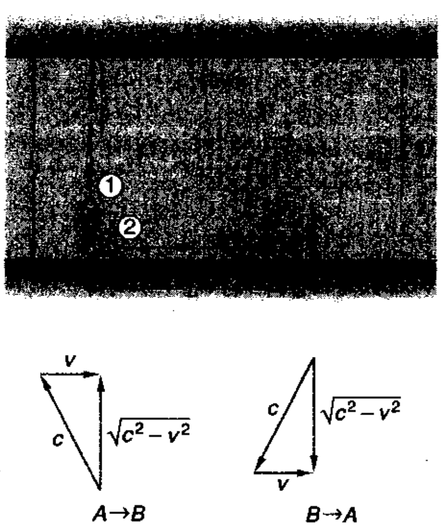


Fig. 1-7 (*a*) The rowers both row at speed c in still water. (See Example 1-3.) The current in the river moves at speed v. Rower 1 goes from A to B and back to A, while rower 2 goes from A to C and back to A. (*b*) Rower 1 must point the bow upstream so that the sum of the velocity vectors $\mathbf{c} + \mathbf{v}$ results in the boat moving from A directly to B. His speed relative to the banks (i.e., points A and B) is then $(c^2 - v^2)^{1/2}$. The same is true on the return trip.

where we have again used the binomial expansion. Boat 2 moves downstream at speed $c + v$ relative to the ground and returns at $c - v$, also relative to the ground. The round-trip time t_2 is thus

$$t_2 = \frac{L}{c+v} + \frac{L}{c-v} = \frac{2Lc}{c^2 - v^2}$$
$$= \frac{2L}{c}\frac{1}{1 - \frac{v^2}{c^2}} \approx \frac{2L}{c}\left(1 + \frac{v^2}{c^2} + \ldots\right) \tag{1-7}$$

which, you may note, is the same result that we obtained in our discussion of the speed-of-light experiment (Equation 1-5).

The difference Δt between the round-trip times of the boats is then

$$\Delta t = t_2 - t_1 \approx \frac{2L}{c}\left(1 + \frac{v^2}{c^2}\right) - \frac{2L}{c}\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) \approx \frac{Lv^2}{c^3} \tag{1-8}$$

The quantity Lv^2/c^3 is always positive; therefore, $t_2 > t_1$ and the rower of boat 1 has the faster average speed and wins the race.

The Results Michelson and Morley carried out the experiment in 1887, repeating with a much improved interferometer an inconclusive experiment that Michelson alone had performed in 1881 in Potsdam. The path length L on the new interferometer (see Figure 1-8) was about 11 m, obtained by a series of multiple reflections. Michelson's interferometer is shown schematically in Figure 1-9*a* (page 14). The field of view seen by the observer consists of parallel alternately bright and dark interference bands, called *fringes,* as illustrated in Figure 1-9*b.* The two light beams in the interferometer are exactly analogous to the two boats in Example 1-3, and Earth's motion through the ether was expected to introduce a time (phase) difference as given by Equation 1-8. Rotating the interferometer through 90° doubles the time difference and changes the phase, causing the fringe pattern to shift by an amount ΔN. An improved system for rotating the apparatus was used in which the massive stone slab on which the interferometer was mounted floated on a pool of mercury. This dampened vibrations and enabled the experimenters to rotate the interferometer without introducing mechanical strains, both of which would cause changes in L, and hence a shift in the fringes. Using a sodium light source with $\lambda = 590$ nm and assuming $v = 30$ km/s (i.e., Earth's orbital speed), ΔN was expected to be about 0.4 of the width of a fringe, about 40 times the minimum shift (0.01 fringe) that the interferometer was capable of detecting.

To Michelson's immense disappointment, and that of most scientists of the time, the expected shift in the fringes did not occur. Instead, the shift observed was only about 0.01 fringe, i.e., approximately the experimental uncertainty of the apparatus. With characteristic reserve, Michelson described the results thus:[6]

> The actual displacement [of the fringes] was certainly less than the twentieth part [of 0.4 fringe], and probably less than the fortieth part. But since the displacement is proportional to the square of the velocity, the relative velocity of the earth and the ether is probably less than one-sixth the earth's orbital velocity and certainly less than one-fourth.

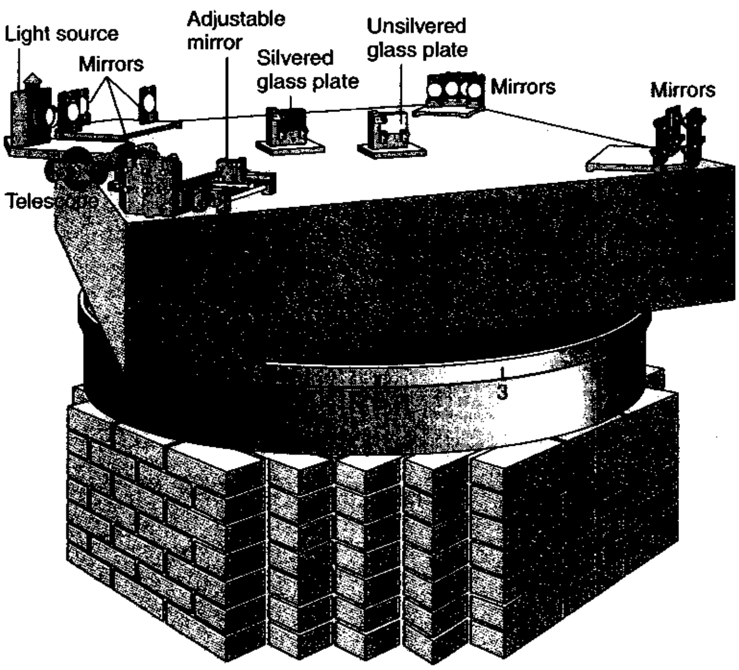


Fig. 1-8 Drawing of Michelson-Morley apparatus used in their 1887 experiment. The optical parts were mounted on a sandstone 5 ft square slab, which was floated in mercury, thereby reducing the strains and vibrations during rotation that had affected the earlier experiments. Observations could be made in all directions by rotating the apparatus in the horizontal plane. *[From R. S. Shankland, "The Michelson-Morley Experiment." Copyright © November 1964 by Scientific American, Inc. All rights reserved.]*

Michelson and Morley had placed an upper limit on Earth's motion relative to the ether of about 5 km/s. From this distance in time it is difficult for us to appreciate the devastating impact of this result. The then accepted theory of light propagation could not be correct, and the ether as a favored frame of reference for Maxwell's equations was not tenable. The experiment was repeated by a number of people more than a dozen times under various conditions and with improved precision, and no shift has ever been found. In the most precise attempt, the upper limit on the relative velocity was lowered to 1.5 km/s by Georg Joos in 1930 using an interferometer with light paths much longer than Michelson's. Recent, high-precision variations of the experiment using laser beams have lowered the upper limit to 15 m/s.

Michelson interferometers with arms as long as 4 km are currently being used in the search for gravity waves. See Section 2-5.

Maxwell: 1860
c = constant

More generally, on the basis of this and other experiments, we must conclude that Maxwell's equations are correct and that the speed of electromagnetic radiation is the same in all inertial reference systems independent of the motion of the source relative to the observer. This invariance of the speed of light between inertial reference frames means that there must be some relativity principle that applies to electromagnetism as well as to mechanics. That principle cannot be Newtonian relativity, which implies the dependence of the speed of light on the relative motion of the source and observer. It follows that the Galilean transformation of coordinates between inertial frames cannot be correct, but must be replaced with a new coordinate transformation whose application preserves the invariance of the laws of electromagnetism. We then expect that the fundamental laws of mechanics, which were consistent with the old Galilean transformation, will require modification in order to be invariant under the new transformation. The theoretical derivation of that new transformation was a cornerstone of Einstein's development of special relativity.

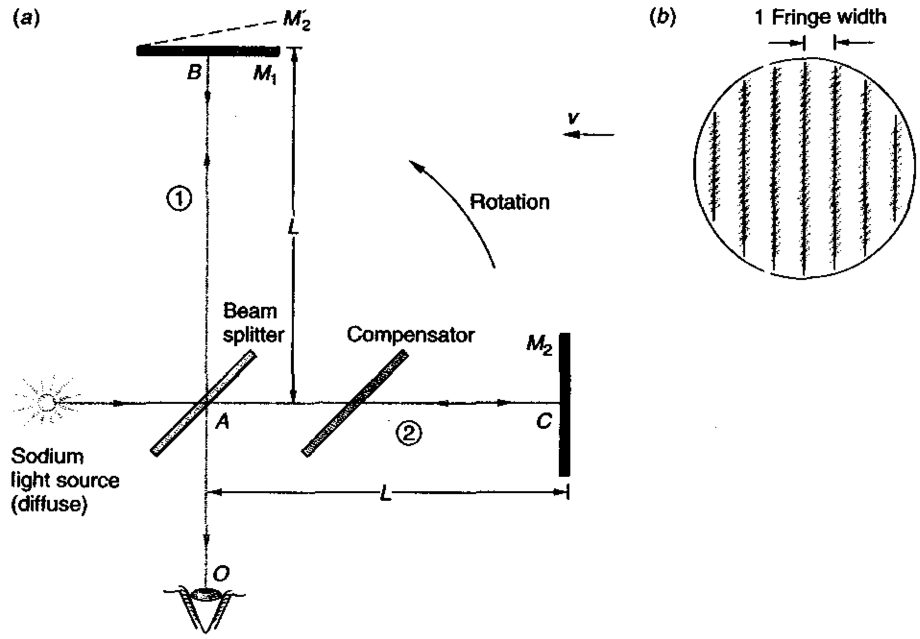


Fig. 1-9 Michelson interferometer. (*a*) Yellow light from the sodium source is divided into two beams by the second surface of the partially reflective beam splitter at *A*, at which point the two beams are exactly in phase. The beams travel along the mutually perpendicular paths 1 and 2, reflect from mirrors M_1 and M_2, and return to *A*, where they recombine and are viewed by the observer. The compensator's purpose is to make the two paths of equal optical length, so that the lengths *L* contain the same number of light waves, by making both beams pass through two thicknesses of glass before recombining. M_2 is then tilted slightly so that it is not quite perpendicular to M_1. Thus, the observer *O* sees M_1 and M_2', the image of M_2 formed by the partially reflecting second surface of the beam splitter, forming a thin wedge-shaped film of air between them. The interference of the two recombining beams depends on the number of waves in each path, which in turn depends on (1) the length of each path and (2) the speed of light (relative to the instrument) in each path. Regardless of the value of that speed, the wedge-shaped air film between M_1 and M_2' results in an increasing path length for beam 2 relative to beam 1, looking from left to right across the observer's field of view; hence, the observer sees a series of parallel interference fringes as in (*b*), alternately yellow and black from constructive and destructive interference, respectively.

More

A more complete description of the *Michelson-Morley experiment,* its interpretation, and the results of very recent versions can be found on the home page: www.whfreeman.com/modphysics4e See also Figures 1-10 through 1-12 here, as well as Equations 1-9 through 1-12.

1-2 Einstein's Postulates

In 1905, at the age of 26, Albert Einstein published several papers, among which was one on the electrodynamics of moving bodies.[11] In this paper, he postulated a more general principle of relativity which applied to both electrodynamic and mechanical laws. A consequence of this postulate is that absolute motion cannot be detected by any experiment. We can then consider the Michelson apparatus and Earth to be at rest. No fringe shift is expected when the interferometer is rotated 90°, since all directions are equivalent. The null result of the Michelson-Morley experiment is therefore to be expected. It should be pointed out that Einstein did not set out to explain the Michelson-Morley experiment. His theory arose from his considerations of the theory of electricity and magnetism and the unusual property of electromagnetic waves that they propagate in a vacuum. In his first paper, which contains the complete theory of special relativity, he made only a passing reference to the experimental attempts to detect Earth's motion through the ether, and in later years he could not recall whether he was aware of the details of the Michelson-Morley experiment before he published his theory.

1867

The theory of special relativity was derived from two postulates proposed by Einstein in his 1905 paper:

Postulate 1. The laws of physics are the same in all inertial reference frames.

Poincaré ~1880

Postulate 2. The speed of light in a vacuum is equal to the value c, independent of the motion of the source.

Maxwell 1860

Postulate 1 is an extension of the Newtonian principle of relativity to include all types of physical measurements (not just measurements in mechanics). It implies that no inertial system is preferred over any other; hence, absolute motion cannot be detected. Postulate 2 describes a common property of all waves. For example, the speed of sound waves does not depend on the motion of the sound source. When an approaching car sounds its horn, the frequency heard increases according to the Doppler effect, but the speed of the waves traveling through the air does not depend on the speed of the car. The speed of the waves depends only on the properties of the air, such as its temperature. The force of this postulate was to include light waves, for which experiments had found no propagation medium, together with all other waves whose speed *was* known to be independent of the speed of the source. Recent analysis of the light curves of gamma-ray bursts that occur near the edge of the observable universe have shown the speed of light to be independent of the speed of the source to a precision of one part in 10^{20}.

Although each postulate seems quite reasonable, many of the implications of the two together are surprising and seem to contradict common sense. One important implication of these postulates is that every observer measures the same value for the speed of light independent of the relative motion of the source and observer. Consider a light source S and two observers R_1, at rest relative to S, and R_2, moving toward S with speed v, as shown in Figure 1-13a. The speed of light measured by R_1 is $c = 3 \times 10^8$ m/s. What is the speed measured by R_2? The answer is *not* $c + v$, as one would expect based on Newtonian relativity. By postulate 1, Figure 1-13a is equivalent to Figure 1-13b, in which R_2 is at rest and the source S and R_1 are moving with speed v. That is, since absolute motion cannot be detected, it is not possible to say which is really moving and which is at rest. By postulate 2, the speed of light from a moving source is independent of the motion of the source. Thus, looking at Figure 1-13b, we see that R_2 measures the speed of light to be c, just as R_1 does. This result, that all observers measure the same value c for the speed of light, is often considered an alternative to Einstein's second postulate.

This result contradicts our intuition. Our intuitive ideas about relative velocities are approximations that hold only when the speeds are very small compared with the

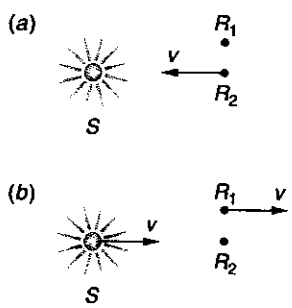


Fig. 1-13 (a) Stationary light source S and a stationary observer R_1, with a second observer R_2 moving toward the source with speed v. (b) In the reference frame in which the observer R_2 is at rest, the light source S and observer R_1 move to the right with speed v. If absolute motion cannot be detected, the two views are equivalent. Since the speed of light does not depend on the motion of the source, observer R_2 measures the same value for that speed as observer R_1.

Albert Einstein in 1905, at the time of his greatest productivity. [*Courtesy of Lotte Jacobi.*]

speed of light. Even in an airplane moving at the speed of sound, it is not possible to measure the speed of light accurately enough to distinguish the difference between the results c and $c + v$, where v is the speed of the plane. In order to make such a distinction, we must either move with a very great velocity (much greater than that of sound) or make extremely accurate measurements, as in the Michelson-Morley experiment, and when we do, we will find, as Einstein pointed out in his original relativity paper, that the contradictions are "only apparently irreconcilable."

Events and Observers

In considering the consequences of Einstein's postulates in greater depth, i.e., in developing the theory of special relativity, we need to be certain that meanings of some important terms are crystal clear. First, there is the concept of an *event*. A physical event is something that happens, like the closing of a door, a lightning strike, the collision of two particles, your birthday, or the explosion of a star. Every event occurs at some point in space and at some instant in time, but it is very important to recognize that events are independent of the particular inertial reference frame that we might use to describe them. Events do not "belong" to any reference frame.

Events are described by *observers* who do belong to particular inertial frames of reference. Observers could be people (as in Section 1-1), electronic instruments, or other suitable recorders, but for our discussions in special relativity we are going to be very specific. Strictly speaking, the observer will be an array of recording clocks located throughout the inertial reference system. It may be helpful for you to think of the observer as a person who goes around reading out the memories of the recording clocks or receives records that have been transmitted from distant clocks, but always keep in mind that in reporting events such a person is strictly limited to summarizing the data collected from the clock memories. The travel time of light precludes him from including in his report distant events that he may have seen by eye! It is in this sense that we will be using the word *observer* in our discussions.

Each inertial reference frame may be thought of as being formed by a cubic three-dimensional lattice made of identical measuring rods (e.g., meter sticks) with a recording clock at each intersection as illustrated in Figure 1-14. The clocks are all identical, and we, of course, want them all to read the "same time" as one another at any instant, i.e., they must be *synchronized.* There are many ways to accomplish synchronization of the clocks, but a very straightforward way, made possible by the second postulate, is to use one of the clocks in the lattice as a standard, or *reference clock.* For convenience we will also use the location of the reference clock in the lattice as the coordinate origin for the reference frame. The reference clock is started with its indicator (hands, pointer, digital display) set at zero. At the instant it starts it also sends out a flash of light that spreads out as a spherical wave in all directions. When the flash from the reference clock reaches the lattice clocks 1 m away (notice that in Figure 1-14 there are six of them, two of which are off the edges of the figure), we want their indicators to read the time required for light to travel 1 m (=1/299,792,458 s). This can be done simply by having an observer at each clock set that time on the indicator and then having the flash from the reference clock start them as it passes. The clocks 1 m from the origin now display the same time as the reference clock, i.e., they are all synchronized. In a similar fashion, all of the clocks throughout the inertial frame can be synchronized, since the distance of any clock from the reference clock can be calculated from the space coordinates of its position in the lattice and the initial setting of its indicator will be the corresponding travel time for the reference light flash. This procedure can be used to synchronize the

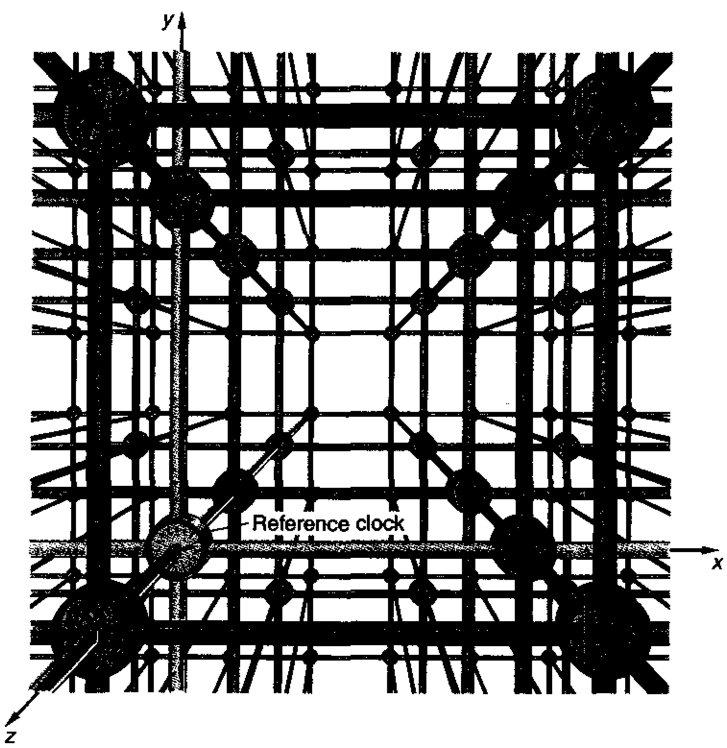


Fig. 1-14 Inertial reference frame formed from a lattice of measuring rods with a clock at each intersection. The clocks are all synchronized using a reference clock. In this diagram the measuring rods are shown to be 1 m long, but they could all be 1 cm, 1 μm, or 1 km as required by the scale and precision of the measurements being considered. The three space dimensions are the clock positions. The fourth spacetime dimension, time, is shown by indicator readings on the clocks.

Poincaré's
Synch
System

clocks in any inertial frame, *but* it does not synchronize the clocks in reference frames that move with respect to one another. Indeed, as we shall see shortly, clocks in relatively moving frames cannot in general be synchronized with one another.

When an event occurs, its location and time are recorded instantly by the nearest clock. Suppose that an atom located at $x = 2$ m, $y = 3$ m, $z = 4$ m in Figure 1-14 emits a tiny flash of light at $t = 21$ s on the clock at that location. That event is recorded in space and in time, or, as we will henceforth refer to it, the *spacetime* coordinate system with the numbers (2,3,4,21). The observer may read out and analyze these data at his leisure, within the limits set by the information transmission time (i.e., the light travel time) from distant clocks. For example, the path of a particle moving through the lattice is revealed by analysis of the records showing the particle's time of passage at each clock's location. Distances between successive locations and the corresponding time differences enable the determination of the particle's velocity. Similar records of the spacetime coordinates of the particle's path can, of course, also be made in any inertial frame moving relative to ours, but to compare the distances and time intervals measured in the two frames requires that we consider carefully the relativity of simultaneity.

Relativity of Simultaneity

Einstein's postulates lead to a number of predictions regarding measurements made by observers in inertial frames moving relative to one another that initially seem very strange, including some that appear paradoxical. Even so, these predictions have been experimentally verified; and nearly without exception, every paradox is resolved by an understanding of the *relativity of simultaneity,* which states that

> Two spatially separated events simultaneous in one reference frame are not simultaneous in any other inertial frame moving relative to the first.

A corollary to this is that

> Clocks synchronized in one reference frame are not synchronized in any other inertial frame moving relative to the first.

