the total kinetic energy cannot be converted into rest energy because of conservation of momentum. However, in the zero-momentum reference frame in which the two initial protons are moving toward each other with equal speed u, the total kinetic energy can be converted into rest energy. (*a*) Find the speed of each proton u such that the total kinetic energy in the zero-momentum frame is $2mc^2$. (*b*) Transform to the laboratory's frame in which one proton is at rest, and find the speed u' of the other proton. (*c*) Show that the kinetic energy of the moving proton in the laboratory's frame is $E_k = 6mc^2$.

2-48. In a simple thought experiment, Einstein showed that there is mass associated with electromagnetic radiation. Consider a box of length L and mass M resting on a frictionless surface. At the left wall of the box is a light source that emits radiation of energy E, which is absorbed at the right wall of the box. According to classical electromagnetic theory, this radiation carries momentum of magnitude $p = E/c$. (*a*) Find the recoil velocity of the box such that momentum is conserved when the light is emitted. (Since p is small and M is large, you may use classical [illegible] (*b*) When the light is absorbed at the right wall of the box, the box stops, so the total momentum remains zero. If we neglect the very small velocity of the box, the time it takes for the radiation to travel across the box is $\Delta t = L/c$. Find the distance moved by the box in this time. (*c*) Show that if the center of mass of the system is to remain at the same place, the radiation must carry mass $m = E/c^2$.

2-49. A pion spontaneously decays into a muon and an antineutrino according to (among other processes) $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$. Current experimental evidence indicates that the mass m of the $\bar{\nu}_\mu$ is no greater that about 190 keV and may, in fact, be zero. Assuming that the pion decays at rest in the laboratory, compute the energies and momenta of the muon and muon antineutrino (*a*) if the mass of the antineutrino is zero and (*b*) if its mass is 190 keV. The mass of the pion is 139.56755 MeV/c^2 and the mass of the muon is 105.65839 MeV/c^2.

2-50. Use Equation 2-47 to obtain the gravitational redshift in terms of the wavelength λ. Use that result to determine the shift in wavelength of light emitted by a white dwarf star at 720.00 nm. Assume the white dwarf has the same mass as the sun (1.99×10^{30} kg), but a radius equal to only 1 percent of the solar radius $R_\odot$. ($R_\odot = 6.96 \times 10^8$ m.)

2-51. For a particle moving in the xy plane of S, show that the y' component of the acceleration is given by

$$a_y' = \frac{a_y}{\gamma^2(1 - u_x v/c^2)} + \frac{a_x u_y v/c^2}{\gamma^2(1 - u_x v/c^2)^3}$$

2-52. Consider an object of mass m at rest in S acted upon by a force **F** with components F_x and F_y. System S' moves with instantaneous velocity **v** in the $+x$ direction. Defining the force with Equation 2-8 and using the Lorentz velocity transformation, show that (*a*) $F_x' = F_x$ and (*b*) $F_y' = F_y/\gamma$. (*Hint*: See Problem 2-51.)

2-53. An unstable particle of mass M decays into two identical particles, each of mass m. Obtain an expression for the velocities of the two decay particles in the lab frame (*a*) if M is at rest in the lab and (*b*) if M has total energy $4mc^2$ when it decays and the decay particles move along the direction of M.

Chapter 3 Quantization of Charge, Light, and Energy

3-1 Quantization of Electric Charge
3-2 Blackbody Radiation
3-3 The Photoelectric Effect
3-4 X Rays and the Compton Effect

The idea that all matter is composed of tiny particles, or atoms, dates back to the speculations of the Greek philosopher Democritus[1] and his teacher Leucippus about 450 B.C. However, there was little attempt to correlate such speculations with observations of the physical world until the seventeenth century. Pierre Gassendi, in the middle of the seventeenth century, and Robert Hooke, somewhat later, attempted to explain states of matter and the transitions between them with a model of tiny, indestructible solid objects flying in all directions. But it was Avogadro's hypothesis, advanced in 1811, that all gases at a given temperature contain the same number of molecules per unit volume, which led to great success in the interpretation of chemical reactions and to development of the kinetic theory, around 1900. It enabled quantitative understanding of many bulk properties of matter and led to general (though not unanimous) acceptance of the molecular theory of matter. Thus, matter is not continuous, as it appears, but is *quantized* (i.e., discrete) on the microscopic scale. It was understood that the small size of the atom prevented the discreteness of matter from being readily observable.

In this chapter, we shall study how three additional great quantization discoveries were made: (1) electric charge, (2) light energy, and (3) energy of oscillating mechanical systems. The quantization of electric charge was not particularly surprising to scientists in 1900; it was quite analogous to the quantization of mass. However, the quantizations of light energy and mechanical energy, which are of central importance in modern physics, were revolutionary ideas.

3-1 Quantization of Electric Charge

Early Measurements of e and e/m

The first estimates of the order of magnitude of the electric charges found in atoms were obtained from Faraday's law. The work of Michael Faraday (1791–1867) in the early to mid-1800s stands out even today for its vision, experimental ingenuity, and thoroughness. The story of this self-educated blacksmith's son who rose from errand boy and bookbinder's apprentice to become the director of the distinguished Royal Institution of London and the foremost experimental investigator of his time is a fascinating one. One aspect of his work concerned the study of the conduction of electricity in liquids. His results and his subsequent statement of the law of electrolysis

(1833) were of great importance for the evidence they gave of the electrical nature of atomic forces. The phenomenon still has interest in that it provides the basis for the study of the field of electrochemistry.

In his experiments, Faraday passed a direct current (dc) through weakly conducting solutions and observed the subsequent liberation of the components of the solution at the electrodes. Quantitatively, Faraday discovered that the same quantity of electricity, F, called the faraday and equal to about 96,500 C, always decomposes 1 gram-ionic weight of monovalent ions. For example, if 96,500 C pass through a solution of NaCl, 23 g of Na appear at the cathode and 35.5 g of Cl at the anode. For ions of valence 2, such as Cu or SO_4, it takes 2 faradays to decompose 1 gram-ionic weight. Since a gram-ionic weight is just Avogadro's number of ions N_A, it is reasonable to assume that each monovalent ion contains the same charge, e, and

$$F = N_A e \qquad \text{3-1}$$

Equation 3-1 is called Faraday's law of electrolysis. Since the faraday could be measured quite accurately, N_A or e could be determined if the other were known. Faraday was aware of this but could not determine either quantity. Even so, it seemed logical to expect that electric charge, like matter, was not continuous but consisted of particles of some discrete minimum charge. In 1874, G. J. Stoney[2] suggested that the apparent minimum amount of charge be called an *electron e* and used an estimate of N_A from kinetic theory to compute the value of e to be about 10^{-20} C. Based on accumulating experimental evidence, Helmholtz[3] pointed out in 1880 that it is apparently impossible to obtain a subunit of this charge. The first discrete measurement of this smallest unit of charge was made by Townsend in 1897, by an ingenious method that was the forerunner of the famous Millikan oil-drop experiment.

Pieter Zeeman, in 1896, obtained the first evidence for the existence of atomic particles with a specific charge-to-mass ratio by looking at the light emitted by atoms placed in a strong magnetic field. When viewed through a spectroscope without the magnetic field, this light appears as a discrete set of lines called *spectral lines*. According to classical electromagnetic theory, a charge oscillating in simple harmonic motion will emit electromagnetic radiation at the frequency of oscillation. If the moving charge is placed in a magnetic field, there will be an additional force on the charge which, to a first approximation, merely changes the frequency of oscillation. The frequency is either slightly increased, slightly decreased, or unchanged from its original value, depending on the orientation of the line of oscillation relative to the direction of the magnetic field. Then, according to classical theory, if a spectral line from atoms is due to the oscillation of charged particles in the atoms, that line will be split into three very closely spaced lines of slightly different frequencies when the atom is placed in a magnetic field. (This phenomenon is called the *Zeeman effect* and will be discussed further in Chapter 7.) The magnitude of the frequency difference depends on the charge-to-mass ratio q/m of the oscillating particle. Zeeman measured such a splitting and calculated q/m to be about 1.6×10^{11} C/kg, which compares favorably with the presently accepted value for the electron of 1.759×10^{11} C/kg. From the polarization of the spectral lines, Zeeman concluded that the oscillating particles were negatively charged.

Discovery of the Electron: J. J. Thomson's Experiment

Many studies of electrical discharges in gases were done in the late nineteenth century. It was found that the ions responsible for gaseous conduction carried the same charge as did those in electrolysis. The year following Zeeman's work, J. J. Thomson[4]

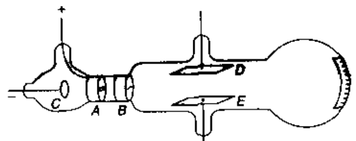


Fig. 3-1 J. J. Thomson's tube for measuring e/m. Electrons from the cathode C pass through the slits at A and B and strike a phosphorescent screen. The beam can be deflected by an electric field between the plates D and E or by a magnetic field (not shown) whose direction is perpendicular to the electric field between D and E. From measurements of the deflections measured on a scale on the tube at the screen, e/m can be determined. [*From J. J. Thomson, "Cathode Rays,"* Philosophical Magazine *(5), 44, 293 (1897).*]

measured the q/m value for the so-called cathode rays and pointed out that if their charge was Faraday's minimum charge e as determined by Stoney, then their mass was only a small fraction of the mass of a hydrogen atom. He had, in fact, discovered the *electron*. The cathode-ray tube used by J. J. Thomson (the apparatus is shown in Figure 3-1) is typical of those used by his contemporaries. It was the forerunner of the television picture tube, the oscilloscope, and a host of video display terminals on everything from word processors and personal computers to video games and radar screens. At sufficiently low pressure, the space near the cathode becomes dark, and as the pressure is lowered, this dark space extends across the tube until it finally reaches the glass, which then glows as a result of the energy absorbed from the cathode rays. When apertures are placed at A and B, the glow is limited to a well-defined spot on the glass. This spot can then be deflected by electrostatic or magnetic fields.[5] In 1895, J. Perrin had collected these "cathode rays" on an electrometer and found them to carry a negative electric charge. That direct measurement of the charge-to-mass ratio e/m of electrons by J. J. Thomson in 1897 can be justly considered the beginning of our understanding of atomic structure.