What do we mean by simultaneous events? Suppose two observers both in the inertial frame S at different locations A and B, agree to explode bombs at time t_0 (remember, we have synchronized all of the clocks in S). The clock at C, equidistant from A and B, will record the arrival of light from the explosions at the same instant, i.e., simultaneously. Other clocks in S will record the arrival of light from A or B first, depending on their locations, but after correcting for the time the light takes to reach each clock, the data recorded by each would lead an observer to conclude that the explosions were simultaneous. We will thus define two events to be simultaneous in an inertial reference frame if the light signals from the events reach an observer halfway between them at the same time as recorded by a clock at that location, called a local clock.

Einstein's Example To show that two events which are simultaneous in frame S are not simultaneous in another frame S' moving relative to S, we use an example introduced by Einstein. A train is moving with speed v past a station platform. We have observers located at A', B', and C' at the front, back, and middle of the train. (We consider the train to be at rest in S' and the platform in S.) We now suppose that the train and platform are struck by lightning at the front and back of the train and that the lightning bolts are simultaneous in the frame of the platform (S; Figure 1-15*a*). That is, an observer located at C halfway between positions A and B, where lightning strikes, observes the two flashes at the same time. It is convenient to suppose that the lightning scorches both the train and the platform so that the events can be easily located in each reference frame. Since C' is in the middle of the train, halfway between the places on the train which are scorched, the events are simultaneous in S' only if the clock at C' records the flashes at the same time. However, the clock at C' records the flash from the front of the train before the flash from the back. In frame S, when the light from the front flash reaches the observer at C', the train has moved some distance toward A, so that the flash from the back has not yet reached C', as indicated in Figure 1-15*b*. The observer at C' must therefore conclude that the events are not simultaneous, but that the front of the train was struck before the back. Figures 1-15*c* and 1-15*d* illustrate, respectively, the subsequent simultaneous arrival of the flashes at C and the still later arrival of the flash from the rear of the train at C'. As we have discussed, all observers in S' on the train will agree with the observer at C' when they have corrected for the time it takes light to reach them.

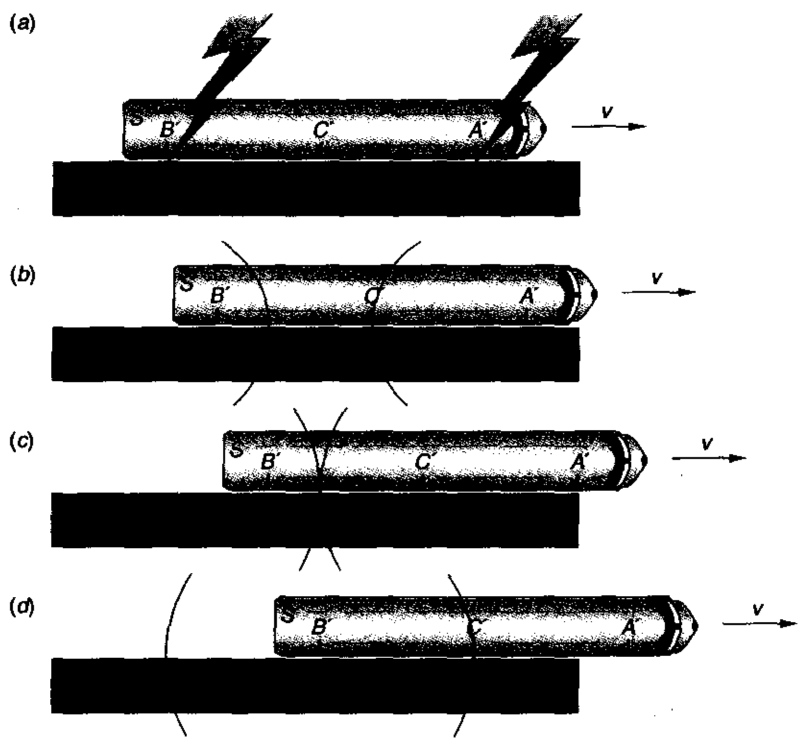


Fig. 1-15 Lightning bolts strike the front and rear of the train, scorching both the train and the platform, as the train (frame S') moves past the platform (system S) at speed v. (*a*) The strikes are simultaneous in S, reaching the C observer located midway between the events at the same instant as recorded by the clock at C as shown in (*c*). In S' the flash from the front of the train is recorded by the C' clock, located midway between the scorch marks on the train, before that from the rear of the train (*b* and *d*, respectively). Thus, the C' observer concludes that the strikes were not simultaneous.

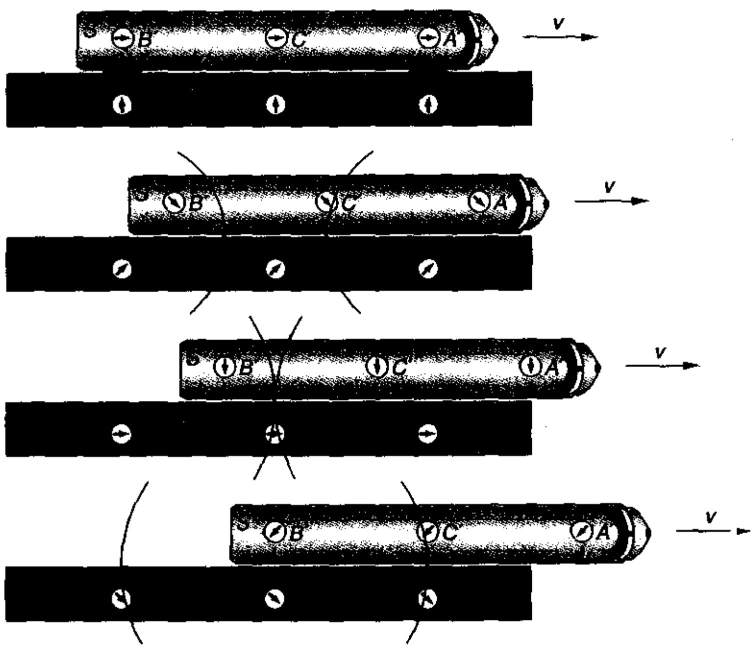


Fig. 1-16 (*a*) Light flashes originate simultaneously at clocks *A* and *B*, synchronized in *S*. (*b*) The clock at *C'*, midway between *A'* and *B'* on the moving train, records the arrival of the flash from *A* before the flash from *B* shown in (*d*). Since the observer in *S* announced that the flashes were triggered at t_0 on the local clocks, the observer at *C'* concludes that the local clocks at *A* and *B* did not read t_0 simultaneously; i.e., they were not synchronized. The simultaneous arrival of the flashes at *C* is shown in (*c*).

The corollary can also be demonstrated with a similar example. Again consider the train to be at rest in *S'*, which moves past the platform, at rest in *S*, with speed *v*. Figure 1-16 shows three of the clocks in the *S* lattice and three of those in the *S'* lattice. The clocks in each system's lattice have been synchronized in the manner that was described earlier, but those in *S* are not synchronized with those in *S'*. The observer at *C* midway between *A* and *B* on the platform announces that light sources at *A* and *B* will flash when the clocks at those locations read t_0 (Figure 1-16*a*). The observer at *C'*, positioned midway between *A'* and *B'*, notes the arrival of the light flash from the front of the train (Figure 1-16*b*) *before* the arrival of the one from the rear (Figure 1-16*d*). Observer *C'* thus concludes that, if the flashes were each emitted at t_0 on the local clocks, as announced, then the clocks at *A* and *B* are not synchronized. All observers in *S'* would agree with that conclusion after correcting for the time of light travel. The clock located at *C* records the arrival of the two flashes simultaneously, of course, since the clocks in *S* are synchronized (Figure 1-16*c*). Notice, too, in Figure 1-16 that *C'* also concludes that the clock at *A* is ahead of the clock at *B*. This is important, and we will return to it in more detail in the next section.

QUESTIONS

3. In addition to that described above, what would be another possible method of synchronizing all of the clocks in an inertial reference system?
4. In the demonstration of the validity of the corollary, how do observers at *A'* and *B'* reach the same conclusion as the observer at *C'* regarding the synchronization of the clocks at *A* and *B*?

1-3 The Lorentz Transformation

We now consider a very important consequence of Einstein's postulates, the general relation between the spacetime coordinates x, y, z, and t of an event as seen in reference frame S and the coordinates x', y', z', and t' of the same event as seen in reference frame S', which is moving with uniform velocity relative to S. For simplicity we shall consider only the special case in which the origins of the two coordinate systems are coincident at time $t = t' = 0$ and S' is moving, relative to S, with speed v along the x (or x') axis and with the y' and z' axes parallel, respectively, to the y and z axes, as shown in Figure 1-17. As we discussed earlier, the classical Galilean coordinate transformation is

$$x' = x - vt \qquad y' = y \qquad z' = z \qquad t' = t \qquad \textbf{1-3}$$

which expresses coordinate measurements made by an observer in S' in terms of those measured by an observer in S. The inverse transformation is

$$x = x' + vt' \qquad y = y' \qquad z = z' \qquad t = t'$$

and simply reflects the fact that the sign of the relative velocity of the reference frames is different for the two observers. The corresponding classical velocity transformation was given in Equation 1-2 and the (invariant) acceleration transformation in Equation 1-4. (For the rest of the discussion we shall ignore the equations for y and z, which do not change in this special case of motion along the x and x' axes.) These equations are consistent with experiment as long as v is much less than c.

It should be clear that the classical velocity transformation is not consistent with the Einstein postulates of special relativity. If light moves along the x axis with speed c in S, Equation 1-2 implies that the speed in S' is $u'_x = c - v$ rather than $u'_x = c$. The Galilean transformation equations must therefore be modified to be consistent with Einstein's postulates, but the result must reduce to the classical equations when v is much less than c. We shall give a brief outline of one method of obtaining the relativistic transformation which is called the *Lorentz transformation,* so named because of its original discovery by H. A. Lorentz.[12] We assume the equation for x' to be of the form 1890 s

$$x' = \gamma (x - vt) \qquad \textbf{1-13}$$

Almost Galilean

where γ is a constant which can depend upon v and c but not on the coordinates. If this equation is to reduce to the classical one, γ must approach 1 as v/c approaches 0. The inverse transformation must look the same except for the sign of the velocity:

$$x = \gamma (x' + vt') \qquad \textbf{1-14}$$

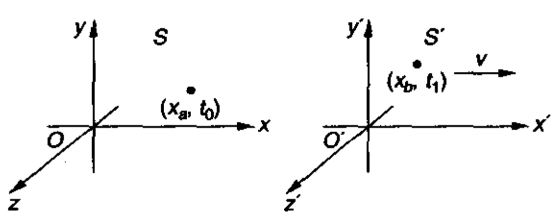


Fig. 1-17 Two inertial frames S and S' with the latter moving at speed v in $+x$ direction of system S. Each set of axes shown is simply the coordinate axes of a lattice like that in Figure 1-14. Remember, there is a clock at each intersection. A short time before the times represented by this diagram O and O' were coincident and the lattices of S and S' were intermeshed.

With the arrangement of the axes in Figure 1-17, there is no relative motion of the frames in the y and z directions; hence $y' = y$ and $z' = z$. However, insertion of the as yet unknown multiplier γ modifies the classical transformation of time, $t' = t$. To see this, we substitute x' from Equation 1-13 into Equation 1-14 and solve for t'. The result is

$$t' = \gamma\left[t + \frac{(1-\gamma^2)}{\gamma^2}\frac{x}{v}\right] \qquad \textbf{1-15}$$

Now let a flash of light start from the origin of S at $t = 0$. Since we have assumed that the origins coincide at $t = t' = 0$, the flash also starts at the origin of S' at $t' = 0$. The flash expands from *both* origins as a spherical wave. The equation for the wave front according to an observer in S is

$$x^2 + y^2 + z^2 = c^2t^2 \qquad \textbf{1-16}$$

and according to an observer in S' it is

$$x'^2 + y'^2 + z'^2 = c^2t'^2 \qquad \textbf{1-17}$$

where both equations are consistent with the second postulate. Consistency with the first postulate means that the relativistic transformation that we seek must transform Equation 1-16 into Equation 1-17, and vice versa. For example, substituting Equations 1-13 and 1-15 into 1-17 results in Equation 1-16, *if*

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\beta^2}} \qquad \textbf{1-18}$$

where $\beta = v/c$. Notice that $\gamma = 1$ for $v = 0$ and $\gamma \rightarrow \infty$ for $v = c$. How this is done is illustrated in Example 1-4.

EXAMPLE 1-4 Relativistic Transformation Multiplier γ Show that γ must be given by Equation 1-18, if Equation 1-17 is to be transformed into Equation 1-16 consistent with Einstein's first postulate.

Solution

Substituting Equations 1-13 and 1-15 into 1-17 and noting that $y' = y$ and $z' = z$ in this case yield

$$\gamma^2(x - vt)^2 + y^2 + z^2 = c^2\gamma^2\left[t + \frac{1-\gamma^2}{\gamma^2}\frac{x}{v}\right]^2 \qquad \textbf{1-19}$$

To be consistent with the first postulate, Equation 1-19 must be identical to 1-16. This requires that the coefficient of the x^2 term in Equation 1-19 be equal to 1, that of the t^2 term equal to c^2, and that of the xt term equal to 0. Any of these conditions can be used to determine γ and all yield the same result. Using, for example, the coefficient of x^2, we have from Equation 1-19 that

$$\gamma^2 - c^2\gamma^2 \frac{(1-\gamma^2)^2}{\gamma^4 v^2} = 1$$

which can be rearranged to

$$-c^2 \frac{(1-\gamma^2)^2}{\gamma^2 v^2} = (1-\gamma^2)$$

Canceling $1 - \gamma^2$ on both sides and solving for γ yield

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

With the value for γ found in Example 1-4, Equation 1-15 can be written in a somewhat simpler form, and with it the complete Lorentz transformation becomes

$$x' = \gamma(x - vt) \qquad y' = y$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad z' = z \tag{1-20}$$

length contraction

and the inverse

$$x = \gamma(x' + vt') \qquad y = y'$$
$$t = \gamma\left(t' + \frac{vx'}{c^2}\right) \qquad z = z' \tag{1-21}$$

with

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

EXAMPLE 1-5 Transformation of Time Intervals **The arrivals of two cosmic ray μ mesons (muons) are recorded by detectors in the laboratory, one at time t_0 at location x_a and the second at time t_1 at location x_b in the laboratory reference frame, S in Figure 1-17. What is the time interval between those two events in system S' which moves relative to S at speed v?**

Solution

Applying the time coordinate transformation from Equation 1-20,

$$t_1' - t_0' = \gamma\left(t_1 - \frac{vx_b}{c^2}\right) - \gamma\left(t_0 - \frac{vx_a}{c^2}\right)$$
$$\tau = t_1' - t_0' = \gamma(t_1 - t_0) - \frac{\gamma v}{c^2}(x_b - x_a) \tag{1-22}$$

proper time

→ p. 35

We see that the time interval measured in S' depends not just on the corresponding time interval in S, but also on the spatial separation of the clocks in S that measured the interval. This result should not come as a total surprise, since we have already discovered that, although the clocks in S are synchronized with each other, they are not synchronized for observers in other inertial frames.

Special Case 1

If it should happen that the two events occur at the same location in S, i.e., $x_a = x_b$, then $(t_1 - t_0)$, the time interval measured on a clock located at the events, is called the *proper time interval.* Notice that, since $\gamma > 1$ for all frames moving relative to S, the proper time interval is the ***minimum*** time interval that can be measured between those events.

Special Case 2

Does there exist an inertial frame for which the events described above would be measured to be simultaneous? Since the question has been asked, you probably suspect that the answer is yes, and you are right. The two events will be simultaneous in a system S'' for which $t''_1 - t''_0 = 0$, i.e., when

$$\gamma (t_1 - t_0) = \frac{\gamma v}{c^2}(x_b - x_a)$$

or when

$$\beta = \frac{v}{c} = \left(\frac{t_1 - t_0}{x_b - x_a}\right)c \qquad \textbf{1-23}$$

Notice that $(x_b - x_a)/c$ = time for a light beam to travel from x_a to x_b; thus we can characterize S'' as being that system whose speed relative to S is that fraction of c given by the time interval between the events divided by the travel time of light between them.

While it is possible for us to get along in special relativity without the Lorentz transformation, it has an application that is quite valuable: it enables the spacetime coordinates of events measured by the measuring rods and clocks in the reference frame of one observer to be translated into the corresponding coordinates determined by the measuring rods and clocks of an observer in another inertial frame. As we will see in Section 1-4, such transformations lead to some startling results.

Relativistic Velocity Transformations

The transformation for velocities in special relativity can be obtained by differentiation of the Lorentz transformation, keeping in mind the definition of the velocity. Suppose a particle moves in S with velocity **u** whose components are $u_x = dx/dt$, $u_y = dy/dt$, and $u_z = dz/dt$. An observer in S' would measure the components $u'_x = dx'/dt'$, $u'_y = dy'/dt'$, $u'_z = dz'/dt'$. Using the transformation equations, we obtain

$$dx' = \gamma(dx - v\,dt) \qquad dy' = dy$$
$$dt' = \gamma\left(dt - \frac{v\,dx}{c^2}\right) \qquad dz' = dz$$

from which we see that u_x' is given by

$$u_x' = \frac{dx'}{dt'} = \frac{\gamma(dx - v\,dt)}{\gamma\left(dt - \frac{v\,dx}{c^2}\right)} = \frac{(dx/dt - v)}{1 - \frac{v}{c^2}\frac{dx}{dt}}$$

or

$$u_x' = \frac{u_x - v}{1 - \frac{vu_x}{c^2}} \qquad \textbf{1-24}$$

and, if a particle has velocity components in the y and z directions, it is not difficult to find the components in S' in a similar manner.

$$u_y' = \frac{u_y}{\gamma\left(1 - \frac{vu_x}{c^2}\right)} \qquad u_z' = \frac{u_z}{\gamma\left(1 - \frac{vu_x}{c^2}\right)}$$

$\gamma > 1$

$\left(1 - \frac{vu}{c^2}\right) < 1$

$u_y' \gtrless u_y$

Remember that this form of the velocity transformation is specific to the arrangement of the coordinate axes in Figure 1-17. Note, too, that when $v \ll c$, i.e., when $\beta = v/c \approx 0$, the relativistic velocity transforms reduce to the classical velocity addition of Equation 1-2. Likewise the inverse velocity transformation is

$$u_x = \frac{u_x' + v}{\left(1 + \frac{vu_x'}{c^2}\right)} \qquad u_y = \frac{u_y'}{\gamma\left(1 + \frac{vu_x'}{c^2}\right)} \qquad u_z = \frac{u_z'}{\gamma\left(1 + \frac{vu_x'}{c^2}\right)} \qquad \textbf{1-25}$$

EXAMPLE 1-6 Relative Speeds of Cosmic Rays Suppose that two cosmic ray protons approach Earth from opposite directions as shown in Figure 1-18*a*. The speeds relative to Earth are measured to be $v_1 = 0.6c$ and $v_2 = -0.8c$. What is

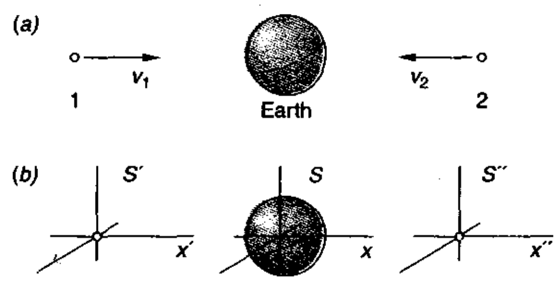


Fig. 1-18 (*a*) Two cosmic ray protons approach Earth from opposite directions at speeds v_1 and v_2 with respect to Earth. (*b*) Attaching an inertial frame to each particle and Earth enables one to visualize the several relative speeds involved and apply the velocity transformation correctly.

Earth's velocity relative to each proton, and what is the velocity of each proton relative to the other?

Solution

Consider each particle and Earth to be inertial reference frames S', S'', and S with their respective x axes parallel, as in Figure 1-18*b*. With this arrangement $v_1 = u_{1x} = 0.6c$ and $v_2 = u_{2x} = -0.8c$. Thus, the speed of Earth measured in S' is $v'_{Ex} = -0.6c$ and the speed of Earth measured in S'' is $v''_{Ex} = 0.8c$.

To find the speed of proton 2 with respect to proton 1, we apply Equation 1-24 to compute u'_{2x}, i.e., the speed of particle 2 in S'. Its speed in S has been measured to be $u_{2x} = -0.8c$, where the S' system has relative speed $v_1 = 0.6c$ with respect to S. Thus, substituting into Equation 1-24, we obtain

$$u'_{2x} = \frac{-0.8c - (0.6c)}{1 - (0.6c)(-0.8c)/c^2} = \frac{-1.4c}{1.48} = -0.95c$$

and the first proton measures the second to be approaching (moving in the $-x'$ direction) at $0.95c$.

The observer in S'' must of course make a consistent measurement, i.e., find the speed of proton 1 to be $0.95c$ in the $+x''$ direction. This can be readily shown by a second application of Equation 1-24 to compute u''_{1x}.

$$u''_{1x} = \frac{0.6c - (-0.8c)}{1 - (0.6c)(-0.8c)/c^2} = \frac{1.4c}{1.48} = 0.95c$$

QUESTIONS

5. The Lorentz transformation for y and z is the same as the classical result: $y = y'$ and $z = z'$. Yet the relativistic velocity transformation does not give the classical result $u_y = u'_y$ and $u_z = u'_z$. Explain.
6. Since the velocity components of a moving particle are different in relatively moving frames, the *directions* of the velocity vectors are also different, in general. Explain why the fact that observers in S and S' measure different directions for a particle's motion is not an inconsistency in their observations.