Measurement of e/m When a uniform magnetic field of strength B is established perpendicular to the direction of motion of charged particles, the particles move in a circular path. The radius R of the path can be obtained from Newton's second law, by setting the magnetic force quB equal to the mass m times the centripetal acceleration u^2/R.

$$quB = \frac{mu^2}{R} \quad \text{or} \quad R = \frac{mu}{qB} \qquad \text{3-2}$$

Present-day particle physicists routinely use the modern equivalent of Thomson's experiment to measure the momenta of elementary particles. Equation 3-2 is the nonrelativistic version of Equation 2-37, i.e., with $\gamma = 1$; Thomson, who didn't know about relativity at the time, of course, was fortunate in that the speeds of his "cathode rays" (electrons) were decidedly nonrelativistic; that is, the electron speeds u were much smaller that the speed of light c, with $u/c \ll 0.2$. (See Figure 2-2.) In his first measurement, Thomson determined the velocity from measurements of the total charge and the temperature change occurring when the beam struck an insulated collector. For N particles, the total charge is $Q = Ne$, while the temperature rise is

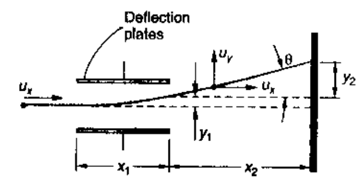


Fig. 3-2 Deflection of the electron beam in Thomson's apparatus. The deflection plates are D and E in Figure 3-1. Deflection of the beam is shown with the magnetic field off and the top plate positive. Thomson used up to about 200 V between D and E. A magnetic field was applied perpendicular to the plane of the diagram directed into the page to bend the beam back down to its undeflected position.

proportional to the energy loss $W = N(\frac{1}{2}mu^2)$. Eliminating N and u from these equations, we obtain

$$\frac{e}{m} = \frac{2W}{B^2R^2Q} \qquad \text{3-3}$$

In his second measurement, which came to be known as the *J. J. Thomson experiment*, he adjusted perpendicular B and $\mathcal{E}$ fields so that the particles were *undeflected*. This allowed him to determine the speed by equating the magnitudes of the magnetic and electric forces:

$$quB = q\mathcal{E} \quad \text{or} \quad u = \frac{\mathcal{E}}{B} \qquad \text{3-4}$$

He then turned off the B field and measured the deflection of the particles on the screen. This deflection is made up of two parts (see Figure 3-2). While the particles are between the plates they undergo a vertical deflection y_1, given by

J. J. Thomson in his laboratory. He is facing the screen end of an e/m tube; an older cathode-ray tube is visible in front of his left shoulder. [*Courtesy of Cavendish Laboratory.*]

$$y_1 = \frac{1}{2}at_1^2 = \frac{1}{2}\frac{e\mathcal{E}}{m}\left(\frac{x_1}{u_x}\right)^2 \qquad \text{3-5}$$

where x_1 is the horizontal distance traveled. After they leave the plates they undergo additional deflection y_2, given by

$$y_2 = u_y t_2 = at_1\left(\frac{x_2}{u_x}\right) = \frac{e\mathcal{E}}{m}\left(\frac{x_1}{u_x}\right)\left(\frac{x_2}{u_x}\right) = \frac{e\mathcal{E}}{m}\frac{x_1x_2}{u_x^2} \qquad \text{3-6}$$

where x_2 is the horizontal distance traveled beyond the deflection plates. The total deflection $(y_1 + y_2)$ is proportional to e/m. Combining Equations 3-4, 3-5, and 3-6 and noting that $u = u_x$ for the undeflected beam, we have

$$y_1 + y_2 = \frac{e}{m}\left(\frac{B^2}{\mathcal{E}}\right)\left(\frac{x_1^2}{2} + x_1x_2\right) \qquad \text{3-7}$$

Note the "direct" character of the measurement. Thomson needed only a voltmeter, an ammeter, and a measuring rod to determine e/m. It is also interesting to note that his original values of e/m from his first method, about 2×10^{11} C/kg, were closer to the present value of 1.76×10^{11} C/kg than those from his second method, 0.7×10^{11} C/kg. The inaccuracy of the results obtained from the second method was due to his having neglected the magnetic field outside the region of the deflecting plates. Despite this inaccuracy, however, the second method had the advantage of reproducibility and is considered the superior experiment.

Thomson's technique of controlling the direction of the electron beam with "crossed" electric and magnetic fields was subsequently applied in the development of cathode-ray tubes used in oscilloscopes and the picture tubes of television receivers.

Thomson repeated the experiment with different gases in the tube and different metals for cathodes and always obtained the same value of e/m within his experimental accuracy, thus showing that these particles were common to all metals. The agreement of these results with Zeeman's led to the unmistakable conclusion that these particles—called *corpuscles* by Thomson and later called *electrons* by Lorentz—having one unit of negative charge e and about 2000 times less mass than the lightest known atom, were constituent in all atoms.

Questions

1. One advantage of Thomson's evidence over others' (such as Faraday's or Zeeman's) was its directness. Another was that it was not just a statistical inference. How is it shown in the Thomson experiment that e/m is the same for a large number of particles?
2. Thomson noted that his values for e/m were about 2000 times larger than those for the lightest known ion, that of hydrogen. Could he distinguish from his data between the possibility that this was a result of the electron having either a greater charge or smaller mass than the hydrogen ion?

The Mass Spectrometer One of several devices currently used to measure the charge-to-mass ratio q/m of charged atoms and molecules is the mass spectrometer. The mass spectrometer is used to find the charge-to-mass ratio of ions of known charge by measuring the radius of their circular orbits in a uniform magnetic field.

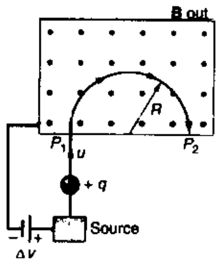


Fig. 3-3 Schematic drawing of a mass spectrometer. Ions from an ion source are accelerated through a potential difference ΔV and enter a uniform magnetic field. The magnetic field is directed out of the plane of the page as indicated by the dots. The ions are bent into circular arcs and strike a photographic plate or exit through an aperture to an ion detector at P_2. The radius of the circle is proportional to the mass of the ion.

Equation 3-2 gives the radius of R for the circular orbit of a particle of mass m and charge q moving with speed u in a magnetic field B that is perpendicular to the velocity of the particle. Figure 3-3 shows a simple schematic drawing of a mass spectrometer. Ions from an ion source are accelerated by an electric field and enter a uniform magnetic field produced by an electromagnet. If the ions start from rest and move through a potential drop ΔV, their kinetic energy when they enter the magnetic field equals their loss in potential energy, $q\Delta V$:

$$\frac{1}{2}mu^2 = q\Delta V \qquad \text{3-8}$$

The ions move in a semicircle of radius R given by Equation 3-2 and strike a photographic plate or exit through a narrow aperture to an ion detector at point P_2, a distance $2R$ from the point where they enter the magnet. The speed u can be eliminated from Equations 3-2 and 3-8 to find q/m in terms of ΔV, B, and R. The result is

$$\frac{q}{m} = \frac{2\Delta V}{B^2R^2} \qquad \text{3-9}$$

In the original mass spectrometer, invented by F. W. Aston (who was a student of Thomson's) in 1919, mass differences could be measured to a precision of about 1 part in 10,000. The precision has been improved by introducing a velocity selector between the ion source and the magnet, which makes it possible to limit the range of velocities of the incoming ions and to determine the velocities of the ions more accurately. Today, values of atomic and molecular masses are typically measured with mass spectrometers to precisions of better than 1 part in 10^9. The method normally used is to measure the differences in R between standard masses and the ions of interest, as illustrated in the following example.

EXAMPLE 3-1 Mass Spectrometer Measurements A ^{58}Ni ion of charge $+e$ and mass 9.62×10^{-26} kg is accelerated through a potential difference of 3 kV and deflected in a magnetic field of 0.12 T. (*a*) Find the radius of curvature of the orbit of the ion. (*b*) Find the difference in the radii for curvature of ^{58}Ni ions and ^{60}Ni ions. (Assume that the mass ratio is 58/60.)

Solution

1. For question (*a*), the radius of the ion's orbit is given by rearranging Equation 3-9:

$$R^2 = \frac{2m\Delta V}{qB^2}$$

2. Noting that in this case $q = +e$ and substituting the values yield:

$$R^2 = \frac{(2)(9.62 \times 10^{-26}\,\text{kg})(3000\,\text{V})}{(1.60 \times 10^{-19}\,\text{C})(0.12\,\text{T})^2}$$
$$= 0.251\ \text{m}^2$$
$$R = \sqrt{0.251\ \text{m}^2} = 0.501\ \text{m}$$

3. For question (*b*), note that according to Equation 3-9 an ion's orbit radius is proportional to the square root of its mass. For identical values of q, V, and B, if R_1 is the radius for the ^{58}Ni ion and R_2 is the radius for the ^{60}Ni ion, their ratio is:

$$\frac{R_2}{R_1} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{60}{58}} = 1.017$$