Spacetime Diagrams

The relativistic discovery that time intervals between events are not the same for all observers in different inertial reference frames underscores the four-dimensional character of spacetime. With the diagrams that we have used thus far, it is difficult to depict and visualize on the two-dimensional page events that occur at different times, since each diagram is equivalent to a snapshot of the spacetime at a particular instant. Showing events as a function of time typically requires a series of diagrams, such as Figures 1-15 and 1-16; but even then our attention tends to be drawn to the space coordinate systems, rather than the events, whereas it is the *events* that are fundamental. This difficulty is removed in special relativity with a simple yet powerful graphing method called the *spacetime diagram*. (This is just a new name given to the t versus x graphs that you first began to use when you discussed motion in introductory physics.) On the spacetime diagram we can graph both the space and time coordinates of many events in

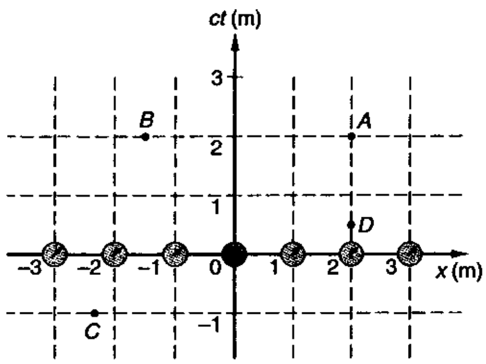


Fig. 1-19 Spacetime diagram for an inertial reference frame S. Two of the space dimensions (y and z) are suppressed. The units on both the space and time axes are the same, meters. A meter of time means the time required for light to travel 1 meter, i.e., 3.3×10^{-9} s.

one or more inertial frames, albeit with one limitation. Since the page offers only two dimensions for graphing, we suppress, or ignore for now, two of the space dimensions, in particular y and z. With our choice of the relative motion of inertial frames along the x axis, $y' = y$ and $z' = z$ anyhow. (This is one of the reasons we made that convenient choice a few pages back, the other reason being mathematical simplicity.) This means that for the time being we are limiting our attention to one space dimension and to time, i.e., to events that occur, regardless of when, along one line in space. Should we need the other two dimensions, e.g., in a consideration of velocity vector transformations, we can always use the Lorentz transformation equations.

In a spacetime diagram the space location of each event is plotted along the x axis horizontally and the time is plotted vertically. From the three-dimensional array of measuring rods and clocks in Figure 1-14, we will use only those located on the x axis as in Figure 1-19. (See, things are simpler already!) Since events that exhibit relativistic effects generally occur at high speeds, it will be convenient to multiply the time scale by the speed of light (a constant), which enables us to use the same units and scale on both the space and time axes, e.g., meters of distance and meters of light travel time.[13] The time axis is, therefore, c times the time t in seconds, i.e., ct. As we will see shortly, this choice prevents events from clustering about the axes and enables the straightforward addition of other inertial frames into the diagram.

As time advances, notice that in Figure 1-19 each clock in the array moves vertically upward along the dotted lines. Thus, as events A, B, C, and D occur in spacetime, one of the clocks of the array is at (or very near) each event when it happens. Extending our previous definition a bit, the clock located at each event records *proper time*. (See Example 1-5.) In the figure, events A and D occur at the same place ($x = 2$ m), but at different times. The time interval between them measured on clock 2 is the proper time, since clock 2 is located at *both* events. Events A and B occur at different locations, but at the same time (i.e., simultaneously in this frame). Event C occurred before the present ($ct = 0 =$ present), since $ct = -1$ m.

Worldlines in Spacetime Particles moving in space trace out a line in the spacetime diagram called the *worldline* of the particle. The worldline is the "trajectory" of the particle on a ct versus x graph. To illustrate, consider four particles moving in space (not spacetime), as shown in Figure 1-20a, which shows the array of synchronized clocks on the x axis and the space trajectories of four particles, each starting at $x = 0$ and moving at some constant speed, during 3 m of time. Figure 1-20b shows the worldline for each of the particles in spacetime. Notice that constant speed means that the worldline has

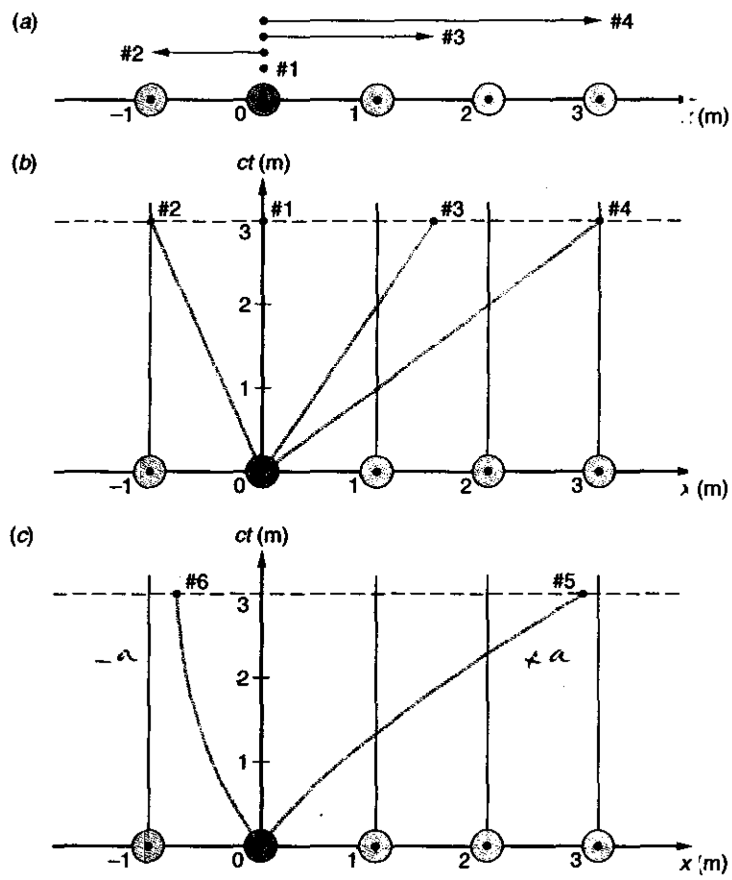


Fig. 1-20 (*a*) The space trajectories of four particles with various constant speeds. Note that particle 1 has a speed of zero and particle 2 moves in the $-x$ direction. The worldlines of the particles are straight lines. (*b*) The worldline of particle 1 is also the *ct* axis, since that particle remains at $x = 0$. The constant slopes are a consequence of the constant speeds. (*c*) For accelerating particles 5 and 6 [not shown in (*a*)], the worldlines are curved, the slope at any point yielding the instantaneous speed.

constant slope; i.e., it is a straight line (slope $= \Delta t/\Delta x = 1/(\Delta x/\Delta t) = 1/\text{speed}$). That was also the case when you first encountered elapsed time versus displacement graphs in introductory physics. Even then, you were plotting spacetime graphs and drawing worldlines! If the particle is accelerating—either speeding up as particle 5 in Figure 1-20*c*, or slowing down, like particle 6—the worldlines are curved. Thus, the worldline is the record of the particle's travel through spacetime, giving its speed (= 1/slope) and acceleration (= 1/rate at which the slope changes) at every instant.

EXAMPLE 1-7 Computing Speeds in Spacetime Find the speed u of particle 3 in Figure 1-20.

Solution

The speed $u = \Delta x/\Delta t = 1/\text{slope}$ where we have $\Delta x = 1.5 - 0 = 1.5$ m and $\Delta ct = c \cdot \Delta t = 3.0 - 0 = 3.0$ m (from Figure 1-20). Thus, $\Delta t = (3.0/c) = (3.0/3.0 \times 10^8) = 10^{-8}$ s and $u = 1.5\ \text{m}/10^{-8}\ \text{s} = 0.5c$.

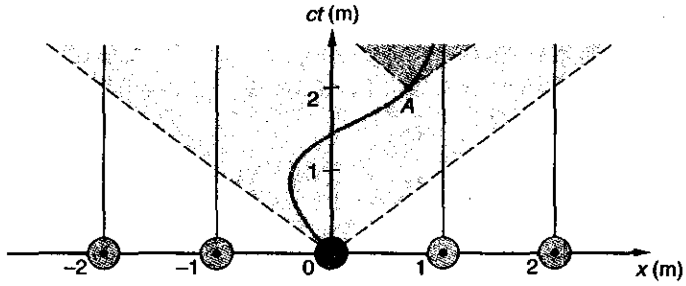


Fig. 1-21 The speed-of-light limit to the speeds of particles limits the slopes of worldlines for particles that move through $x = 0$ at $ct = 0$ to the shaded area of spacetime, i.e., to slopes < -1 and $> +1$. The dashed lines are worldlines of light flashes moving in the $-x$ and $+x$ directions. The curved worldline of the particle shown has the same limits at every instant. Notice that the particle's speed = 1/slope.

The speed of particle 4, computed as shown in Example 1-7, turns out to be c, the speed of light. (Particle 4 is a light pulse.) The slope of its worldline $\Delta(ct)/\Delta x =$ 3 m/3 m = 1. Similarly, the slope of the worldline of a light pulse moving in the $-x$ direction is -1. Since relativity limits the speed of particles with mass to less than c, as we will see in Chapter 2, the slopes of worldlines for particles that move through $x = 0$ at $ct = 0$ are limited to the larger shaded triangle in Figure 1-21. The same limits to the slope apply at every point along a particle's worldline, such as point A on the curved spacetime trajectory in Figure 1-21. This means that the particle's possible worldlines for times greater than $ct = 2$ m must lie within the heavily shaded triangle.

Analyzing events and motion in inertial systems that are in relative motion can now be accomplished more easily than with diagrams such as Figures 1-15 through 1-18. Suppose we have two inertial frames S and S' with S' moving in the $+x$ direction of S at speed v as in those figures. The clocks in both systems are started at $t = t' = 0$ (the present) as the two origins $x = 0$ and $x' = 0$ coincide, and, as before, observers in each system have synchronized the clocks in their respective systems. The spacetime diagram for S is, of course, like that in Figure 1-19, but how does S' appear in that diagram, i.e., with respect to an observer in S? Consider that, as the origin of S' (i.e., the point where $x' = 0$) moves in S, its worldline is the ct' axis, since the ct' axis is the locus of all points with $x' = 0$ (just as the ct axis is the locus of points with $x = 0$). Thus, the slope of the ct' axis as seen by an observer in S can be found from Equation 1-20, the Lorentz transformation, as follows:

$$x' = \gamma(x - vt) = 0 \qquad \text{for} \qquad x' = 0$$

or

$$x = vt = (v/c)(ct) = \beta ct$$

and

$$ct = (1/\beta)x$$

which says that the slope (in S) of the worldline of the point $x' = 0$, the ct' axis, is $1/\beta$. (See Figure 1-22*a*.)

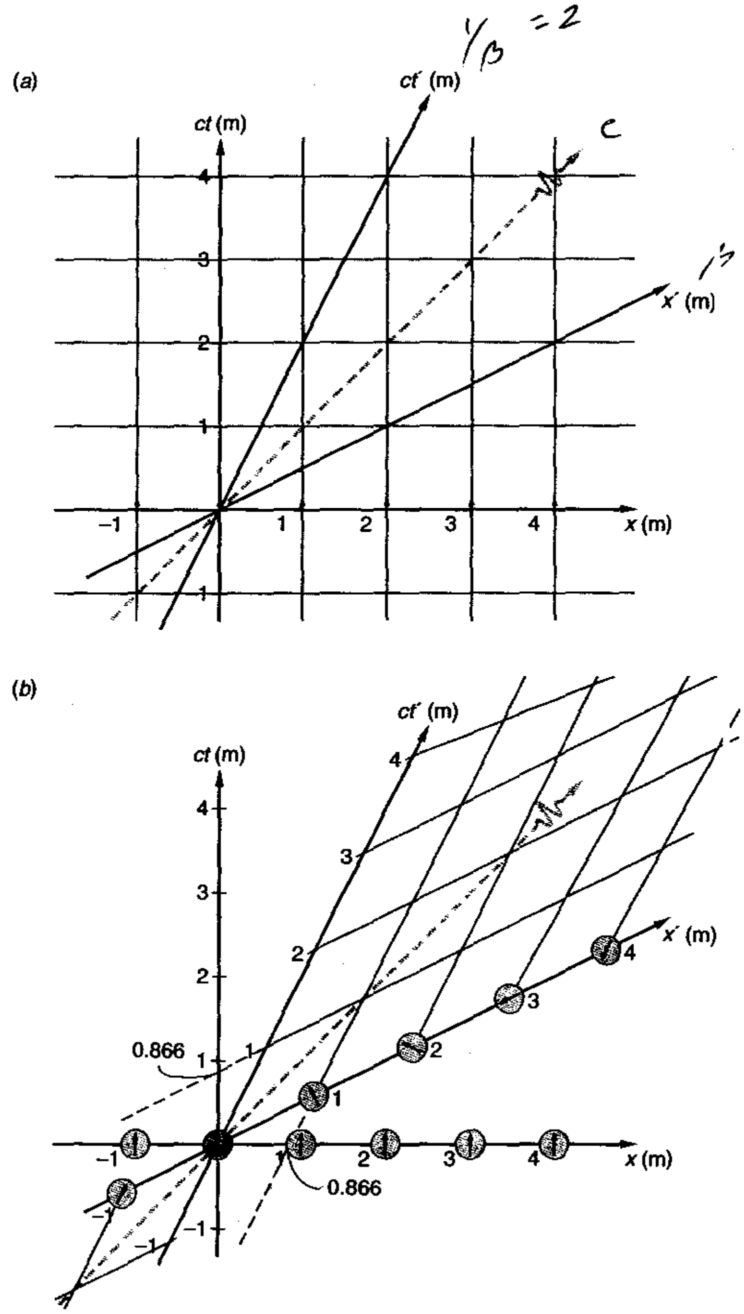


Fig. 1-22 (*a*) Spacetime diagram of *S* showing *S′* moving at speed $v = 0.5c$ in the $+x$ direction. The diagram is drawn with $t = t' = 0$ when the origins of *S* and *S′* coincided. The dashed line shows the worldline of a light flash that passed through the point $x = 0$ at $t = 0$ heading in the $+x$ direction. Its slope equals 1 in both *S* and *S′*. The ct' and x' axes of *S′* have slopes of $1/\beta = 2$ and $\beta = 0.5$, respectively. (*b*) Calibrating the axes of *S′* as described in the text allows the grid of coordinates to be drawn on *S′*. Interpretation is facilitated by remembering that (*b*) shows the system *S′* as it is observed in the spacetime diagram of *S*.

In the same manner, the x' axis can be located using the fact that it is the locus of points for which $ct' = 0$. The Lorentz transformation once again provides the slope:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) = 0$$

or

$$t = \frac{vx}{c^2} \quad \text{and} \quad ct = \frac{v}{c}x = \beta x$$

Thus, the slope of the x' axis as measured by an observer in S is β, as shown in Figure 1-22*a*. Don't be confused by the fact that the x axes don't look parallel anymore. They are still parallel in *space*, but this is a *spacetime* diagram. It shows motion in both space and time. For example, the clock at $x' = 1$ m in Figure 1-22*b* passed the point $x = 0$ at about $ct = -1.5$ m as the x' axis of S' moved both upward and to the right in S. Remember, as time advances, the array of synchronized clocks and measuring rods that are the x axis also moves upward, so that, for example, when $ct = 1$, the origin of S' ($x' = 0$, $ct' = 0$) has moved $vt = (v/c)ct = \beta ct$ to the right along the x axis.

QUESTION

7. Explain how the spacetime diagram in Figure 1-22*b* would appear drawn by an observer in S'.

EXAMPLE 1-8 Simultaneity in Spacetime Use the train-platform example of Figure 1-16 and a suitable spacetime diagram to show that events simultaneous in one frame are not simultaneous in a frame moving relative to the first. (This is the corollary to the relativity of simultaneity that we first demonstrated in the previous section using Figure 1-16.)

Solution

Suppose a train is passing a station platform at speed v and an observer C at the midpoint of the platform, system S, announces that light flashes will be emitted at clocks A and B located at opposite ends of the platform at $t = 0$. Let the train, system S', be a rocket train with $v = 0.5c$. As in the earlier discussion, clocks at C and C' both read 0 as C' passes C. Figure 1-23 shows this situation. It is the spacetime equivalent of Figure 1-16.

Two events occur: the light flashes. The flashes are simultaneous in S, since both occur at $ct = 0$. In S', however, the event at A occurred at ct' (A') (see Figure 1-23), about 1.2 ct' units *before* $ct' = 0$, and the event at B occurred at ct' (B'), about 1.2 ct' units *after* $ct' = 0$. Thus, the flashes are not simultaneous in S' and A occurs before B, as we also saw in Figure 1-16.

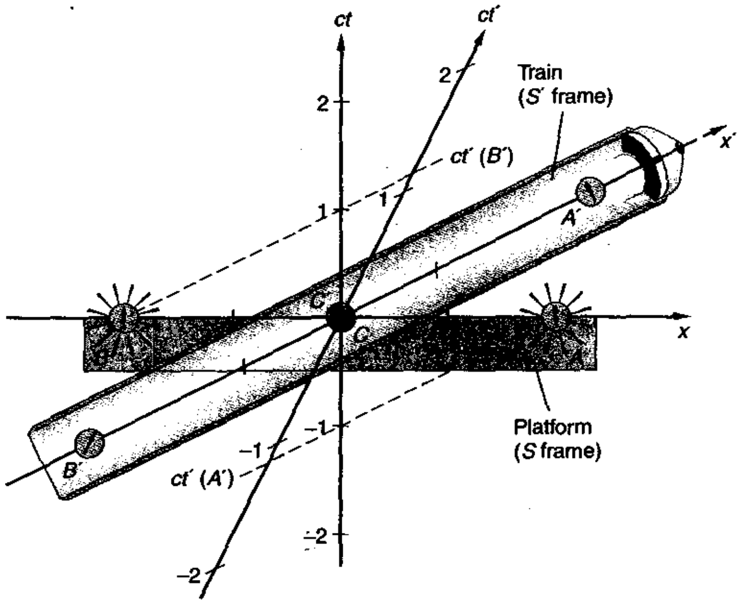


Fig. 1-23 Spacetime equivalent of Figure 1-16, showing the spacetime diagram foı the system S in which the platform is at rest. Measurements made by observers in S' are read f om the primed axes.

Exploring

Calibrating the Spacetime Axes

By calibrating the coordinate axes of S' consistent with the Lorentz transfo mation we will be able to read the coordinates of events and calculate space and tir e intervals between events as measured in both S and S' directly from the diag am, in addition to calculating them from Equations 1-20 and 1-21. The calibratioı of the S' axes is straightforward and is accomplished as follows. The locus of poin s, e.g., with $x' = 1$ m, is a line parallel to the ct' axis through the point $x' = 1$, $ct' = 0$, just as we saw earlier that the ct' axis was the locus of those points with $x' = 0$ through the point $x' = 0$, $ct' = 0$. Substituting these values into the Lorent: transformation for x', we see that the line through $x' = 1$ m intercepts the x axis, i.e., the line where $ct = 0$, at

$$x' = \gamma\,(x - vt) = \gamma\,(x - \beta ct)$$

$$1 = \gamma x \qquad \text{or} \qquad x = \frac{1}{\gamma} = \sqrt{1 - \beta^2} \tag{1-26}$$

or, in general,

$$x = x'\sqrt{1 - \beta^2}$$

In Figure 1-22*b*, where $\beta = 0.5$, the line $x' = 1$ m intercepts the x axis at $x = 0.866$ m. Similarly, if $x' = 2$ m, $x = 1.73$ m; if $x' = 3$ m, $x = 2.60$ m; and so on.

The ct' axis is calibrated in a precisely equivalent manner. The locus of points with $ct' = 1$ m is a line parallel to the x' axis through the point $ct' = 1$, $x' = 0$. Using the Lorentz transformation, the intercept of that line with the ct axis (where $x = 0$) is found as follows:

$$t' = \gamma(t - vx/c^2)$$

which can also be written as

$$ct' = \gamma(ct - \beta x) \qquad \textbf{1-27}$$

or $ct' = \gamma ct$ for $x = 0$. Thus, for $ct' = 1$, $1 = \gamma ct$ or $ct = (1 - \beta^2)^{1/2}$ and, again, in general, $ct = ct'(1 - \beta^2)^{1/2}$. The $x' \cdot ct'$ coordinate grid is shown in Figure 1-22*b*.

Notice in Figure 1-22*b* that the clocks located in S' are *not* found to be synchronized by observers in S, even though they are synchronized in S'. This is exactly the conclusion that we arrived at in the discussion of the lightning striking the train and platform. In addition, those with positive x' coordinates are behind the S' reference clock and those with negative x' coordinates are ahead, the difference being greatest for those clocks farthest away. This is a direct consequence of the Lorentz transformation of the time coordinate—i.e., when $ct = 0$ in Equation 1-27, $ct' = -\gamma\beta x$. Note, too, that the slope of the worldline of the light beam equals 1 in S', as well as in S, as required by the second postulate.