4. Substituting the value for the ^{58}Ni radius computed above gives:

$$R_2 = 1.017\,R_1 = (1.017)(0.501\ \text{m}) = 0.510\ \text{m}$$

5. The difference ΔR in the radii is then:

$$\Delta R = R_2 - R_1 = 0.510\ \text{m} - 0.501\ \text{m} = 0.009\ \text{m} = 9\ \text{mm}$$

Measuring the Electric Charge: Millikan's Experiment

The fact that Thomson's *e/m* measurements always yielded the same results regardless of the materials used for the cathodes or the kind of gas in the tube was a persuasive argument that the electrons all carried one unit e of negative electric charge. Thomson initiated a series of experiments to determine the value of e. The first of these experiments, which turned out to be very difficult to do with high precision, was carried out by his student J. S. E. Townsend. The idea was simple: a small (but visible) cloud of identical water droplets, each carrying a single charge e, was observed to drift downward in response to the gravitational force. The total charge on the cloud $Q = Ne$ was measured, as was the mass of the cloud and the radius of a single drop. Finding the radius allowed calculation of N, the total number of drops in the cloud, and, hence, the value of e.

The accuracy of Thomson's method was limited by the uncertain rate of evaporation of the cloud, and the assumption that each droplet contained a single charge could not be verified. R. A. Millikan tried to eliminate the evaporation problem by using a field strong enough to hold the top surface of the cloud stationary so that he could observe the rate of evaporation, and correct for it. That, too, turned out to be very difficult, but then he made a discovery of enormous importance, one that allowed him to measure directly the charge of a single electron! Millikan described his discovery in the following words:

> It was not found possible to balance the cloud as had been originally planned, but it was found possible to do something much better: namely, to hold individual charged drops suspended by the field for periods varying from 30 to 60 seconds. I have never actually timed drops which lasted more than 45 seconds,

although I have several times observed drops which in my judgement lasted considerably longer than this. The drops which it was found possible to balance by an electric field always carried multiple charges, and the difficulty experienced in balancing such drops was less than had been anticipated.[6]

The discovery that he could see individual droplets and that droplets suspended in a vertical electric field sometimes suddenly moved upward or downward, evidently because they had picked up a positive or negative ion, led to the possibility of observing the charge of a single ion. In 1909, Millikan began a series of experiments which not only showed that charges occurred in integer multiples of an elementary unit e, but measured the value of e to about 1 part in 1000. To eliminate evaporation, he used oil drops sprayed into dry air between the plates of a capacitor. These drops were already charged by the spraying process, i.e., friction in the spray nozzle, and during the course of observation they picked up or lost additional charges. By switching the field between the plates, a drop could be moved up or down and observed for several hours. When the charge on a drop changed, the velocity of the drop with the field "on" changed. Assuming only that the terminal velocity of the drop was proportional to the force acting on it (this assumption was carefully checked experimentally), Millikan's experiment gave conclusive evidence that charges always occur in multiples of a fundamental unit e, whose value he determined to be 1.601×10^{-19} C. The currently accepted value is, to three decimal places, 1.602×10^{-19} C. The expanded discussion of Millikan's experiment on the home page includes the value to eight places.

More

Millikan's Oil-Drop Experiment,[7] one of the few truly crucial experiments in physics, is also remarkable for its simple directness and its excellent precision. The discussion of Millikan's experiment on our home page includes a portion of the data on drop number 6, one of several thousand oil drops he used in determining the value of the electron's charge. See also Equations 3-10 through 3-18 and Figures 3-4 and 3-5 on the home page: www.whfreeman.com/modphysics4e

3-2 Blackbody Radiation

The first clue to the quantum nature of radiation came from the study of thermal radiation emitted by opaque bodies. When radiation falls on an opaque body, part of it is reflected and the rest absorbed. Light-colored bodies reflect most of the visible radiation incident on them, whereas dark bodies absorb most of it. The absorption part of the process can be described briefly as follows. The radiation absorbed by the body increases the kinetic energy of the constituent atoms which oscillate about their equilibrium positions. Recalling that the average translational kinetic energy of the atoms determines the temperature of the body, the absorbed energy causes the temperature to rise. However, the atoms contain charges (the electrons) and they are accelerated by the oscillations. Consequently, as required by electromagnetic theory, the atoms emit electromagnetic radiation which reduces the kinetic energy of the oscillations and tends to reduce the temperature. When the rate of absorption equals that of emission, the temperature is constant and we say that the body is in thermal equilibrium with its surroundings. A good absorber of radiation is therefore also a good emitter.

The electromagnetic radiation emitted under these circumstances is called *thermal radiation*. At ordinary temperatures (below about 600°C) the thermal radiation emitted by a body is not visible; most of the energy is concentrated in wavelengths much longer than those of visible light. As a body is heated, the quantity of thermal radiation emitted increases, and the energy radiated extends to shorter and shorter wavelengths. At about 600–700°C there is enough energy in the visible spectrum so that the body glows and becomes a dull red, and at higher temperatures it becomes bright-red or even "white-hot."