1-4 Time Dilation and Length Contraction

The results of correct measurements of the time and space intervals between events do not depend upon the kind of apparatus used for the measurements or on the events themselves. We are free therefore to choose any events and measuring apparatus that will help us understand the application of the Einstein postulates to the results of measurements. As you have already seen from previous examples, convenient events in relativity are those that produce light flashes. A convenient clock is a *light clock*, pictured schematically in Figure 1-24. A photocell detects the light pulse and sends a

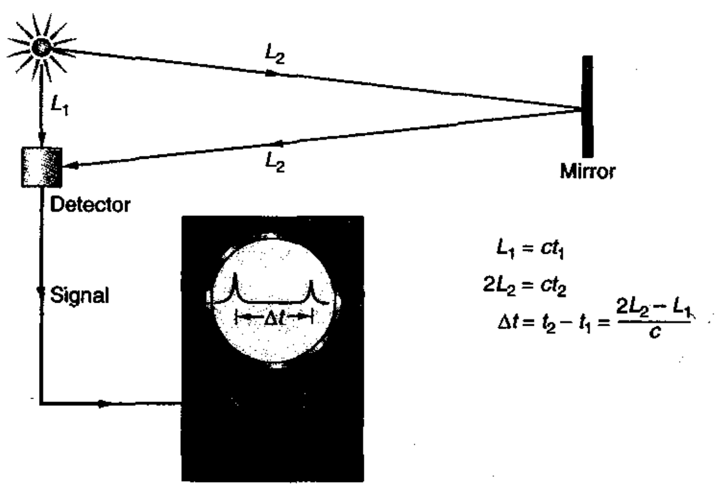


Fig. 1-24 Light clock for measuring time intervals. The time is measured by reading the distance between pulses on the oscilloscope after calibrating the sweep speed.

voltage pulse to an oscilloscope, which produces a vertical deflection of the oscilloscope's trace. The phosphorescent material on the face of the oscilloscope tube gives a persistent light that can be observed visually, photographed, or recorded electronically. The time between two light flashes is determined by measuring the distance between pulses on the scope and knowing the sweep speed. Such a clock, which can easily be calibrated and compared with other types of clocks, is often used in nuclear physics experiments. Although not drawn as in Figure 1-24, the clocks used in explanations in this section may be thought of as light clocks.

Time Dilation (or Time Stretching)

We first consider an observer A' at rest in frame S' a distance D from a mirror, also in S', as shown in Figure 1-25*a*. He triggers a flash gun and measures the time interval $\Delta t'$ between the original flash and the return flash from the mirror. Since light travels with speed c, this time is $\Delta t' = (2D)/c$.

We now consider these same two events, the original flash of light and the returning flash, as observed in reference frame S, with respect to which S' is moving to the right with speed v. The events happen at two different places, x_1 and x_2, in frame S because between the original flash and the return flash observer A' has moved a horizontal distance $v\Delta t$, where Δt is the time interval between the events measured in S. In Figure 1-25*b*, a space diagram, we see that the path traveled by the light is longer in S than in S'. However, by Einstein's postulates, light travels with the same speed c in frame S as it does in frame S'. Since it travels farther in S at the same speed, it takes longer in S to reach the mirror and return. The time interval between flashes in S is thus longer than it is in S'. We can easily calculate Δt in terms of $\Delta t'$. From the triangle in Figure 1-25*c*, we see that

$$\left(\frac{c\,\Delta t}{2}\right)^2 = D^2 + \left(\frac{v\,\Delta t}{2}\right)^2$$

or

$$\Delta t = \frac{2D}{\sqrt{c^2 - v^2}} = \frac{2D}{c}\frac{1}{\sqrt{1 - v^2/c^2}}$$

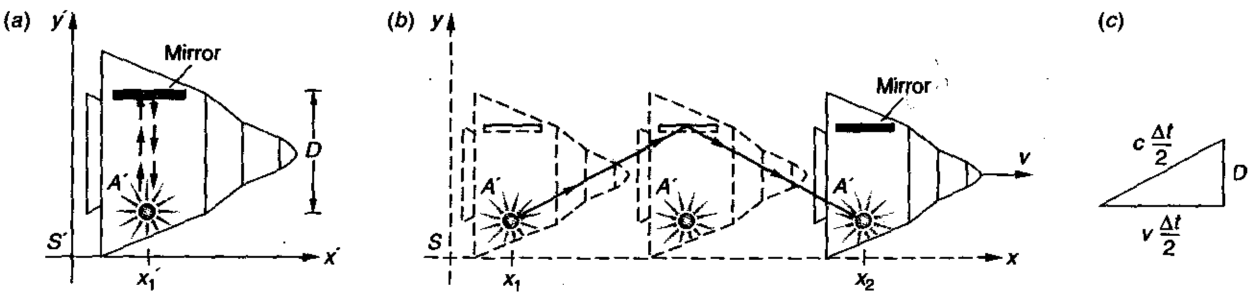


Fig. 1-25 (*a*) Observer A' and the mirror are in a spaceship at rest in frame S'. The time it takes for the light pulse to reach the mirror and return is measured by A' to be $2D/c$. (*b*) In frame S, the spaceship is moving to the right with speed v. If the speed of light is the same in both frames, the time it takes for the light to reach the mirror and return is longer than $2D/c$ in S because the distance traveled is greater than $2D$. (*c*) A right triangle for computing the time Δt in frame S.

Using $\Delta t' = 2D/c$, we have

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}} = \gamma \Delta t' = \gamma \tau \qquad \text{1-28}$$

where $\tau = \Delta t'$ is the *proper time interval* that we first encountered in Example 1-5. Equation 1-28 describes *time dilation*; i.e., it tells us that the observer in frame S always measures the time interval between two events to be longer (since $\gamma > 1$) than the corresponding interval measured on the clock located at both events in the frame where they occur at the same location. Thus, observers in S conclude that the clock at A' in S' runs slow, since that clock measures a smaller time interval between the two events. Notice that the faster S' moves with respect to S, the larger is γ, and the slower the S' clocks will tick. It appears to the S observer that time is being stretched out in S'.

Be careful! The *same* clock must be located at each event for $\Delta t'$ to be the proper time interval τ. We can see why this is true by noting that Equation 1-28 can be obtained directly from the inverse Lorentz transformation for t. Referring again to Figure 1-25 and calling the emission of the flash event 1 and its return event 2, we have that

$$\Delta t = t_2 - t_1 = \gamma\left(t_2' + \frac{vx_2'}{c^2}\right) - \gamma\left(t_1' + \frac{vx_1'}{c^2}\right)$$

$$\Delta t = \gamma\,(t_2' - t_1') + \frac{\gamma v}{c^2}(x_2' - x_1')$$

or

$$\Delta t = \gamma \Delta t' + \frac{\gamma v}{c^2}\Delta x' \qquad \text{1-29}$$

If the clock that records t_2' and t_1' is located at the events, then $\Delta x' = 0$. If that is not the case, however, $\Delta x' \neq 0$ and $\Delta t'$, though certainly a valid measurement, is not a proper time interval. Only a clock located *at* an event *when* it occurs can record proper time.

EXAMPLE 1-9 Spatial Separation of Events Two events occur at the same point x_0' at times t_1' and t_2' in S', which moves with speed v relative to S. What is the spatial separation of these events measured in S?

Solution

1. The location of the events in S is given by the Lorentz inverse transformation Equation 1-21:

$$x = \gamma\,(x' + vt')$$

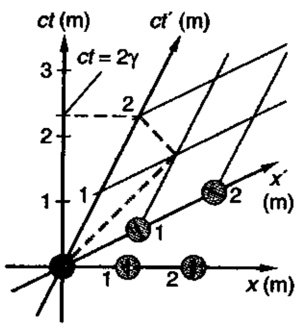


Fig. 1-26 Spacetime diagram illustrating time dilation. The dashed line is the worldline of a light flash emitted at $x' = 0$ and reflected back to that point by a mirror at $x' = 1$ m. $\beta = 0.5$.

2. The spatial separation of the two events $\Delta x = x_2 - x_1$ is then:

$$\Delta x = \gamma(x_0' + vt_2') - \gamma(x_0' + vt_1')$$

3. The $\gamma x_0'$ terms cancel:

$$\Delta x = \gamma v(t_2' - t_1') = \gamma v \Delta t'$$

4. Since $\Delta t'$ is the proper time interval τ, Equation 1-28 yields:

$$\Delta x = v\gamma\tau = v\Delta t$$

5. Using the situation in Figure 1-26 as a numerical example, where $\beta = 0.5$ and $\gamma = 1.15$, we have:

$$\Delta x = \gamma \frac{v}{c} \Delta(ct') = (1.15)(0.5)(2)$$
$$= 1.15 \text{ m}$$

EXAMPLE 1-10 The Pregnant Elephant[14] Elephants have a gestation period of 21 months. Suppose that a freshly impregnated elephant is placed on a spaceship and sent toward a distant space jungle at $v = 0.75c$. If we monitor radio transmissions from the spaceship, how long after launch might we expect to hear the first squealing trumpet from the newborn calf?

Solution

1. In S', the rest frame of the elephant, the time interval from launch to birth is $\tau = 21$ months. In the Earth frame S, the time interval is Δt, given by Equation 1-28:

$$\Delta t_1 = \gamma\tau = \frac{1}{\sqrt{1 - \beta^2}}\tau$$
$$= \frac{1}{\sqrt{1 - (0.75)^2}}(21 \text{ months})$$
$$= 31.7 \text{ months}$$

2. At that time the radio signal announcing the happy event starts toward Earth at speed c, but from where? Using the result of Example 1-9, since launch the spaceship has moved Δx in S given by:

$$\Delta x = \gamma v\tau = \gamma\beta c\tau$$
$$= (1.51)(0.75)(21c \cdot \text{months})$$
$$= 23.8c \cdot \text{months}$$

where $c \cdot$ month is the distance light travels in one month.

3. Notice that there is no need to convert Δx into meters, since our interest is in how long it will take the radio signal to travel this distance in S. That time is Δt_2, given by:

$$\begin{aligned}\Delta t_2 &= \Delta x/c \\ &= 23.8c \cdot \text{months}/c \\ &= 23.8 \text{ months}\end{aligned}$$

4. Thus, the good news will arrive at Earth at time Δt after launch where:

$$\begin{aligned}\Delta t &= \Delta t_1 + \Delta t_2 \\ &= 31.7 + 23.8 \\ &= 55.5 \text{ months}\end{aligned}$$

Remarks: *This result, too, is readily obtained from a spacetime diagram. Figure 1-27 illustrates the general appearance of the spacetime diagram for this example, showing the elephant's worldline and the worldline of the radio signal.*

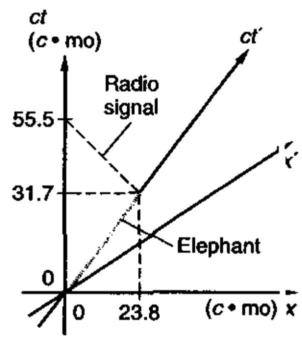


Fig. 1-27 Sketch of the spacetime diagram for Example 1–10. $\beta = 0.75$. The colored line is the worldline of the pregnant elephant. The worldline of the radio signal is the dashed line at 45° toward the upper left.

QUESTION

8. You are standing on a corner and a friend is driving past in an automobile. Both of you note the times when the car passes two different intersections and determine from your watch readings the time that elapses between the two events. Which of you has determined the proper time interval?

The time dilation of Equation 1-28 is easy to see in a spacetime diagram such as Figure 1-26, using the same round trip for a light pulse used above. Let the light flash leave $x' = 0$ at $ct' = 0$ when the S and S' origins coincided. The flash travels to $x' = 1$ m, reflects from a mirror located there, and returns to $x' = 0$. Let $\beta = 0.5$. The dotted line shows the worldline of the light beam, reflecting at ($x' = 1$, $ct' = 1$) and returning to $x' = 0$ at $ct' = 2$ m. Note that the S observer records the latter event at $ct > 2$ m; i.e., the observer in S sees the S' clock running slow.

Experimental tests of the time dilation prediction have been performed using macroscopic clocks, in particular, accurate atomic clocks. In 1975, C. O. Alley conducted a test of both general and special relativity in which a set of atomic clocks were carried by a U.S. Navy antisubmarine patrol aircraft while it flew back and forth over the same path for 15 hours at altitudes between 8000 m and 10,000 m over Chesapeake Bay. The clocks in the plane were compared by laser pulses with an identical group of clocks on the ground. (See Figure 1-14 for one way such a comparison might be done.) Since the experiment was primarily intended to test the gravitational effect on clocks predicted by general relativity (see Section 2-5), the aircraft was deliberately flown at the rather sedate average speed of 270 knots (140 m/s) $= 4.7 \times 10^{-7}c$ so as to minimize the time dilation due to the relative speeds of the clocks. Even so, after deducting the effect of gravitation as predicted by general relativity, the airborne clocks lost an average of 5.6×10^{-9} s due to the relative speed during the 15-hour flight. This result agrees with the prediction of special relativity, 5.7×10^{-9} s to within 2 percent, even at this low relative speed. The experimental results leave little basis for further debate as to whether traveling clocks of all kinds lose time on a round trip. They do.

Length Contraction

A phenomenon closely related to time dilation is *length contraction.* The length of an object measured in the reference frame in which the object is at rest is called its *proper length* L_p. In a reference frame in which the object is moving, the measured length parallel to the direction of motion is shorter than its proper length. Consider a rod at rest in the frame S' with one end at x'_2 and the other end at x'_1 as illustrated in Figure 1-28. The length of the rod in this frame is its proper length $L_p = x'_2 - x'_1$. Some care must be taken to find the length of the rod in frame S. In this frame, the rod is moving to the right with speed v, the speed of frame S'. The length of the rod in frame S is *defined* as $L = x_2 - x_1$, where x_2 is the position of one end at some time t_2, and x_1 is the position of the other end *at the same time* $t_1 = t_2$ as measured in frame S. Since the rod is at rest in S', t'_2 need not equal t'_1. Equation 1-20 is convenient to use to calculate $x_2 - x_1$ at some time t because it relates x, x', and t, whereas Equation 1-21 is not convenient because it relates x, x', and t':

$$x'_2 = \gamma(x_2 - vt_2) \qquad \text{and} \qquad x'_1 = \gamma(x_1 - vt_1)$$

Since $t_2 = t_1$, we obtain

$$x'_2 - x'_1 = \gamma(x_2 - x_1)$$

$$x_2 - x_1 = \frac{1}{\gamma}(x'_2 - x'_1) = \sqrt{1 - \frac{v^2}{c^2}}\,(x'_2 - x'_1)$$

or

$$L = \frac{1}{\gamma}L_p = \sqrt{1 - \frac{v^2}{c^2}}\,L_p \qquad \text{1-30}$$

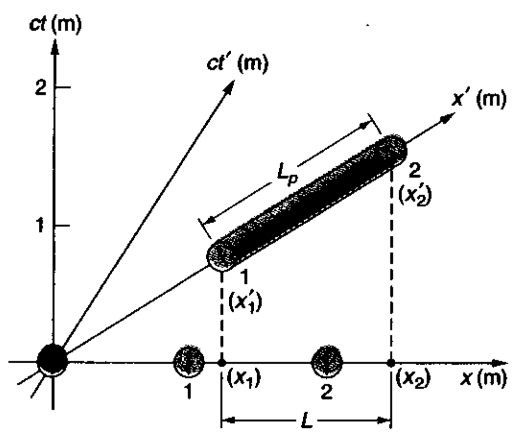


Fig. 1-28 A measuring rod, a meterstick in this case, lies at rest in S' between $x'_2 = 2$ m and $x'_1 = 1$ m. System S' moves with $\beta = 0.62$ relative to S. Since the rod is in motion, S must measure the locations of the ends of the rod x_2 and x_1 simultaneously in order to have made a valid length measurement. L is obviously shorter than L_p. By direct measurement from the diagram (use a millimeter scale) $L/L_p = 0.78 = 1/\gamma$.

Thus the length of a rod is smaller when it is measured in a frame with respect to which it is moving. Before Einstein's paper was published, Lorentz and FitzGerald had independently shown that the null result of the Michelson-Morley experiment could be explained by assuming that the lengths in the direction of the interferometer's motion contracted by the amount given in Equation 1-30. For that reason, the length contraction is often called the *Lorentz-FitzGerald contraction.* 1890s

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EXAMPLE 1-11 Speed of S' A stick that has a proper length of 1 m moves in a direction parallel to its length with speed v relative to you. The length of the stick as measured by you is 0.914 meter. What is the speed v?

Solution

1. The length of the stick measured in a frame relative to which it is moving with speed v is related to its proper length by Equation 1-30:

$$L = \frac{L_p}{\gamma}$$

2. Rearranging to solve for γ:

$$\gamma = \frac{L_p}{L}$$

3. Substituting the values of L_p and L:

$$\gamma = \frac{1\text{ m}}{0.914\text{ m}} = \frac{1}{\sqrt{1 - v^2/c^2}}$$

4. Solving for v:

$$\begin{aligned}\sqrt{1 - v^2/c^2} &= 0.914\\ 1 - v^2/c^2 &= (0.914)^2 = 0.835\\ v^2/c^2 &= 1 - 0.835 = 0.165\\ v^2 &= 0.165c^2\\ v &= 0.406c\end{aligned}$$

It is important to remember that the relativistic contraction of moving lengths occurs only parallel to the relative motion of the reference frames. In particular, observers in relatively moving systems measure the same values for lengths in the y and y' and in the z and z' directions perpendicular to their relative motion. The result is that observers measure different shapes and angles for two- and three-dimensional objects. (See Example 1-12 and Figures 1-29 and 1-30.)

EXAMPLE 1-12 The Shape of a Moving Square Consider the square in the $x'y'$ plane of S' with one side making a 30° angle with the x' axis as in Figure 1-30*a*. If S' moves with $\beta = 0.5$ relative to S, what are the shape and orientation of the figure in S?

Solution

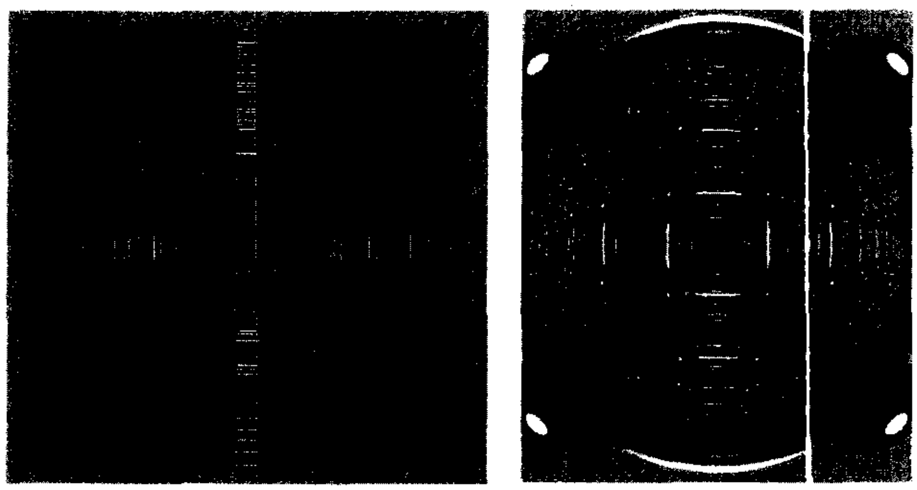
The S observer measures the x components of each side to be shorter by a factor $1/\gamma$ than those measured in S'. Thus, S measures

$$A = [\cos^2 30 + \sin^2 30/\gamma^2]^{1/2} A' = 0.968A'$$
$$B = [\sin^2 30 + \cos^2 30/\gamma^2]^{1/2} B' = 0.901B'$$

Since the figure is a square in S', $A' = B'$. In addition, the angles between B and the x axis and between A and the x axis are given by, respectively,

$$\theta = \tan^{-1}\left[\frac{B'\sin 30}{B'\cos 30/\gamma}\right] = \tan^{-1}\left[\gamma\frac{\sin 30}{\cos 30}\right] = 33.7°$$
$$\phi = \tan^{-1}\left[\frac{A'\cos 30}{A'\sin 30/\gamma}\right] = \tan^{-1}\left[\gamma\frac{\cos 30}{\sin 30}\right] = 63.4°$$

Thus, S concludes from geometry that the interior angle at vertex 1 is not 90°, but 180° − (63.4° + 33.7°) = 82.9°—i.e., the figure is not a square, but a parallelogram whose shorter sides make 33.7° angles with the x axis! Its shape and orientation in S are shown in Figure 1-30*b*.



(*a*) (*b*)

Fig. 1-29 The appearance of rapidly moving objects depends on both length contraction in the direction of motion and the time when the observed light left the object. (*a*) The array of clocks and measuring rods that represents S' as viewed by an observer in S with $\beta = 0$. (*b*) When S' approaches the S observer with $\beta = 0.9$, the distortion of the lattice becomes apparent. This is what an observer on a cosmic ray proton might see as it passes into the lattice of a cubic crystal such as NaCl. [*P.-K. Hsiung, R. Dunn, and C. Cox. Courtesy of C. Cox, Adobe Systems, Inc., San Jose, CA.*]

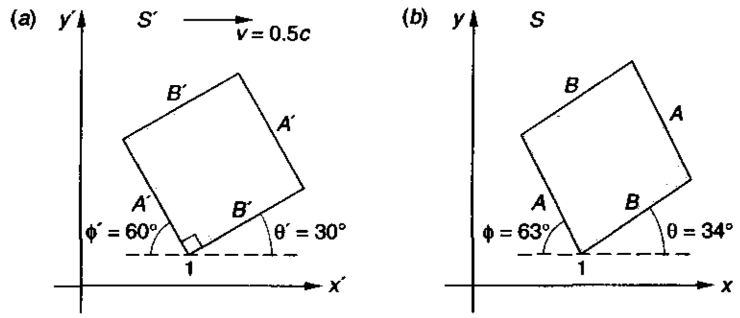


Fig. 1-30 Length contraction distorts the shape and orientation of two- and three-dimensional objects. The observer in S measures the square shown in S' as a rotated parallelogram.

Muon Decay

An interesting example of both time dilation and length contraction is afforded by the appearance of muons as secondary radiation from cosmic rays. Muons decay according to the statistical law of radioactivity:

$$N(t) = N_0 e^{-t/\tau} \qquad \textbf{1-31}$$

where N_0 is the original number of muons at time $t = 0$, $N(t)$ is the number remaining at time t, and τ is the mean lifetime (a proper time interval), which is about 2 μs for muons. Since muons are created (from the decay of pions) high in the atmosphere, usually several thousand meters above sea level, few muons should reach sea level. A typical muon moving with speed $0.998c$ would travel only about 600 m in 2 μs. However, the lifetime of the muon measured in Earth's reference frame is increased according to time dilation (Equation 1-28) by the factor $1/(1 - v^2/c^2)^{1/2}$, which is 15 for this particular speed. The mean lifetime measured in Earth's reference frame is therefore 30 μs, and a muon with speed $0.998c$ travels about 9000 m in this time. From the muon's point of view, it lives only 2 μs, but the atmosphere is rushing past it with a speed of $0.998c$. The distance of 9000 m in Earth's frame is thus contracted to only 600 m in the muon's frame, as indicated in Figure 1-31.

Experiments with muons moving near the speed of light are performed at many accelerator laboratories throughout the world despite their short mean life. Time dilation results in much longer mean lives relative to the laboratory, providing plenty of time to do experiments.

It is easy to distinguish experimentally between the classical and relativistic predictions of the observations of muons at sea level. Suppose that we observe 10^8 muons at an altitude of 9000 m in some time interval with a muon detector. How many would we expect to observe at sea level in the same time interval? According to the nonrelativistic prediction, the time it takes for these muons to travel 9000 m is $(9000\text{ m})/0.998c \approx 30$ μs, which is 15 lifetimes. Substituting $N_0 = 10^8$ and $t = 15\tau$ into Equation 1-31, we obtain

$$N = 10^8 e^{-15} = 30.6$$

We would thus expect all but about 31 of the original 100 million muons to decay before reaching sea level.

According to the relativistic prediction, Earth must travel only the contracted distance of 600 m in the rest frame of the muon. This takes only 2 μs = 1τ. Therefore the number of muons expected at sea level is

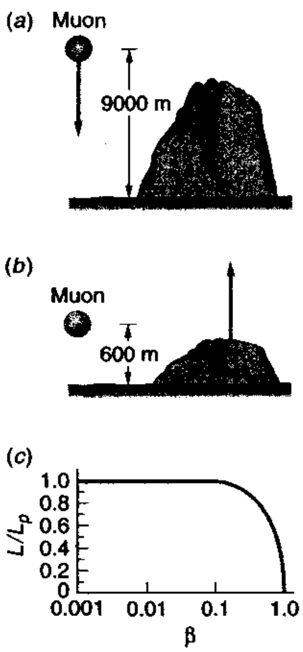


Fig. 1-31 Although muons are created high above Earth, and their mean lifetime is only about 2 μs when at rest, many appear at Earth's surface. (*a*) In Earth's reference frame, a typical muon moving at 0.998*c* has a mean lifetime of 30 μs and travels 9000 m in this time. (*b*) In the reference frame of the muon, the distance traveled by Earth is only 600 m in the muon's lifetime of 2 μs. (*c*) L varies only slightly from L_p until v is of the order of 0.1*c*. $L \to 0$ as $v \to c$.

$$N = 10^8 e^{-1} = 3.68 \times 10^7$$

Thus relativity predicts that we would observe 36.8 million muons in the same time interval. Experiments of this type have confirmed the relativistic predictions.

The Spacetime Interval = INVARIANT

We have seen earlier in this section that time intervals and lengths (= space intervals), quantities that were absolutes, or invariants, for relatively moving observers using the classical Galilean coordinate transformation, are not invariants in special relativity. The Lorentz transformation and the relativity of simultaneity lead observers in inertial frames to conclude that lengths moving relative to them are contracted and time intervals are stretched, both by the factor γ. The question naturally arises: Is there *any* quantity involving the space and time coordinates that is invariant under a Lorentz transformation? The answer to that question is yes, and as it happens, we have already dealt with a special case of that invariant quantity when we first obtained the correct form of the Lorentz transformation. It is called the *spacetime interval,* or usually just the *interval,* Δs, and is given by

$$(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2] \qquad \textbf{1-32}$$

or, specializing it to the one-space-dimensional systems that we have been discussing,

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 \qquad \textbf{1-33}$$

It may help to think of Equations 1-32 and 1-33 like this:

$$[\text{interval}]^2 = [\text{separation in time}]^2 - [\text{separation in space}]^2$$

The interval Δs is the only measurable quantity describing pairs of events in spacetime for which observers in all inertial frames will obtain the same numerical value. The negative sign in Equations 1-32 and 1-33 implies that $(\Delta s)^2$ may be positive, negative, or zero depending on the relative sizes of the time and space separations. With the sign of $(\Delta s)^2$ nature is telling us about the causal relation between the two events. Notice that whichever of the three possibilities characterizes a pair for one observer, it does so for all observers, since Δs is invariant. The interval is called *timelike* if the time separation is the larger and *spacelike* if the space separation predominates. If the two terms are equal, so that $\Delta s = 0$, then it is called *lightlike.*

Timelike Interval Consider a material particle[15] or object, e.g., the elephant in Figure 1-27, that moves relative to S. Since no material particle has ever been measured traveling faster than light, particles always travel less than 1 m of distance in 1 m of light travel time. We saw that to be the case in Example 1–10, where the time interval between launch and birth of the baby was 31.7 months on the S clock, during which time the elephant had moved a distance of 23.8 $c \cdot$ months. Equation 1-33 then yields $(c\Delta t)^2 - (\Delta x)^2 = (31.7c)^2 - (23.8c)^2 = (21.0c)^2 = (\Delta s)^2$ and the interval in S is $\Delta s = 21.0\ c \cdot$ months. The time interval term being the larger, Δs is a timelike interval and we say that material particles have *timelike worldlines.* Such worldlines lie within the shaded area of the spacetime diagram in Figure 1-21. Note that in the elephant's frame S' the separation in space between the launch and birth is zero and Δt is 21.0 months. Thus $\Delta s = 21.0\ c \cdot$ months in S', too. That is what we mean by the interval being invariant: observers in both S and S' measure the same number for the separation of the two events in spacetime.

The proper time interval τ between two events can be determined from Equation 1-33 using space and time measurements made in *any* inertial frame, since we can write that equation as

$$\frac{\Delta s}{c} = \sqrt{(\Delta t)^2 - (\Delta x/c)^2}$$

Since $\Delta t = \tau$ when $\Delta x = 0$—i.e., for the time interval recorded on a clock in a system moving such that the clock is located at each event as it occurs—in that case

$$\sqrt{(\Delta t)^2 - (\Delta x/c)^2} = \sqrt{\tau^2 - 0} = \tau = \frac{\Delta s}{c} \qquad \textbf{1-34}$$

Notice that this yields the correct proper time $\tau = 21.0$ months in the elephant example.

Spacelike Interval When two events are separated in space by an interval whose square is greater than the value of $(c\Delta t)^2$, then Δs is called *spacelike.* In that case it is convenient for us to write Equation 1-33 in the form

$$(\Delta s)^2 = (\Delta x)^2 - (c\Delta t)^2 \qquad \textbf{1-35}$$

so that, as with timelike intervals, $(\Delta s)^2$ is not negative.[16] Events that are spacelike occur sufficiently far apart in space and close together in time that no inertial frame could move fast enough to carry a clock from one event to the other. For example, suppose two observers in Earth frame S, one in San Francisco and one in London, agree to each generate a light flash at the same instant, so that $c\Delta t = 0$ m in S and $\Delta x = 1.08 \times 10^7$ m. For *any* other inertial frame $(c\Delta t)^2 > 0$ and we see from Equation 1-35 that $(\Delta x)^2$ must be greater than $(1.08 \times 10^7)^2$ in order that Δs be invariant. In other words, 1.08×10^7 m is as close in space as the two events can be in any system; consequently, it will not be possible to find a system moving fast enough to move a clock from one event to the other. A speed greater than c, in this case infinitely greater, would be needed. Notice that the value of $\Delta s = L_p$, the proper length. Just as with the proper time τ, measurements of space and time intervals in any inertial system can be used to determine L_p.

Lightlike (or Null) Interval The relation between two events is *lightlike* if Δs in Equation 1-33 equals zero. In that case

$$c\Delta t = \Delta x \qquad \textbf{1-36}$$

and a light pulse that leaves the first event as it occurs will just reach the second as it occurs.

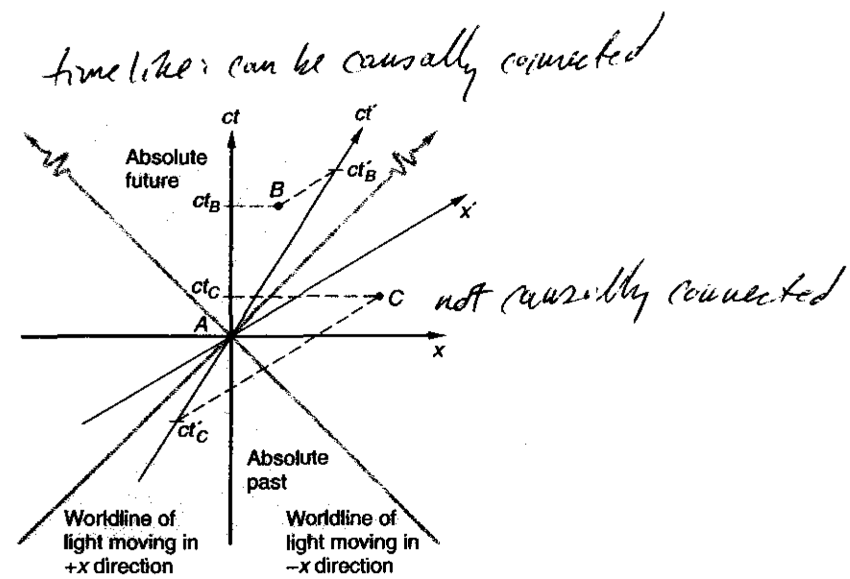


Fig. 1-32 The relative temporal order of events for pairs characterized by timelike intervals, such as *A* and *B*, is the same for all inertial observers. Events in the upper shaded area will all occur in the future of *A*; those in the lower shaded area occurred in *A*'s past. Events whose intervals are spacelike, such as *A* and *C*, can be measured as occurring in either order, depending on the relative motion of the frames. Thus, *C* occurs after *A* in *S*, but before *A* in *S′*.

The existence of the lightlike interval in relativity has no counterpart in the world of our everyday experience, where the geometry of space is Euclidean. In order for the distance between two points in space to be zero, the separation of the points in each of the three space dimensions must be zero. However, in spacetime the interval between two events may be zero, even though the intervals in space and time may individually be quite large. Notice, too, that pairs of events separated by lightlike intervals have both the proper time interval and proper length equal to zero, since $\Delta s = 0$.

Things that move at the speed of light[17] have lightlike worldlines. As we saw earlier (see Figure 1-22), the worldline of light bisects the angles between the *ct* and *x* axes in a spacetime diagram. Timelike intervals lie in the shaded areas of Figure 1-32 and share the common characteristic that their relative order in time is the same for observers in all inertial systems. Events *A* and *B* in Figure 1-32 are such a pair. Observers in both *S* and *S′* agree that *A* occurs *before B,* although they of course measure different values for the space and time separations. Causal events, i.e., events that depend upon or affect one another in some fashion, such as your birth and that of your mother, have timelike intervals. On the other hand, the temporal order of events with spacelike intervals, such as *A* and *C* in Figure 1-32, depends upon the relative motion of the systems. As you can see in the diagram, *A* occurs before *C* in *S*, but *C* occurs first in *S′*. Thus, the relative order of pairs of events is absolute in the shaded areas, but elsewhere may be in either order.

QUESTION

9. In 1987 light arrived at Earth from the explosion of a star (a supernova) in the Large Magellanic Cloud, a small companion galaxy to the Milky Way, located about 170,000 $c \cdot y$ away. Describe events that together with the

explosion of the star would be separated from it by (*a*) a spacelike interval, (*b*) a lightlike interval, and (*c*) a timelike interval.

EXAMPLE 1-13 Characterizing Spacetime Intervals Figure 1-33 is the spacetime diagram of a laboratory showing three events, the emission of light from an atom in each of three samples.

1. Determine whether the interval between each of the three possible pairs of events is timelike, spacelike, or lightlike.
2. Would it have been possible in any of the pairs for one of the events to have been caused by the other? If so, which?

Solution

1. The spacetime coordinates of the events are:

event	ct	x
1	2	1
2	5	9
3	8	6

and for the three possible pairs 1 and 2, 2 and 3, and 1 and 3 we have

pair	$c\Delta t$	Δx	$(c\Delta t)^2$	$(\Delta x)^2$	[illegible]
1 & 2	5−2	9−1	9	64	spacelike
2 & 3	8−5	6−9	9	9	lightlike
1 & 3	8−2	6−1	36	25	timelike

2. Yes, event 3 may possibly have been caused by either event 1, since 3 is in the absolute future of 1, or event 2, since 2 and 3 can just be connected by a flash of light.

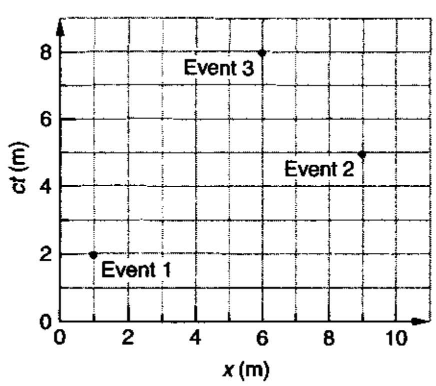


Fig. 1-33 A spacetime diagram of three events whose intervals Δs are found in Example 1-13.

1-5 The Doppler Effect

In the Doppler effect for sound the change in frequency for a given velocity v depends on whether it is the source or receiver that is moving with that speed. Such a distinction is possible for sound because there is a medium (the air) relative to which the motion takes place, and so it is not surprising that the motion of the source or the receiver relative to the still air can be distinguished. Such a distinction between motion of the source or receiver cannot be made for light or other electromagnetic waves in a vacuum as a consequence of Einstein's second postulate; therefore, the classical expressions for the Doppler effect cannot be correct for light. We will now derive the relativistic Doppler effect equations that are correct for light.

Consider a light source moving toward an observer or receiver at A in Figure 1-34*a* at velocity v. The source is emitting a train of light waves toward receivers A and B while approaching A and receding from B. Figure 1-34*b* shows the spacetime diagram of S,

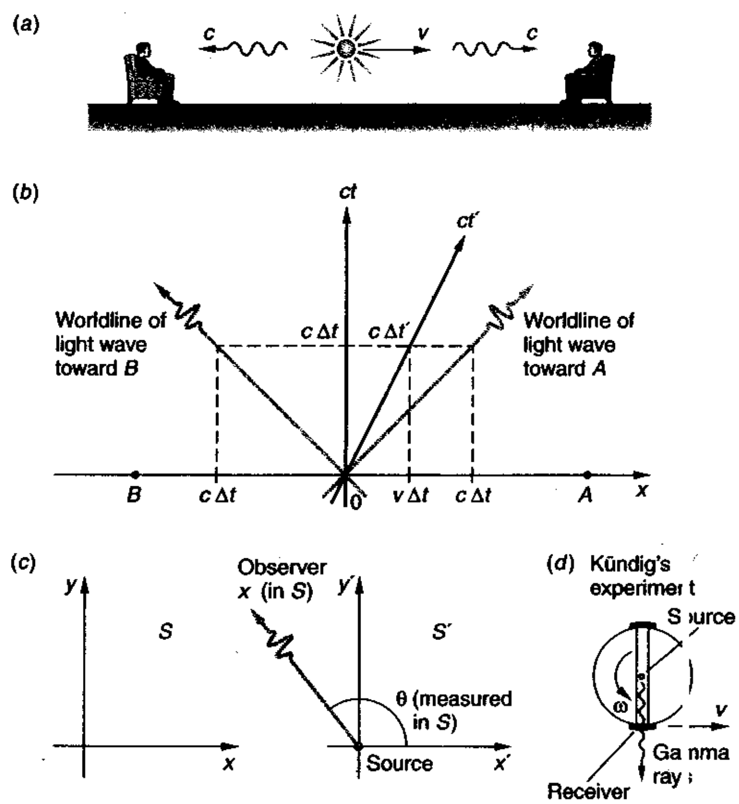


Fig. 1-34 Doppler effect in light, as in sound, arises from the relative motion of the source and receiver; however, the independence of the speed of light on that motion leads to different expressions for the frequency shift. (*a*) A source approaches observer A and recedes from observer B. The spacetime diagram of the system S in which A and B are at rest and the source moves at velocity v illustrates the two situations. (*b*) The source located at $x' = 0$ (the x' axis is omitted) moves along its worldline, the ct' axis. The N waves emitted toward A in time Δt occupy space $\Delta x = c\Delta t - v\Delta t$, whereas those headed for B occupy $\Delta x = c\Delta t + v\Delta t$. In three dimensions the observer in S may see light emitted at some angle θ with respect to the x axis as in (*c*). In that case a transverse Doppler effect occurs. (*d*) Kündig's apparatus for measuring the transverse Doppler effect.

the system in which A and B are at rest. The source is located at $x' = 0$ (the x' axis is not shown), and, of course, its worldline is the ct' axis. Let the source emit a train of N electromagnetic waves in each direction beginning when the S and S' origins were coincident. First, let's consider the train of waves headed toward A. During the time Δt over which the source emits the N waves, the first wave emitted will have traveled a distance $c\Delta t$ and the source itself a distance $v\Delta t$ in S. Thus, the N waves are seen by the observer at A to occupy a distance $c\Delta t - v\Delta t$ and, correspondingly, their wavelength λ is given by

$$\lambda = \frac{c\Delta t - v\Delta t}{N}$$

and the frequency $f = c/\lambda$ is

$$f = \frac{c}{\lambda} = \frac{cN}{(c - v)\Delta t} = \frac{1}{1 - \beta}\frac{N}{\Delta t}$$

The frequency of the source in S', called the *proper frequency*, is given by $f_0 = c/\lambda' = N/\Delta t'$, where $\Delta t'$ is measured in S', the rest system of the source. The time interval $\Delta t' = \tau$ is the proper time, since the light waves, in particular the first and the Nth, are all emitted at $x' = 0$; hence $\Delta x' = 0$ between the first and the Nth in S'. Thus, Δt and $\Delta t'$ are related by Equation 1-28 for time dilation, so $\Delta t = \gamma\Delta t'$. Thus, when the source and receiver are moving toward each other, the observer A in S measures the frequency

$$f = \frac{1}{1 - \beta}\frac{f_0\Delta t'}{\Delta t} = \frac{f_0}{1 - \beta}\frac{1}{\gamma} \qquad \text{1-37}$$

or

$$f = \frac{\sqrt{1 - \beta^2}}{1 - \beta}f_0 = \sqrt{\frac{1 + \beta}{1 - \beta}}f_0 \qquad \text{(approaching)} \qquad \text{1-38}$$

This differs from the classical equation only in the addition of the time dilation factor. Note that $f > f_0$ for the source and observer approaching one another. Since for visible light this corresponds to a shift toward the blue part of the spectrum, it is called a *blueshift*.

The use of Doppler radar to track weather systems is a direct application of special relativity.

Suppose the source and receiver are moving away from one another, as for observer B in Figure 1-34*b*. Observer B, in S, sees the N waves occupying a distance $c\Delta t + v\Delta t$, and the same analysis shows that observer B in S measures the frequency

$$f = \frac{\sqrt{1 - \beta^2}}{1 + \beta}f_0 = \sqrt{\frac{1 - \beta}{1 + \beta}}f_0 \qquad \text{(receding)} \qquad \text{1-39}$$

Notice that $f < f_0$ for the observer and source receding from one another. Since for visible light this corresponds to a shift toward the red part of the spectrum, it is called a *redshift*. It is left as a problem for you to show that the same results are obtained when the analysis is done in the frame in which the source is at rest.

In the event that $v \ll c$ (i.e., $\beta \ll 1$), as is often the case for light sources moving on Earth, useful (and easily remembered) approximations of Equations 1-38 and 1-39 can be obtained. Using Equation 1-38 as an example and rewriting it in the form

$$f = f_0 (1 + \beta)^{1/2} (1 - \beta)^{-1/2}$$

the two quantities in parentheses can be expanded by the binomial theorem to yield

$$f = f_0\left(1 + \frac{1}{2}\beta - \frac{1}{8}\beta + \dots\right)\left(1 + \frac{1}{2}\beta + \frac{3}{8}\beta^2 + \dots\right)$$

Multiplying out and discarding terms of higher order than β yield

$$f/f_0 \approx 1 + \beta \qquad \text{(approaching)}$$

and, similarly,

$$f/f_0 \approx 1 - \beta \qquad \text{(receding)}$$

and $|\Delta f/f_0| \approx \beta$ in both situations, where $\Delta f = f_0 - f$.

EXAMPLE 1-14. Rotation of the Sun The sun rotates at the equator once in about 25.4 days. The sun's radius is 7.0×10^8 m. Compute the Doppler effect that you would expect to observe at the left and right edges (limbs) of the sun near the equator for light of wavelength $\lambda = 550$ nm $= 550 \times 10^{-9}$ m (yellow light). Is this a redshift or a blueshift?