A body that absorbs *all* radiation incident on it is called an *ideal blackbody*. In 1879 Josef Stefan found an empirical relation between the power per unit area radiated by a blackbody and the temperature:

$$R = \sigma T^4 \qquad \text{3-19}$$

where R is the power radiated per unit area, T is the absolute temperature, and $\sigma = 5.6703 \times 10^{-8}$ W/m^{2}K^4 is a constant called Stefan's constant. This result was also derived on the basis of classical thermodynamics by Ludwig Boltzmann about five years later, and Equation 3-19 is now called the Stefan-Boltzmann law. Note that the power per unit area radiated by a blackbody depends only on the temperature, and not on any other characteristic of the object, such as its color or the material of which it is composed. Note, too, that R tells us the *rate* at which energy is emitted by the object. For example, doubling the absolute temperature of an object increases the energy flow out of the object by a factor of $2^4 = 16$. An object at room temperature (300 K) will double the rate at which it radiates energy as a result of a temperature increase of only 57°C. Thus, the Stefan-Boltzmann law has an enormous effect on the establishment of thermal equilibrium in physical systems.

Objects that are not blackbodies radiate energy per unit area at a rate less than that of a blackbody at the same temperature. The rate does depend on properties in addition to the temperature, such as color and composition of the surface. The effects of those dependencies are combined into a factor called the *emissivity* ϵ which multiplies the right side of Equation 3-19. The values of ϵ, which is itself temperature dependent, are always less than unity.

Like the total radiated power R, the *spectral distribution* of the radiation emitted by a blackbody is found empirically to depend *only* on the absolute temperature T. The spectral distribution is determined experimentally as illustrated schematically in Figure 3-6. Let $R(\lambda)d\lambda$ be the power emitted per unit area with wavelength between

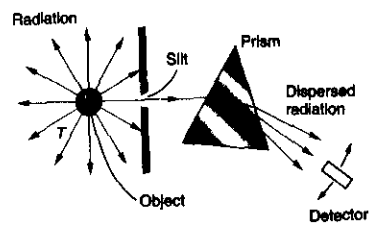


Fig. 3-6 Radiation emitted by the object at temperature T that passes through the slit is dispersed according to its wavelength. The prism shown would be an appropriate device for that part of the emitted radiation in the visible region. In other spectral regions other types of devices or wavelength-sensitive detectors would be used.

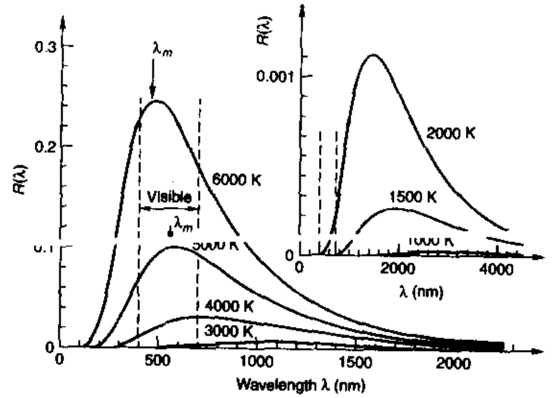


Fig. 3-7 Spectral distribution function $R(\lambda)$ measured at different temperatures. The $R(\lambda)$ axis is in arbitrary units for comparison only. Notice the range in λ of the visible spectrum. The sun emits radiation very close to that of a blackbody at 5800 K. λ_m is indicated for the 5000-K and 6000-K curves.

λ and $\lambda + d\lambda$. Figure 3-7 shows the measured spectral distribution function $R(\lambda)$ versus λ for several values of T ranging from 1000 K to 6000 K.

The $R(\lambda)$ versus λ curves in Figure 3-7 are quite remarkable in several respects. One is that the wavelength at which the distribution is maximum varies inversely with the temperature:

$$\lambda_m \propto \frac{1}{T}$$

or

$$\lambda_m T = \text{constant} = 2.898 \times 10^{-3}\ \text{mK} \qquad \textbf{3-20}$$

This result is known as Wien's displacement law. It was obtained by Wilhelm Wien in 1893. Examples 3-2 and 3-3 illustrate its application.

EXAMPLE 3-2 How Big Is a Star? Measurement of the wavelength at which the spectral distribution $R(\lambda)$ from a certain star is maximum indicates that the star's surface temperature is 3000 K. If that star is also found to radiate 100 times the power radiated by the sun $P_\odot$, how big is the star? (The symbol $\odot$ = sun.) The sun's surface temperature is found to be 5800 K.