Solution

The speed of limbs v = (circumference)/(time for one revolution) or

$$v = \frac{2\pi R}{T} = \frac{2\pi\,(7.0 \times 10^8)\text{ m}}{25.4\text{ d} \cdot 3600\text{ s/h} \cdot 24\text{ h/d}} = 2000\text{ m/s}$$

$v \ll c$, so we may use the approximation equations. Using $\Delta f/f_0 \approx \beta$ we have $\Delta f \approx \beta f_0 = \beta c/\lambda_0 = v/\lambda_0$ or $\Delta f \approx 2000/550 \times 10^{-9} = 3.64 \times 10^9$ Hz. Since $f_0 = c/\lambda_0 = (3 \times 10^8\text{ m/s})/(550 \times 10^{-9}) = 5.45 \times 10^{14}$ Hz, Δf represents a fractional change in frequency of β, or about one part in 10^5. It is a redshift for the receding limb, a blueshift for the approaching one.

Doppler Effect of Starlight

In 1929, E. P. Hubble became the first astronomer to suggest that the universe is expanding.[18] He made that suggestion and offered a simple equation to describe the expansion on the basis of measurements of the Doppler shift of the frequencies of light emitted toward us by distant galaxies. Light from distant galaxies is always shifted toward frequencies lower than those emitted by similar sources nearby. Since the general expression connecting the frequency f and wavelength λ of light is $c = f\lambda$, the shift corresponds to longer wavelengths. As noted above, the color red is on the longer-wavelength side of the visible spectrum (see Chapter 4), so the *redshift* is used to describe the Doppler effect for a receding source. Similarly, *blueshift* describes light emitted by stars, typically stars in our galaxy, that are approaching us.

Astronomers define the redshift of light from astronomical sources by the expression $z = (f_0 - f)/f$, where f_0 = frequency measured in the frame of the star or galaxy and f = frequency measured at the receiver on Earth. This allows us to write $\beta = v/c$ in terms of z as

$$\beta = \frac{(z+1)^2 - 1}{(z+1)^2 + 1} \qquad \textbf{1-40}$$

Equation 1-39 is the appropriate one to use for such calculations, rather than the approximations, since galactic recession velocities can be quite large. For example, the quasar 2000-330, has a measured $z = 3.78$, which implies from Equation 1-40 that it is receding from Earth at $0.91c$.

EXAMPLE 1-15 **Redshift of Starlight** The longest wavelength of light emitted by hydrogen in the Balmer series (see Chapter 4) has a wavelength of $\lambda_0 = 656$ nm. In light from a distant galaxy, this wavelength is measured as $\lambda = 1458$ nm. Find the speed at which the galaxy is receding from Earth.

Solution

1. The recession speed is the v in $\beta = v/c$. Since $\lambda > \lambda_0$, this is a redshift and Equation 1-39 applies:

$$f = \sqrt{\frac{1-\beta}{1+\beta}}\, f_0$$

2. Rewriting Equation 1-39 in terms of the wavelengths:

$$\sqrt{\frac{1-\beta}{1+\beta}} = \frac{f}{f_0} = \frac{\lambda_0}{\lambda}$$

3. Squaring both sides and substituting values for λ_0 and λ:

$$\frac{1-\beta}{1+\beta} = \left(\frac{\lambda_0}{\lambda}\right)^2 = \left(\frac{656 \text{ nm}}{1458 \text{ nm}}\right)^2 = 0.202$$

4. Solving for β:

$$1 - \beta = (0.202)(1+\beta)$$
$$1.202\beta = 1 - 0.202 = 0.798$$
$$\beta = \frac{0.798}{1.202} = 0.664$$

5. The galaxy is thus receding at speed v, where:

$$v = c\beta = 0.664c$$

Exploring

Transverse Doppler Effect

Our discussion of the Doppler effect in Section 1-5 involved only one space dimension wherein the source, observer, and direction of the relative motion all lie on the x axis. In three space dimensions, where they may not be colinear, a more complete analysis, though beyond the scope of our discussion, makes only a small change in Equation 1-37. If the source moves along the positive x axis, but the observer views the light emitted at some angle θ with the x axis, as shown in Figure 1-34c, Equation 1-37 becomes

$$f = \frac{f_0}{\gamma} \frac{1}{1 - \beta \cos \theta} \qquad \textbf{1-37}a$$

When $\theta = 0$, this becomes the equation for the source and receiver approaching, and when $\theta = \pi$ it becomes that for them receding. Equation 1-37a also makes the quite surprising prediction that even when viewed perpendicular to the direction of motion, where $\theta = \pi/2$, the observer will still see a frequency shift, the so-called *transverse Doppler effect*, $f = f_0/\gamma$. Note that $f < f_0$, since $\gamma > 1$. It is sometimes referred to as the second-order Doppler effect. It is the result of time dilation of the moving source.

Following a suggestion first made by Einstein in 1907, Kündig in 1962 made an excellent quantitative verification of the transverse Doppler effect.[19] He used 14.4-keV gamma rays emitted by a particular isotope of Fe as the light source (see Chapter 11). The source was at rest in the laboratory, on the axis of an ultracentrifuge, and the receiver (an Fe absorber foil) was mounted on the ultracentrifuge rim, as shown in Figure 1-34d. Using the extremely sensitive frequency measuring technique called the Mössbauer effect (see Chapter 11), Kündig found a transverse Doppler effect in agreement with the relativistic prediction within ±1 percent over a range of relative speeds up to about 400 m/s.

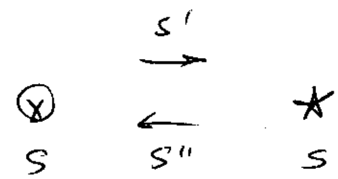
1-6 The Twin Paradox and Other Surprises

The consequences of Einstein's postulates—the Lorentz transformation, relativistic velocity addition, time dilation, length contraction, and the relativity of simultaneity—lead to a large number of predictions which are unexpected and even startling when compared with our experiences in a macroscopic world where $\beta \approx 0$ and geometry obeys the Euclidean rules. Still other predictions seem downright paradoxical, with relatively moving observers obtaining equally valid but apparently totally inconsistent results. This chapter concludes with the discussion of a few such examples that will help you hone your understanding of special relativity.

Twin Paradox

Perhaps the most famous of the paradoxes in special relativity is that of the twins, or, as it is sometimes called, the clock paradox. It arises out of the time dilation

(Equation 1-28) and goes like this. Homer and Ulysses are identical twins. Ulysses travels at a constant high speed to a star beyond our solar system and returns to Earth while his twin Homer remains at home. When the traveler Ulysses returns home, he finds his twin brother much aged compared to himself—in agreement, we shall see, with the prediction of relativity. The paradox arises out of the contention that the motion is relative and either twin could regard the other as the traveler, in which case each twin should find the other to be younger than he and we have a logical contradiction—a paradox. Let's illustrate the paradox with a specific example. Let Earth and the destination star be in the same inertial frame S. Two other frames S' and S'' move relative to S at $v = +0.8c$ and $v = -0.8c$, respectively. Thus $\gamma = 5/3$ in both cases. The spaceship carrying Ulysses accelerates quickly from S to S', then coasts with S' to the star, again accelerates quickly from S' to S'', coasts with S'' back to Earth, and brakes to a stop alongside Homer.



It is easy to analyze the problem from Homer's point of view on Earth. Suppose, according to Homer's clock, Ulysses coasts in S' for a time interval $\Delta t = 5$ y and in S'' for an equal time. Thus Homer is 10 y older when Ulysses returns. The time interval in S' between the events of Ulysses' leaving Earth and arriving at the star is shorter because it is proper time. The time it takes to reach the star by Ulysses' clock is

$$\Delta t' = \frac{\Delta t}{\gamma} = \frac{5\text{ y}}{5/3} = 3\text{ y}$$

Ulysses is 6 y older

Since the same time is required for the return trip, Ulysses will have recorded 6 y for the round trip and will be 4 y younger than Homer upon his return.

The difficulty in this situation seems to be for Ulysses to understand why his twin aged 10 y during his absence. If we consider Ulysses as being at rest and Homer as moving away, Homer's clock should run slow and measure only $3/\gamma = 1.8$ y, and it appears that Ulysses should expect Homer to have aged only 3.6 y during the round trip. This is, of course, the paradox. Both predictions can't be right. However, this approach makes the incorrect assumption that the twins' situations are symmetrical and interchangeable. They are not. Homer remains in a single inertial frame, whereas Ulysses *changes* inertial frames, as illustrated in Figure 1-35*a*, the spacetime diagram for Ulysses' trip. While the turnaround may take only a minute fraction of the total time, it is absolutely essential if the twins' clocks are to come together again so that we can compare their ages (readings).

A correct analysis can be made using the invariant interval Δs from Equation 1-33 rewritten as

$$\tau^2 = \left(\frac{\Delta s}{c}\right)^2 = (\Delta t)^2 - \left(\frac{\Delta x}{c}\right)^2$$

where the left side is constant and equal to $(\tau)^2$, the proper time interval squared, and the right side refers to measurements made in any inertial frame. Thus Ulysses, along each of his worldlines in Figure 1-35*a*, has $\Delta x = 0$ and, of course, measures $\Delta t = \tau = 3$ y, or 6 y for the round trip. Homer, on the other hand, measures

$$(\Delta t)^2 = (\tau)^2 + \left(\frac{\Delta x}{c}\right)^2$$

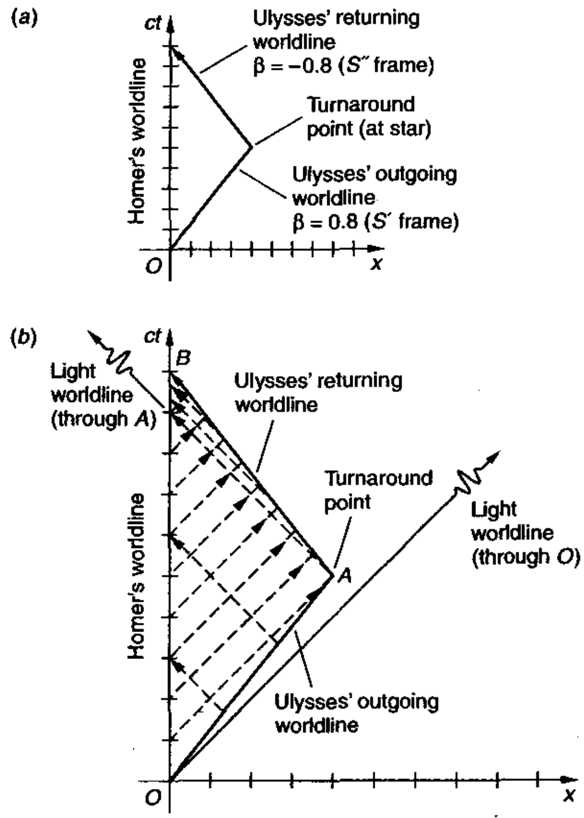


Fig. 1-35 (*a*) The spacetime diagram of Ulysses' journey to a distant star in the inertial frame in which Homer and the star are at rest. (*b*) Divisions on the *ct* axis correspond to years on Homer's clock. The broken lines show the paths (worldlines) of light flashes transmitted by each twin with a frequency of one/year on his clock. Note the markedly different frequencies at the receivers.

and since $(\Delta x/c)^2$ is always positive, he always measures $\Delta t > \tau$. In this situation $\Delta x = 0.8c\Delta t$, so

$$(\Delta t)^2 = (3\ \text{y})^2 + (0.8c\Delta t/c)^2$$

or

$$(\Delta t)^2(0.36) = (3)^2$$

$$\Delta t = \frac{3}{0.6} = 5\ \text{y}$$

or 10 y for the round trip, as we saw earlier. The reason that the twins' situations cannot be treated symmetrically is because the special theory of relativity can predict the behavior of accelerated systems, such as Ulysses at the turnaround, provided that in the formulation of the physical laws we take the view of an inertial, i.e., unaccelerated, observer such as Homer. That's what we have done. Thus, we can not do the same analysis in the rest frame of Ulysses' spaceship because it does not remain in an inertial frame during the round trip; hence, it falls outside of the special theory, and no paradox arises. The laws of physics can be reformulated so as to be invariant for accelerated observers, which is the role of general relativity (see Chapter 2), but the result is the same: Ulysses returns younger that Homer by just the amount calculated.

EXAMPLE 1-16 Twin Paradox and the Doppler Effect This example, first suggested by C. G. Darwin,[20] may help you understand what each twin sees during Ulysses' journey. Homer and Ulysses agree that once each year, on the anniversary of the launch date of Ulysses' spaceship (when their clocks were together), each twin will send a light signal to the other. Figure 1-35*b* shows the light signals each sends. Homer sends 10 light flashes (the *ct* axis, Homer's worldline, is divided into 10 equal intervals corresponding to the 10 years of the journey on Homer's clock) and Ulysses sends 6 light flashes (each of Ulysses' worldlines is divided into 3 equal intervals corresponding to 3 years on Ulysses' clock). Note that each transmits his final light flash as they are reunited at *B*. Although each transmits light signals with a frequency of 1 per year, they obviously do not receive them at that frequency. For example, Ulysses sees no signals from Homer during the first 3 years! How can we explain the observed frequencies?

Solution

The Doppler effect provides the explanation. As the twins (and clocks) recede from each other, the frequency of their signals is reduced from the proper frequency f_0 according to Equation 1-39, and we have

$$\frac{f}{f_0} = \sqrt{\frac{1 - \beta}{1 + \beta}} = \sqrt{\frac{1 - 0.8}{1 + 0.8}} = \frac{1}{3}$$

which is exactly what both twins see (refer to Figure 1-35*b*): Homer receives 3 flashes in the first 9 years and Ulysses 1 flash in his first 3 years; i.e., $f = (1/3) f_0$ for both.

After the turnaround they are approaching each other and Equation 1-38 yields

$$\frac{f}{f_0} = \sqrt{\frac{1 + \beta}{1 - \beta}} = \sqrt{\frac{1 + 0.8}{1 - 0.8}} = 3$$

and again this agrees with what the twins see: Homer receives 3 flashes during the final (10th) year and Ulysses receives 9 flashes during his final 3 years; i.e., $f = 3f_0$ for both.

QUESTION

10. The different ages of the twins upon being reunited are an example of the relativity of simultaneity that was discussed earlier. Explain how that accounts for the fact that their biological clocks are no longer synchronized.

More

It is the relativity of simultaneity that is responsible for the age difference between the twins, not their different accelerations. This is readily illustrated in *The Case of the Identically Accelerated Twins*, which can be found on the home page: www.whfreeman.com/modphysics4e See also Figure 1-36 here.

The Pole and Barn Paradox

An interesting problem involving length contraction developed by E. F. Taylor and J. A. Wheeler[22] involves putting a long pole into a short barn. One version goes as follows. A runner carries a pole 10 m long toward the open front door of a small barn 5 m long. A farmer stands near the barn so that he can see both the front and the back doors of the barn, the latter being a closed swinging door, as shown in Figure 1-37*a*. The runner carrying the pole at speed v enters the barn and at some instant the farmer sees the pole completely contained in the barn and closes the front door, thus putting a 10-m pole into a 5-m barn. The minimum speed of the runner v that is necessary for the farmer to accomplish this feat may be computed from Equation 1-30, giving the relativistic length contraction $L = L_p/\gamma$, where L_p = proper length of the pole (10 m) and L = length of the pole measured by the farmer, to be equal to the length of the barn (5 m). Therefore, we have

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{L_p}{L} = \frac{10}{5}$$
$$1 - v^2/c^2 = (5/10)^2$$
$$v^2/c^2 = 1 - (5/10)^2 = 0.75$$
$$v = 0.866c \qquad \text{or} \qquad \beta = 0.866$$

A paradox seems to arise when this situation is viewed in the rest system of the runner. For him the pole, being at rest in the same inertial system, has its proper length of 10 m. However, the runner measures the length of the barn to be

$$L = L_p/\gamma = 5\sqrt{1 - \beta^2}$$
$$L = 2.5 \text{ m}$$

How can he possibly fit the 10-m pole into the length-contracted 2.5-m barn? The answer is that he can't, and the paradox vanishes, but how can that be? To understand the answer, we need to examine two events—the coincidences of both the front and back ends of the pole, respectively, with the rear and front doors of the barn—in the inertial frame of the farmer and in that of the runner.

These are illustrated by the spacetime diagrams of the inertial frame S of the farmer and barn (Figure 1-37*b*) and that of the runner S' (Figure 1-37*c*). Both diagrams are drawn with the front end of the pole coinciding with the front door of the barn at the instant the clocks are started. In Figure 1-37*b* the worldlines of the barn doors are, of course, vertical, while those of the two ends of the pole make an angle $\theta = \tan^{-1}(1/\beta) = 49.1°$ with the x axis. Note that in S the front of the pole reaches the rear door of the barn at $ct = 5$ m/0.866 = 5.8 m *simultaneously* with the arrival of the back end of the pole at the front door; i.e., at that instant in S the pole is entirely contained in the barn.

In the runner's rest system S' it is the worldlines of the ends of the pole that are vertical, while those of the front and rear doors of the barn make angles of 49.1° with the $-x'$ axis (since the barn moves in the $-x'$ direction at v). Now we see that the rear door passes the front of the pole at $ct' = 2.5$ m/0.866 = 2.9 m, but the front door of the barn doesn't reach the rear of the pole until $ct' = 10$ m/0.866 = 11.5 m. Thus the first of those two events occurs *before* the second, and the runner never sees the pole entirely contained in the barn. Once again, the relativity of simultaneity is the

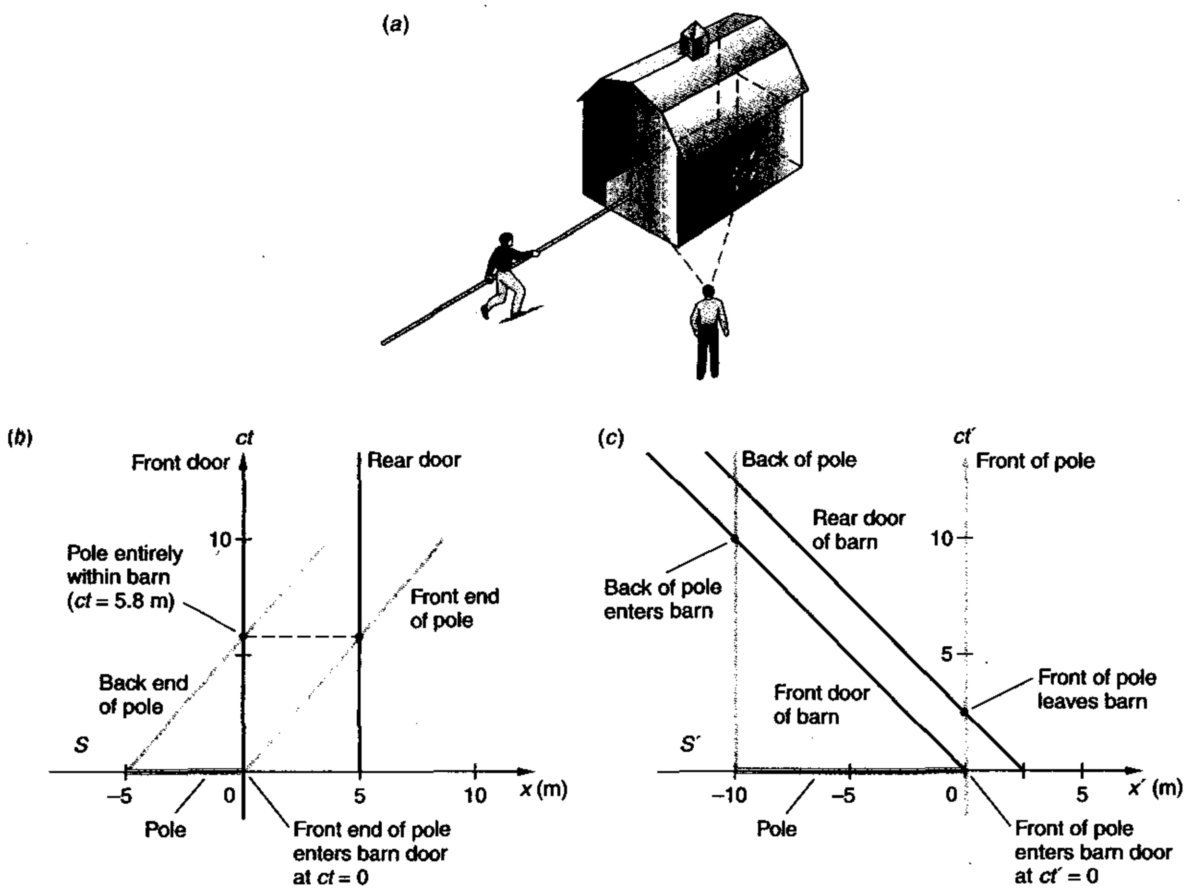


Fig. 1-37 (*a*) A runner carrying a 10-m pole moves quickly enough so that the farmer will see the pole entirely contained in the barn. The spacetime diagrams from the point of view of the farmer's inertial frame (*b*) and that of the runner (*c*). The resolution of the paradox is in the fact that the events of interest, shown by the large dots in each diagram, are simultaneous in S, but not in S'.

key—events simultaneous in one inertial frame are not simultaneous when viewed from another inertial frame.

QUESTION

11. Suppose that the barn's back wall was made from armor-plate steel and had no door. What would the farmer and the runner see then?

Headlight Effect

We have made frequent use of Einstein's second postulate asserting that the speed of light is independent of the source motion for all inertial observers; however, the same is not true for the *direction* of light. Consider a light source in S' that emits light uniformly in all directions. A beam of that light emitted at an angle θ' with

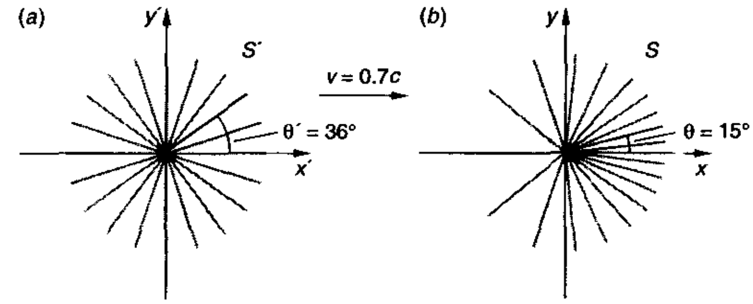


Fig. 1-38 (*a*) The source at rest in S' moves with $\beta = 0.7$ with respect to S. (*b* Light emitted uniformly in S' appears to S concentrated into a cone in the forward direction. Rays shown in (*a*) are 18° apart. Rays shown in (*b*) make angles calculated from Equation 1-4 3. The two colored rays shown are corresponding ones.

respect to the x' axis is shown in Figure 1-38*a*. During a time $\Delta t'$ the x' displacement of the beam is $\Delta x'$, and these are related to θ' by

$$\frac{\Delta x'}{c\Delta t'} = \frac{\Delta x'}{\Delta(ct')} = \cos\theta' \qquad \textbf{1-41}$$

The direction of the beam relative to the x axis in S is similarly given by

$$\frac{\Delta x}{\Delta(ct)} = \cos\theta \qquad \textbf{1-42}$$

Applying the inverse Lorentz transformation to Equation 1-42 yields

$$\cos\theta = \frac{\Delta x}{c\Delta t} = \frac{\gamma(\Delta x' + v\Delta t')}{c\gamma(\Delta t' + v\Delta x'/c^2)}$$

Dividing the numerator and denominator by $\Delta t'$ and then by c, we obtain

$$\cos\theta = \frac{(\Delta x'/\Delta t' + v)}{c\left(1 + \frac{v}{c^2}\Delta x'/\Delta t'\right)} = \frac{\Delta x'/\Delta(ct') + v/c}{1 + \frac{v}{c}\cdot\frac{\Delta x'}{\Delta(ct')}}$$

and substituting from Equation 1-41 yields

$$\cos\theta = \frac{\cos\theta' + \beta}{1 + \beta\cos\theta'} \qquad \textbf{1-43}$$

Considering the half of the light emitted by the source in S' into the forward hemisphere, i.e., rays with θ' between $\pm\pi/2$, note that Equation 1-43 restricts the angle θ measured in S for those rays (50 percent of all the light) to lie between $\theta =$ $\pm\cos^{-1}\beta$. For example, for $\beta = 0.5$, the observer in S would see half of the total light emitted by the source in S' to lie between $\theta = \pm 60°$, i.e., in a cone of half angle 60° whose axis is along the direction of the velocity of the source. For values of β near unity, θ is very small, e.g., $\beta = .99$ yields $\theta = 8.1°$. This means that the observer in S sees half of all the light emitted by the source to be concentrated into a forward cone with that half angle. (See Figure 1-38*b*.) Note, too, that the remaining 50 percent of the emitted light is distributed throughout the remaining 344° of the two-dimensional diagram.[23] As a result of the headlight effect, light from a directly approaching source appears far more intense than that from the same source at rest. For the same reason, light from a directly receding source will appear much dimmer than that from the same source at rest. This result has substantial applications in experimental particle physics and astrophysics.

In determining the brightness of stars and galaxies, a critical parameter in understanding them, astronomers must correc for the headlight effect, particularly at high velocities relative to Earth.

QUESTION

12. Notice from Equation 1-43 that some light emitted by the moving source into the rear hemisphere is seen by the observer in S as having been emitted into the forward hemisphere. Explain how that can be, using physical arguments.

Exploring
Superluminal Speeds

We conclude this chapter with a few comments about things that move faster than light. The Lorentz transformations (Equations 1-20 and 1-21) have no meaning in the event that the relative speeds of two inertial frames exceed the speed of light. This is generally taken to be a prohibition on the moving of mass, energy, and information faster than c. However, it is possible for certain processes to proceed at speeds greater than c and for the speeds of moving objects to appear to be greater than c without contradicting relativity theory. A common example of the first of these is the motion of the point where the blades of a giant pair of scissors intersect as the scissors are quickly closed, sometimes called the scissors paradox. Figure 1-39 shows the situation. A long straight rod (one blade) makes an angle θ with the x axis (the second blade) and moves in the $-y$ direction at constant speed $v_y = \Delta y/\Delta t$. During time Δt,

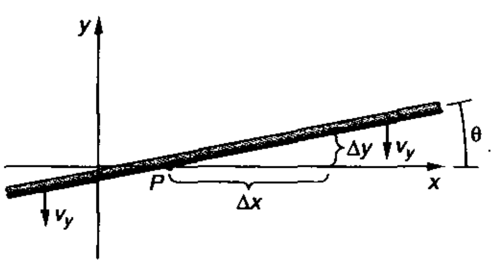


Fig. 1-39 As the long, straight rod moves vertically downward, the intersection of the "blades," point P, moves toward the right at speed $v_p = \Delta x/\Delta t$. In terms of v_y and θ, $v_p = v_y/\tan\theta$.

the intersection of the blades, point P, moves to the right a distance Δx. Note from the figure that $\Delta y/\Delta x = \tan\theta$. The speed with which P moves to the right is

$$v_p = \Delta x/\Delta t = \frac{\Delta x}{\Delta y/v_y} = \frac{v_y \Delta x}{\Delta x \tan\theta} \qquad \textbf{1-44}$$

or

$$v_p = v_y/\tan\theta$$

Since $\tan\theta \rightarrow 0$ as $\theta \rightarrow 0$, it will always be possible to find a value of θ close enough to zero so that $v_p > c$ for any (nonzero) value of v_y. As real scissors are closed, the angle gets progressively smaller, so in principle all that one needs for $v_p > c$ are long blades so that $\theta \rightarrow 0$.

QUESTION

13. Use a diagram like Figure 1-32 to explain why the motion of point P cannot be used to convey information to observers along the blades.

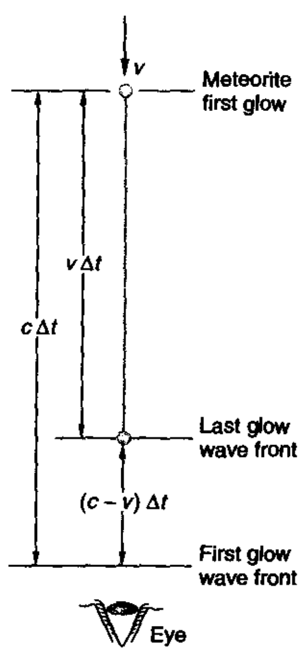


Fig. 1-40 A meteorite moves directly toward the observer's eye at speed v. The spatial distance between the wave fronts is $(c - v)\Delta t$ as they move at c, so the time interval between their arrival at the observer is not Δt, but Δt_{eye}, which is $(c - v)\Delta t/c = (1 - \beta)\Delta t$; and the apparent speed of approach is $v_a = v\Delta t/\Delta t_{eye} = \beta c/(1 - \beta)$.

The point P in the scissors paradox is, of course, a geometric point, not a material object, so it is not surprising that it could appear to move at speeds greater than c. As an example of an object with mass appearing to do so, consider a tiny meteorite moving through space directly toward you at high speed v. As it enters Earth's atmosphere, about 9 km above the surface, frictional heating causes it to glow, and the first light from the glow starts toward your eye. After some time Δt the frictional heating has evaporated all of the meteorite's matter, the glow is extinguished, and its final light starts toward your eye, as illustrated in Figure 1-40. During the time between the first and the final glow, the meteorite traveled a distance $v\Delta t$. During that same time interval light from the first glow has traveled toward your eye a distance $c\Delta t$. Thus, the space interval between the first and final glows is given by

$$\Delta y = c\Delta t - v\Delta t = \Delta t(c - v)$$

and the visual time interval at your eye Δt_{eye} between the arrival of the first and final light is

$$\Delta t_{eye} = \Delta y/c = \frac{\Delta t(c - v)}{c} = \Delta t(1 - \beta)$$

and, finally, the apparent visual speed v_a that you record is

$$v_a = \frac{v\,\Delta t}{\Delta t_{eye}} = \frac{v\,\Delta t}{\Delta t(1 - \beta)} = \frac{\beta c}{1 - \beta} \qquad \textbf{1-45}$$

Clearly, $\beta = 0.5$ yields $v_a = c$ and any larger β yields $v_a > c$. For example, a meteorite approaching you at $v = 0.8c$ is perceived to be moving at $v_a = 4c$. Certain galactic structures may also be observed to move at superluminal speeds, as the sequence of images of galaxy 3C120 in Figure 1-41 illustrates.

As a final example of things that move faster than c, it has been proposed that particles with mass might exist whose speeds would always be faster than light speed. One basis for this suggestion is an appealing symmetry: ordinary particles always have $v < c$, and photons and other massless particles have $v = c$, so the existence of particles with $v > c$ would give a sort of satisfying completeness to the classification of particles. Called *tachyons*, their existence would present relativity with serious but not necessarily insurmountable problems of infinite creation energies and causality paradoxes, e.g., alteration of history. (See the next example.) No compelling theoretical arguments preclude their existence and eventual discovery; however, experimental searches to date for tachyons[24] have failed, and the limits set by those experiments indicate that it is highly unlikely that they exist.

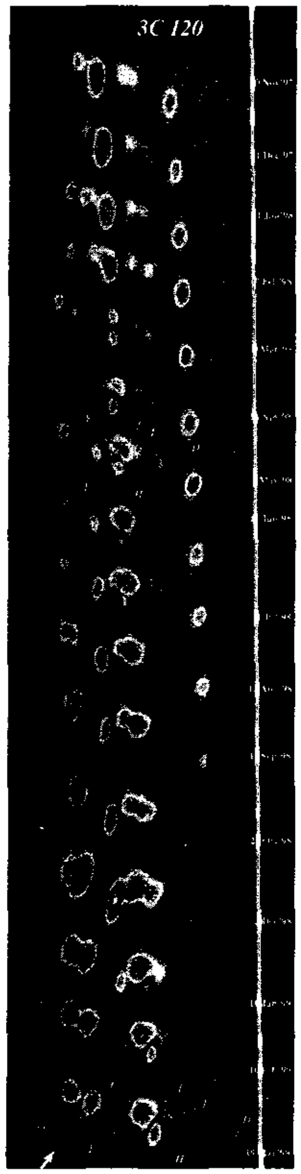


Fig. 1-41 This sequence of 16 images of active galaxy 3C120 made by J.-L. Gomez and co-workers between November 1997 and March 1999 reveals a region in the relativistic jet, marked with an arrow in the lowest image, that flashes on and off over a period of a few months and moves in the plane of the sky at about 4.4 times the speed of light. 3C120 is about $450 \times 10^6\ c \cdot y$ from Earth with a redshift of $z = 0.033$. [*J. L. Gomez et al.*, Science, *September 2000, p. 2317.*]

EXAMPLE 1-17 Tachyons and Reversing History Use tachyons and an appropriate spacetime diagram to show how the existence of such particles might be used to change history and, hence, alter the future, leading to a paradox.

Solution

In a spacetime diagram of the laboratory frame S the worldline of a particle with $v > c$ created at the origin traveling in the $+x$ direction makes an angle less than 45° with the x axis; i.e., it is below the light worldline, as shown in Figure 1-42. After some time the tachyon reaches a tachyon detector mounted on a spaceship moving rapidly away at $v < c$ in the $+x$ direction. The spaceship frame S' is shown in the figure at P. The detector immediately creates a new tachyon, sending it off in the $-x'$ direction and, of course, into the future of S', i.e., with $ct' > 0$. The second tachyon returns to the laboratory at $x = 0$, but at a time ct *before* the first tachyon was emitted, having traveled into the past of S to point M, where $ct < 0$. Having sent an object into our own past, we would then have the ability to alter events that occur after M and produce causal contradictions. For example, the laboratory tachyon detector could be coupled to equipment that created the first tachyon via a computer programmed to cancel emission of the first tachyon if the second tachyon is detected. (Shades of the Terminator!) It is logical contradictions such as this which, together with the experimental results referred to above, lead to the conclusion that faster-than-light particles do not exist.

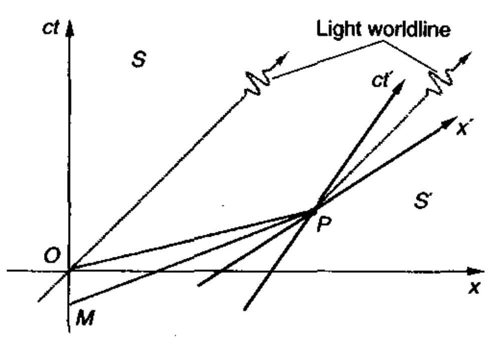


Fig. 1-42 A tachyon emitted at O in S, the laboratory frame, catches up with a spaceship moving at high speed at P. Its detection triggers the emission of a second tachyon at P back toward the laboratory at $x = 0$. The second tachyon arrives at the laboratory at $ct < 0$, i.e., before the emission of the first tachyon.

Summary

TOPIC	RELEVANT EQUATIONS AND REMARKS	
1. Classical relativity		
Galilean transformation	$x' = x - vt \quad y' = y \quad z' = z$	1-3
Newtonian relativity	Newton's laws are invariant in all systems connected by a Galilean transformation.	
2. Einstein's postulates	The laws of physics are the same in all inertial reference frames. The speed of light is c, independent of the motion of the source.	
3. Relativity of simultaneity	Events simultaneous in one reference frame are not simultaneous in any other inertial frame.	
4. Lorentz transformation	$x' = \gamma(x - vt) \quad y' = y \quad z' = z$	1-20
	$t' = \gamma(t - vx/c^2)$ with $\gamma = (1 - v^2/c^2)^{-1/2}$	
5. Time dilation	Proper time is the time interval τ between two events that occur at the same space point. If that interval is $\Delta t' = \tau$, then the time interval in S is	
	$\Delta t = \gamma \Delta t' = \gamma\tau$, where $\gamma = (1 - v^2/c^2)^{-1/2}$	1-28
6. Length contraction	The proper length of a rod is the length L_p measured in the rest system of the rod. In S, moving at speed v with respect to the rod, the length measured is	
	$L = L_p/\gamma$	1-30
7. Spacetime interval	All observers in inertial frames measure the same interval Δs between pairs of events in spacetime, where	
	$(\Delta s)^2 = (c\,\Delta t)^2 - (\Delta x)^2$	1-33
8. Doppler effect		
Source/observer approaching	$f = \sqrt{\frac{1+\beta}{1-\beta}}f_0$	1-38
Source/observer receding	$f = \sqrt{\frac{1-\beta}{1+\beta}}f_0$	1-39

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NOTES

1. Polish astronomer (1473–1543). His book describing heliocentric (i.e., sun-centered) orbits for the planets was published only a few weeks before his death. He had hesitated to release it for many years, fearing that it might be considered heretical. It is not known whether or not he saw the published book.

2. Events are described by measurements made in a coordinate system which defines a frame of reference. The question was, Where is the reference frame in which the law of inertia is valid? Newton knew that no rotating system, e.g., Earth or the sun, would work and suggested the distant "fixed stars" as the fundamental inertial reference frame.

3. The speed of light is exactly 299,792,458 m/s. This value sets the definition of the standard meter as being the distance light travels in 1/299,792,458 s.

4. Over time, an entire continuous spectrum of electromagnetic waves has been discovered, ranging from extremely low-frequency (radio) waves to extremely high-frequency waves (gamma rays), all moving at speed c.

5. Albert A. Michelson (1852–1931), an American experimental physicist whose development of precision optical instruments and their use in precise measurements of the speed of light and the length of the standard meter earned him the Nobel Prize in 1907. Edward W. Morley (1838–1923), American chemist and physicist and professor at Western Reserve College during the period when Michelson was a professor at the nearby Case School of Applied Science.

6. Albert A. Michelson and Edward W. Morley, *American Journal of Science*, **XXXIV**, no. 203 (November 1887).

7. Note that the width depends on the small angle between M_2' and M_1. A very small angle results in relatively few wide fringes, a larger angle in many narrow fringes.

8. Since the source producing the waves, the sodium lamp, was at rest relative to the interferometer, the frequency would be constant.

9. T. S. Jaseja, A. Javan, J. Murray, and C. H. Townes, *Physical Review*, **133**, A1221 (1964).

10. A. Brillet and J. Hall, *Physical Review Letters*, **42**, 549 (1979).

11. *Annalen der Physik*, **17**, 841 (1905). For a translation from the original German, see the collection of original papers by Lorentz, Einstein, Minkowski, and Weyl (New York: Dover, 1923).

12. Hendrik Antoon Lorentz (1853–1928), Dutch theoretical physicist, discovered the Lorentz transformation empirically while investigating the fact that Maxwell's equations are not invariant under a Galilean transformation, although he did not recognize its importance at the time. An expert on electromagnetic theory, he was one of the first to suggest that atoms of matter might consist of charged particles whose oscillations could account for the emission of light. Lorentz used this hypothesis to explain the splitting of spectral lines in a magnetic field discovered by his student Pieter Zeeman, with whom he shared the 1902 Nobel Prize.

13. One meter of light travel time is the *time* for light to travel 1 m, i.e., $ct = 1$ m, or $t = 1\text{ m}/3.00 \times 10^8\text{ m/s} = 3.3 \times 10^{-9}$ s. Similarly, 1 cm of light travel time is $ct = 1$ cm, or $t = 3.3 \times 10^{-11}$ s, and so on.

14. This example is adapted from a problem in H. Ohanian, *Modern Physics* (Englewood Cliffs, N.J.: Prentice Hall, 1987).

15. Any particle that has mass.

16. Equation 1-33 would lead to imaginary values of Δs for spacelike intervals, an apparent problem. However, the geometry of spacetime is not Euclidean, but Lorentzian. While a consideration of Lorentz geometry is beyond the scope of this chapter, suffice it to say that it enables us to write $(\Delta s)^2$ for spacelike intervals as in Equation 1-35.

17. There are only two such things: photons (including those of visible light), to be introduced in Chapter 3, and gravitons, which are the particles that transmit the gravitational force.

18. Edwin P. Hubble, *Proceedings of the National Academy of Sciences*, **15**, 168 (1929).

19. Walter Kündig, *Physical Review*, **129**, 2371 (1963).

20. C. G. Darwin, *Nature*, **180**, 976 (1957).

21. S. P. Boughn, *American Journal of Physics*, **57**, 791 (1989).

22. E. F. Taylor and J. A. Wheeler, *Spacetime Physics*, 2d ed. (New York: W. H. Freeman & Co., 1992).

23. Seen in three space dimensions by the observer in S, 50 percent of the light is concentrated in 0.06 steradians of 4π-steradian solid angle around the moving source.

24. T. Alväger and M. N. Kreisler, "Quest for Faster-Than-Light Particles," *Physical Review*, **171**, 1357 (1968).

25. Paul Ehrenfest (1880–1933), Austrian physicist and professor at the University of Leiden (the Netherlands), longtime friend and correspondent of Einstein about whom, upon his death, Einstein wrote, "[He was] the best teacher in our profession I have ever known."

26. This experiment is described in J. C. Hafele and R. E. Keating, *Science*, **177**, 166 (1972). Although not as accurate as the experiment described in Section 1-4, its results supported the relativistic prediction.

27. R. Shaw, *American Journal of Physics*, **30**, 72 (1962).

PROBLEMS

Level I

Section 1-1 The Experimental Basis of Relativity

1-1. A small airplane takes off from a field into an 18 m/s west wind. After 10 minutes it has moved 25 km west, 16 km north, and 0.5 km upward with respect to the wind. What are its position coordinates at that time relative to the point where it left the ground?

1-2. In one series of measurements of the speed of light, Michelson used a path length L of 27.4 km (17 mi). (*a*) What is the time needed for light to make the round trip of distance $2L$? (*b*) What is the classical correction term in seconds in Equation 1-7, assuming Earth's speed is $v = 10^{-4}c$? (*c*) From about 1600 measurements, Michelson arrived at a result for the speed of light of 299,796 $\pm$ 4 km/s. Is this experimental value accurate enough to be sensitive to the correction term in Equation 1-7?

1-3. A shift of one fringe in the Michelson-Morley experiment would result from a difference of one wavelength or a change of one period of vibration in the round-trip travel of the light when the interferometer is rotated by 90°. What speed would Michelson have computed for Earth's motion through the ether had the experiment seen a shift of one fringe?

1-4. In the "old days" (circa 1935) pilots used to race small, relatively high-powered airplanes around courses marked by a pylon on the ground at each end of the course. Suppose two such evenly matched racers fly at airspeeds of 130 mph. (Remember, this was a long time ago!) Each flies one complete round trip of 25 miles, *but* their courses are perpendicular to one another and there is a 20 mph wind blowing steadily parallel to one course. (*a*) Which pilot wins the race and by how much? (*b*) Relative to the axes of their respective courses, what headings must the two pilots use?

1-5. Paul Ehrenfest[25] suggested the following thought experiment to illustrate the dramatically different observations that might be expected, dependent on whether light moved relative to a stationary ether or according to Einstein's second postulate:

> Suppose that you are seated at the center of a huge dark sphere with a radius of 3×10^8 m and with its inner surface highly reflective. A source at the center emits a very brief flash of light which moves outward through the darkness with uniform intensity as an expanding spherical wave.