Solution

Assuming the sun and the star both radiate as blackbodies (astronomers nearly always make this assumption, based on, among other things, the fact that the solar spectrum is nearly that of a perfect blackbody), their surface temperatures have been determined from Equation 3-20 to be 5800 K and 3000 K, respectively. Measurement also indicates that $P_{\text{star}} = 100\, P_\odot$. Thus, from Equation 3-19 we have that

$$R_{\text{star}} = \frac{P_{\text{star}}}{(\text{area})_{\text{star}}} = \frac{100 P_\odot}{4\pi r_{\text{star}}^2} = \sigma T_{\text{star}}^4$$

and

$$R_\odot = \frac{P_\odot}{(\text{area})_\odot} = \frac{P_\odot}{4\pi r_\odot^2} = \sigma T_\odot^4$$

Thus, we have

$$r_{\text{star}}^2 = 100 r_\odot^2 \left(\frac{T_\odot}{T_{\text{star}}}\right)^4$$

$$r_{\text{star}} = 10 r_\odot \left(\frac{T_\odot}{T_{\text{star}}}\right)^2 = 10\left(\frac{5800}{3000}\right)^2 r_\odot$$

$$r_{\text{star}} = 37.4 r_\odot$$

Since $r_\odot = 6.96 \times 10^8$ m, this star has a radius of about 2.6×10^{10} m, or about half of the radius of the orbit of Mercury.

Rayleigh-Jeans Equation

The calculation of the distribution function $R(\lambda)$ involves the calculation of the energy density of electromagnetic waves in a cavity. Materials such as black velvet or lampblack come close to being ideal blackbodies, but the best practical realization of an ideal blackbody is a small hole leading into a cavity (such as a keyhole in a closet door; see Figure 3-8). Radiation incident on the hole has little chance of being reflected back out of the hole before it is absorbed by the walls of the cavity. The power radiated *out* of the hole is proportional to the total energy density U (energy per unit volume) of the radiation in the cavity. The proportionality constant can be shown to be $c/4$, where c is the speed of light:[9]

$$R = \tfrac{1}{4} cU \qquad \textbf{3-21}$$

Similarly, the spectral distribution of the power emitted from the hole is proportional to the spectral distribution of the energy density in the cavity. If $u(\lambda)d\lambda$ is the fraction

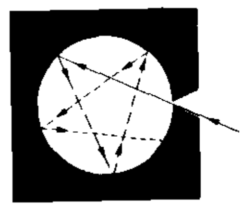


Fig. 3-8 A small hole in the wall of a cavity approximating an ideal blackbody. Radiation entering the hole has little chance of leaving before it is completely absorbed within the cavity.

of the energy per unit volume in the cavity in the range $d\lambda$, then $u(\lambda)$ and $R(\lambda)$ are related by

$$R(\lambda) = \tfrac{1}{4}cu(\lambda) \qquad \textbf{3-22}$$

The energy density distribution function $u(\lambda)$ can be calculated from classical physics in a straightforward way. The method involves finding the number of modes of oscillation of the electromagnetic field in the cavity with wavelengths in the interval $d\lambda$ and multiplying by the average energy per mode. We shall not go into the details of the calculation here. The result is that the number of modes of oscillation per unit volume, $n(\lambda)$, is independent of the shape of the cavity and is given by

$$n(\lambda) = 8\pi\lambda^{-4} \qquad \textbf{3-23}$$

According to classical kinetic theory, the average energy per mode of oscillation is kT, the same as for a one-dimensional harmonic oscillator, where k is the Boltzmann constant. Classical theory thus predicts for the energy density spectral distribution function

$$u(\lambda) = kTn(\lambda) = 8\pi kT\lambda^{-4} \qquad \textbf{3-24}$$

This prediction, initially derived by Lord Rayleigh,[10] is called the *Rayleigh-Jeans law,* and is illustrated in Figure 3-9.

At very long wavelengths the Rayleigh-Jeans law agrees with the experimentally determined spectral distribution, but at short wavelengths this law predicts that $u(\lambda)$ becomes large, approaching infinity as $\lambda \rightarrow 0$, whereas experiment shows (see Figures 3-7 and 3-9) that the distribution actually approaches zero as $\lambda \rightarrow 0$. This enormous disagreement between the experimental measurement of $u(\lambda)$ and the prediction of the fundamental laws of classical physics at short wavelengths was called the *ultraviolet catastrophe.* The word *catastrophe* was not used lightly: Equation 3-24 implies that

$$\int_0^{\infty} u(\lambda)d\lambda \longrightarrow \infty \qquad \textbf{3-25}$$

i.e., every object would have an infinite energy density.