What would you see during the first 3 seconds after the emission of the flash if (*a*) the sphere moved through the ether at a constant 30 km/s, and (*b*) if Einstein's second postulate is correct?

1-6. Einstein reported that as a boy he wondered about the following puzzle. If you hold a mirror at arm's length and look at your reflection, what will happen as you begin to run? In particular, suppose you run with speed $v = 0.99c$. Will you still be able to see yourself? If so, what would your image look like, and why?

1-7. Verify by calculation that the result of the Michelson-Morley experiment places an upper limit on Earth's speed relative to the ether of about 5 km/s.

1-8. Consider two inertial reference frames. When an observer in each frame measures the following quantities, which measurements made by the two observers *must* yield the same results? Explain your reason for each answer.

(*a*) The distance between two events
(*b*) The value of the mass of a proton
(*c*) The speed of light
(*d*) The time interval between two events
(*e*) Newton's first law
(*f*) The order of the elements in the periodic table
(*g*) The value of the electron charge

Section 1-2 Einstein's Postulates

1-9. Assume that the train shown in Figure 1-15 is 1.0 km long as measured by the observer at C' and is moving at 150 km/h. What time interval between the arrival of the wave fronts at C' is measured by the observer at C in S?

1-10. Suppose that A', B', and C' are at rest in frame S', which moves with respect to S at speed v in the $+x$ direction. Let B' be located exactly midway between A' and C'. At $t' = 0$ a light flash occurs at B' and expands outward as a spherical wave. (*a*) According to an observer in S', do the wave fronts arrive at A' and C' simultaneously? (*b*) According to an observer in S, do the wave fronts arrive at A' and C' simultaneously? (*c*) If you answered no to either (*a*) or (*b*), what is the difference in their arrival times and at which point did the front arrive first?

Section 1-3 The Lorentz Transformation

1-11. Make a graph of the relativistic factor $\gamma = 1/(1 - v^2/c^2)^{1/2}$ as a function of $\beta = v/c$. Use at least 10 values of β ranging from 0 up to 0.995.

1-12. Two events happen at the same point x'_0 in frame S' at times t'_1 and t'_2. (*a*) Use Equations 1-21 to show that in frame S the time interval between the events is greater than $t'_2 - t'_1$ by a factor γ. (*b*) Why are Equations 1-20 less convenient than Equations 1-21 for this problem?

1-13. Suppose that an event occurs in inertial frame S with coordinates $x = 75$ m, $y = 18$ m, $z = 4.0$ m at $t = 2.0 \times 10^{-5}$ s. The inertial frame S' moves in the $+x$ direction with $v = 0.85c$. The origins of S and S' coincided at $t = t' = 0$. (*a*) What are the coordinates of the event in S'? (*b*) Use the inverse transformation on the results of (*a*) to obtain the original coordinates.

1-14. Show that the null effect of the Michelson-Morley experiment can be accounted for if the interferometer arm parallel to the motion is shortened by a factor of $(1 - v^2/c^2)^{1/2}$.

1-15. Two spaceships are approaching each other. (*a*) If the speed of each is $0.9c$ relative to Earth, what is the speed of one relative to the other? (*b*) If the speed of each relative to Earth is 30,000 m/s (about 100 times the speed of sound), what is the speed of one relative to the other?

1-16. Starting with the Lorentz transformation for the components of the velocity (Equation 1-24), derive the transformation for the components of the acceleration.

1-17. Consider a clock at rest at the origin of the laboratory frame. (*a*) Draw a spacetime diagram that illustrates that this clock ticks slow when observed from the reference frame of a rocket moving with respect to the laboratory at $v = 0.8c$. (*b*) When 10 s have elapsed on the rocket clock, how many have ticked by on the lab clock?

1-18. A light beam moves along the y' axis with speed c in frame S', which is moving to the right with speed v relative to frame S. (*a*) Find u_x and u_y, the x and y components of the velocity of the light beam in frame S. (*b*) Show that the magnitude of the velocity of the light beam in S is c.

1-19. A particle moves with speed $0.9c$ along the x'' axis of frame S'', which moves with speed $0.9c$ in the positive x' direction relative to frame S'. Frame S' moves with speed $0.9c$ in the positive x direction relative to frame S. (*a*) Find the speed of the particle relative to frame S'. (*b*) Find the speed of the particle relative to frame S.

Section 1-4 Time Dilation and Length Contraction

1-20. Use the binomial expansion to derive the following results for values of $v \ll c$ and use when applicable in the problems that follow.

$$(a) \quad \gamma \approx 1 + \frac{1}{2}\frac{v^2}{c^2}$$

$$(b)\quad \frac{1}{\gamma} \approx 1 - \frac{1}{2}\frac{v^2}{c^2}$$

$$(c)\quad \gamma - 1 \approx 1 - \frac{1}{\gamma} \approx \frac{1}{2}\frac{v^2}{c^2}$$

1-21. How great must the relative speed of two observers be for their time-interval measurements to differ by 1 percent (see Problem 1-20)?

1-22. Supersonic jets achieve maximum speeds of about $3 \times 10^{-6}c$. (*a*) By what percentage would you observe such a jet to be contracted in length? (*b*) During a time of 1 y = 3.16×10^7 s on your clock, how much time would elapse on the pilot's clock? How many minutes are lost by the pilot's clock in 1 year of your time?

1-23. A meterstick moves parallel to its length with speed $v = 0.6c$ relative to you. (*a*) Compute the length of the stick measured by you. (*b*) How long does it take for the stick to pass you? (*c*) Draw a spacetime diagram from the viewpoint of your frame with the front of the meterstick at $x = 0$ when $t = 0$. Show how the answers to (*a*) and (*b*) are obtained from the diagram.

1-24. The proper mean lifetime of π mesons (pions) is 2.6×10^{-8}s. If a beam of such particles has speed $0.9c$, (*a*) What would their mean life be as measured in the laboratory? (*b*) How far would they travel (on the average) before they decay? (*c*) What would your answer be to part (*b*) if you neglected time dilation? (*d*) What is the interval in spacetime between creation of a typical pion and its decay?

1-25. You have been posted to a remote region of space to monitor traffic. Near the end of a quiet shift, a spacecraft streaks past. Your laser-based measuring device reports the spacecraft's length to be 85 m. The identification transponder reports it to be the NCXXB-12, a cargo craft of proper length 100 m. In transmitting your report to headquarters, what speed should you give for this spacecraft?

1-26. A spaceship departs from Earth for the star Alpha Centauri, which is 4 light-years away. The spaceship travels at $0.75c$. How long does it take to get there (*a*) as measured on Earth and (*b*) as measured by a passenger on the spaceship?

1-27. Two spaceships pass each other traveling in opposite directions. A passenger on ship *A*, which she knows to be 100 m long, notes that ship *B* is moving with a speed of $0.92c$ relative to *A* and that the length of *B* is 36 m. What are the lengths of the two spaceships measured by a passenger in *B*?

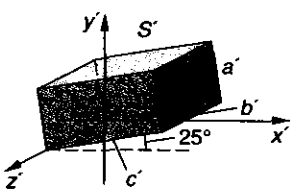

Fig. 1-43 Problem 1-29.

1-28. A meterstick at rest in S' is tilted at an angle of 30° to the x' axis. If S' moves at $\beta = 0.8$, how long is the meterstick as measured in S and what angle does it make with the x axis?

1-29. A rectangular box at rest in S' has sides $a' = 2$ m, $b' = 2$ m, and $c' = 4$ m and is oriented as shown in Figure 1-43. S' moves with $\beta = 0.65$ with respect to the laboratory frame S. (*a*) Compute the volume of the box in S' and in S. (*b*) Draw an accurate diagram of the box as seen by an observer in S.

Section 1-5 The Doppler Effect

1-30. How fast must you be moving toward a red light ($\lambda = 650$ nm) for it to appear yellow ($\lambda = 590$ nm)? green ($\lambda = 525$ nm)? blue ($\lambda = 460$ nm)?

1-31. A distant galaxy is moving away from us at speed 1.85×10^7 m/s. Calculate the fractional redshift $(\lambda' - \lambda_0)/\lambda_0$ of the light from this galaxy.

1-32. The light from a nearby star is observed to be shifted toward the blue by 2 percent, i.e., $f_{obs} = 1.02f_0$. Is the star approaching or receding from Earth? How fast is it moving? (Assume motion is directly toward or away from Earth, so as to avoid superluminal speeds.)

1-33. Stars typically emit the red light of atomic hydrogen with wavelength 656.3 nm (called the H_α spectral line). Compute the wavelength of that light observed at Earth from stars receding directly from us with relative speed $v = 10^{-3}c$, $v = 10^{-2}c$, and $v = 10^{-1}c$.

Section 1-6 The Twin Paradox and Other Surprises

1-34. A friend of yours who is the same age as you travels at $0.999c$ to a star 15 light-years away. She spends 10 years on one of the star's planets and returns at $0.999c$. How long has she been away, (*a*) as measured by you and (*b*) as measured by her?

1-35. You point a laser flashlight at the moon, producing a spot of light on the moon's surface. At what minimum angular speed must you sweep the laser beam in order for the light spot to streak across the moon's surface with speed $v > c$? Why can't you transmit information between research bases on the moon with the flying spot?

1-36. A clock is placed in a satellite that orbits Earth with a period of 108 min. (*a*) By what time interval will this clock differ from an identical clock on Earth after 1 y? (*b*) How much time will have passed on Earth when the two clocks differ by 1.0 s? (Assume special relativity applies and neglect general relativity.)

1-37. Einstein used trains for a number of relativity thought experiments, since they were the fastest objects commonly recognized in those days. Let's consider a train moving at $0.65c$ along a straight track at night. Its headlight produces a beam with an angular spread of 60° according to the engineer. If you are standing alongside the track (rails are 1.5 m apart), how far from you is the train when its approaching headlight suddenly disappears?

Level II

1-38. In 1971 four portable atomic clocks were flown around the world in jet aircraft, two eastbound and two westbound, to test the time dilation predictions of relativity.[26] (*a*) If the westbound plane flew at an average speed of 1500 km/h relative to the surface, how long would it have had to fly for the clock on board to lose 1 s relative to the reference clock on the ground at the U.S. Naval Observatory? (*b*) In the actual experiment the planes circumflew Earth once and the observed discrepancy of the clocks was 273 ns. What was the plane's average speed?

1-39. Show that the spacetime interval Δs is invariant under the Lorentz transformation, i.e., show that

$$(c\,\Delta t)^2 - (\Delta x)^2 = (c\,\Delta t')^2 - (\Delta x')^2$$

1-40. A friend of yours who is the same age as you travels to the star Alpha Centauri, which is $4\ c \cdot y$ away, and returns immediately. He claims that the entire trip took just 6 years. (*a*) How fast did he travel? (*b*) How old are you when he returns? (*c*) Draw a spacetime diagram that verifies your answer to (*a*) and (*b*).

1-41. A clock is placed in a satellite that orbits Earth with a period of 90 min. By what time interval will this clock differ from an identical clock on Earth after 1 year? (Assume that special relativity applies.)

1-42. In frame S, event B occurs 2 μs after event A and at $\Delta x = 1.5$ km from event A. (*a*) How fast must an observer be moving along the $+x$ axis so that events A and B occur simultaneously? (*b*) Is it possible for event B to precede event A for some observer? (*c*) Draw a spacetime diagram that illustrates your answers to (*a*) and (*b*). (*d*) Compute the spacetime interval and proper distance between the events.

1-43. A burst of π^+ mesons travels down an evacuated beam tube at Fermilab moving at $\beta = 0.92$ with respect to the laboratory. (*a*) Compute γ for this group of pions. (*b*) The proper mean lifetime of pions is 2.6×10^{-8} s. What mean lifetime is measured in the lab? (*c*) If the burst contained 50,000 pions, how many remain after the group has traveled 50 m down the beam tube? (*d*) What would be the answer to (*c*) ignoring time dilation?

1-44. H. A. Lorentz suggested 15 years before Einstein's 1905 paper that the null effect of the Michelson-Morley experiment could be accounted for by a contraction of that arm of the interferometer lying parallel to Earth's motion through the ether to a length

$L = L_p(1 - v^2/c^2)^{-1/2}$. He thought of this, incorrectly, as an actual shrinking of matter. By about how many atomic diameters would the material in the parallel arm of the interferometer have had to shrink in order to account for the absence of the expected shift of 0.4 of a fringe width? (Assume the diameter of atoms to be about 10^{-10} m.)

1-45. Observers in reference frame S see an explosion located at $x_1 = 480$ m. A second explosion occurs 5 μs later at $x_2 = 1200$ m. In reference frame S', which is moving along the $+x$ axis at speed v, the explosions occur at the same point in space. (*a*) Draw a spacetime diagram describing this situation. (*b*) Determine v from the diagram. (*c*) Calibrate the ct' axis and determine the separation in time in μs between the two explosions as measured in S'. (*d*) Verify your results by calculation.

1-46. Two spaceships, each 100 m long when measured at rest, travel toward each other with speeds of $0.85c$ relative to Earth. (*a*) How long is each ship as measured by someone on Earth? (*b*) How fast is each ship traveling as measured by an observer on the other? (*c*) How long is one ship when measured by an observer on the other? (*d*) At time $t = 0$ on Earth, the fronts of the ships are together as they just begin to pass each other. At what time on Earth are their ends together? (*e*) Sketch accurately scaled diagrams in the frame of one of the ships showing the passing of the other ship.

1-47. If v is much less than c, the Doppler frequency shift is approximately given by $\Delta f/f_0 = \pm\beta$, both classically and relativistically. A radar transmitter-receiver bounces a signal off an aircraft and observes a fractional increase in the frequency of $\Delta f/f_0 = 8 \times 10^{-7}$. What is the speed of the aircraft? (Assume the aircraft to be moving directly toward the transmitter.)

1-48. Derive Equation 1-38 for the frequency received by an observer moving with speed v toward a stationary source of electromagnetic waves.

1-49. Frames S and S' are moving relative to each other along the x and x' axes. They set their clocks to $t = t' = 0$ when their origins coincide. In frame S, event 1 occurs at $x_1 = 1\ c \cdot \text{y}$ and $t_1 = 1$ y and event 2 occurs at $x_2 = 2.0\ c \cdot \text{y}$ and $t_2 = 0.5$ y. These events occur simultaneously in frame S'. (*a*) Find the magnitude and direction of the velocity of S' relative to S. (*b*) At what time do both of these events occur as measured in S'? (*c*) Compute the spacetime interval Δs between the events. (*d*) Is the interval spacelike, timelike, or lightlike? (*e*) What is the proper distance L_p between the events?

1-50. Do Problem 1-49 parts (*a*) and (*b*) using a spacetime diagram.

1-51. An observer in frame S standing at the origin observes two flashes of colored light separated spatially by $\Delta x = 2400$ m. A blue flash occurs first, followed by a red flash 5 μs later. An observer in S' moving along the x axis at speed v relative to S also observes the flashes 5 μs apart and with a separation of 2400 m, but the red flash is observed first. Find the magnitude and direction of v.

1-52. A cosmic ray proton streaks through the lab with velocity $0.85c$ at an angle of 50° with the $+x$ direction (in the xy plane of the lab). Compute the magnitude and direction of the proton's velocity when viewed from frame S' moving with $\beta = 0.72$.

Level III

1-53. A meterstick is parallel to the x axis in S and is moving in the $+y$ direction at constant speed v_y. Use a spacetime diagram from the viewpoint of S to show that the meterstick will appear tilted at an angle θ' with respect to the x' axis of S' moving in the $+x$ direction at $\beta = 0.65$. Compute the angle θ' measured in S'.

1-54. The equation for the spherical wave front of a light pulse that begins at the origin at time $t = 0$ is $x^2 + y^2 + z^2 - (ct)^2 = 0$. Using the Lorentz transformation, show that such a light pulse also has a spherical wave front in S' by showing that $x'^2 + y'^2 + z'^2 - (ct')^2 = 0$ in S'.

1-55. An interesting paradox has been suggested by R. Shaw[27] that goes like this. A very thin steel plate with a circular hole 1 m in diameter centered on the y axis lies parallel to

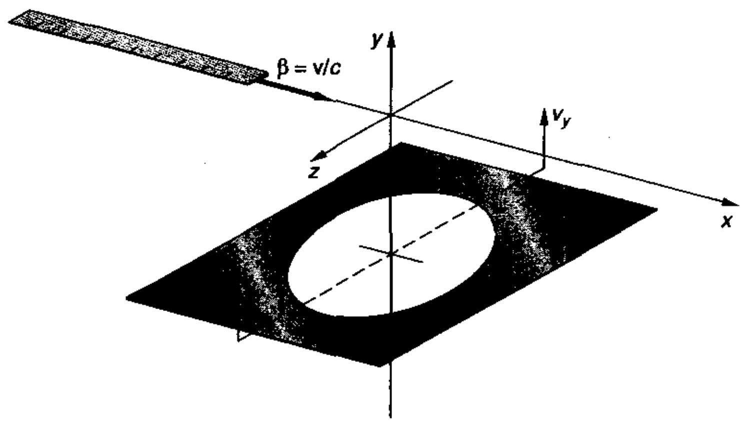

Fig. 1-44 Problem 1-55.

the xz plane in frame S and moves in the $+y$ direction at constant speed v_y as illustrated in Figure 1-44. A meterstick lying on the x axis moves in the $+x$ direction with $\beta = v/c$. The steel plate arrives at the $y = 0$ plane at the same instant that the center of the meterstick reaches the origin of S. Since the meterstick is observed by observers in S to be contracted, it passes through the 1-m hole in the plate with no problem. A paradox appears to arise when one considers that an observer in S', the rest system of the meterstick, measures the diameter of the hole in the plate to be contracted in the x dimension and, hence, becomes too small to pass the meterstick, resulting in a collision. Resolve the paradox. Will there be a collision?

1-56. Two events in S are separated by a distance $D = x_2 - x_1$ and a time $T = t_2 - t_1$. (*a*) Use the Lorentz transformation to show that in frame S', which is moving with speed v relative to S, the time separation is $t'_2 - t'_1 = \gamma(T - vD/c^2)$. (*b*) Show that the events can be simultaneous in frame S' only if D is greater than cT. (*c*) If one of the events is the *cause* of the other, the separation D must be less than cT since D/c is the smallest time that a signal can take to travel from x_1 to x_2 in frame S. Show that if D is less that cT, t'_2 is greater than t'_1 in all reference frames. (*d*) Suppose that a signal could be sent with speed $c' > c$ so that in frame S the cause precedes the effect by the time $T = D/c'$. Show that there is then a reference frame moving with speed v less than c in which the effect precedes the cause.

1-57. Two observers agree to test time dilation. They use identical clocks and one observer in frame S' moves with speed $v = 0.6c$ relative to the other observer in frame S. When their origins coincide, they start their clocks. They agree to send a signal when their clocks read 60 min and to send a confirmation signal when each receives the other's signal. (*a*) When does the observer in S receive the first signal from the observer in S'? (*b*) When does he receive the confirmation signal? (*c*) Make a table showing the times in S when the observer sent the first signal, received the first signal, and received the confirmation signal. How does this table compare with one constructed by the observer in S'?

1-58. The compact disc in a CD-ROM drive rotates with angular speed ω. There is a clock at the center of the disc and one at a distance r from the center. In an inertial reference frame, the clock at distance r is moving with speed $u = r\omega$. Show that from time dilation in special relativity, time intervals Δt_0 for the clock at rest and Δt_r for the moving clock are related by

$$\frac{\Delta t_r - \Delta t_0}{\Delta t_0} \approx \frac{r^2\omega^2}{2c^2} \quad \text{if} \quad r\omega << c$$

1-59. Two rockets A and B leave a space station with velocity vectors $\mathbf{v}_A$ and $\mathbf{v}_B$, relative to the station frame S, perpendicular to one another. (*a*) Determine the velocity of A relative to B, $\mathbf{v}_{BA}$. (*b*) Determine the velocity of B relative to A, $\mathbf{v}_{AB}$. (*c*) Explain why $\mathbf{v}_{AB}$ and $\mathbf{v}_{BA}$ do not point in opposite directions.

1-60. Suppose a system S consisting of a cubic lattice of metersticks and synchronized clocks, e.g., the eight clocks closest to you in Figure 1-14, moves from left to right (the $+x$ direction) at high speed. The metersticks parallel to the x direction are, of course, contracted and the cube would be *measured* by an observer in a system S' to be foreshortened in that direction. However, recalling that your eye constructs images from light waves which reach it simultaneously, not those leaving the source simultaneously, sketch what your eye would *see* in this case. Scale contractions and show any angles accurately. (Assume the moving cube to be farther than 10 m from your eye.)

1-61. Figure 1-12*b* (in the More section about the Michelson-Morley experiment) shows an eclipsing binary. Suppose the period of the motion is T and the binary is a distance L from Earth, where L is sufficiently large so that points A and B in Figure 1-12*b* are a half-orbit apart. Consider the motion of one of the stars and (*a*) show that the star would appear to move from A to B in time $T/2 + 2Lv/(c^2 - v^2)$ and from B to A in time $T/2 - 2Lv/(c^2 - v^2)$, assuming classical velocity addition applies to light, i.e., that emission theories of light were correct. (*b*) What rotational period would cause the star to appear to be at both A and B simultaneously?

1-62. Show that if a particle moves at an angle θ with respect to the x axis with speed u in system S, it moves at an angle θ' with the x' axis in S' given by

$$\tan \theta' = \frac{\sin \theta}{\gamma (\cos \theta - v/u)}$$