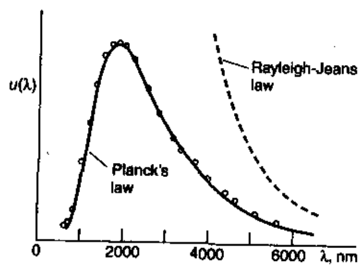


Fig. 3-9 Comparison of Planck's law and the Rayleigh-Jeans law with experimental data at $T = 1600$ K obtained by W. W. Coblentz in about 1915. The $u(\lambda)$ axis is linear. [*Adapted from F. K. Richtmyer, E. H. Kennard, and J. N. Cooper,* Introduction to Modern Physics, *6th ed. (New York: McGraw-Hill Book Company, 1969), by permission.*]

Planck's Law

In 1900 the German physicist Max Planck[11] announced that by making somewhat strange assumptions, he could derive a function $u(\lambda)$ that agreed with the experimental data. He first found an empirical function that fit the data, and then searched for a way to modify the usual calculation so as to predict his empirical formula. We can see the type of modification needed if we note that, for any cavity, the shorter the wavelength, the more standing waves (modes) will be possible. As $\lambda \rightarrow 0$ the number of modes of oscillation approaches infinity, as evidenced in Equation 3-23. In order for the energy density distribution function $u(\lambda)$ to approach zero, we expect the average energy per mode to depend on the wavelength λ and approach zero as λ approaches zero, rather than be equal to the value kT predicted by classical theory.

Parenthetically, we should observe that those working on the ultraviolet catastrophe at the time—and there were many besides Planck—had no a priori way of knowing whether the number of modes $n(\lambda)$ or the average energy per mode kT (or both) was the source of the problem. Both were correct classically. Many attempts were made to rederive each so as to solve the problem. It was the average energy per mode (that is, kinetic theory) that turned out to be at fault.

Classically, the electromagnetic waves in the cavity are produced by accelerated electric charges in the walls of the cavity vibrating like simple harmonic oscillators. Recall that the radiation emitted by such an oscillator has the same frequency as the oscillator itself. The average energy for a one-dimensional simple harmonic oscillator is calculated classically from the energy distribution function, which in turn is found from the Maxwell-Boltzmann distribution function. The energy distribution function has the form (see Chapter 8)

$$f(E) = Ae^{-E/kT} \qquad \textbf{3-26}$$

where A is a constant and $f(E)$ is the fraction of the oscillators with energy equal to E. The average energy is then found, as is any weighted average, from

$$\overline{E} = \int_0^{\infty} Ef(E)dE = \int_0^{\infty} EAe^{-E/kT}dE \qquad \textbf{3-27}$$

with the result $\overline{E} = kT$, as was used by Rayleigh and others.

Planck found that he could derive his empirical function by calculating the average energy $\overline{E}$ assuming the energy of the oscillating charges, and hence the radiation that they emitted, was a discrete variable, i.e., that it could take on only the values 0, ϵ, 2ϵ, . . . , $n\epsilon$ where n is an integer; and further, that ϵ was proportional to the frequency of the oscillators and, thus, the radiation. Planck therefore wrote the energy as

$$E_n = n\epsilon = nhf \qquad n = 0, 1, 2, \ldots \qquad \textbf{3-28}$$

where h is a constant now called *Planck's constant.* The Maxwell-Boltzmann distribution law (Equation 3-26) then becomes

$$f_n = Ae^{-E_n/kT} = Ae^{-n\epsilon/kT} \qquad \textbf{3-29}$$

where A is determined by the normalization condition that the sum of all fractions f_n must, of course, be 1, i.e.,

$$\sum_{n=0}^{\infty} f_n = A \sum_{n=0}^{\infty} e^{-n\epsilon/kT} = 1 \qquad 3\text{-}30$$

The average energy of an oscillator is then given by the discrete-sum equivalent of Equation 3-27,

$$\overline{E} = \sum_{n=0}^{\infty} E_n f_n = \sum_{n=0}^{\infty} E_n A e^{-E_n/kT} \qquad 3\text{-}31$$

Calculating the sums in Equations 3-30 and 3-31 (see Problem 3-58) yields the result:

$$\overline{E} = \frac{\epsilon}{e^{\epsilon/kT} - 1} = \frac{hf}{e^{hf/kT} - 1} = \frac{hc/\lambda}{e^{hc/\lambda kT} - 1} \qquad 3\text{-}32$$

Multiplying this result by the number of oscillators per unit volume in the interval $d\lambda$ given by Equation 3-23, we obtain for the energy density distribution function of the radiation in the cavity:

$$u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1} \qquad 3\text{-}33$$

This function, called *Planck's law*, is sketched in Figure 3-9. It is clear from the figure that the result fits the data quite well.

For very large λ, the exponential in Equation 3-33 can be expanded using $e^x \approx 1 + x + \ldots$ for $x \ll 1$ (see Appendix B4), where $x = hc/\lambda kT$. Then

$$e^{hc/\lambda kT} - 1 \approx \frac{hc}{\lambda kT}$$

and

$$u(\lambda) \longrightarrow 8\pi\lambda^{-4} kT$$

which is the Rayleigh-Jeans formula. For short wavelengths, we can neglect the 1 in the denominator of Equation 3-33, and we have

$$u(\lambda) \longrightarrow 8\pi hc\lambda^{-5} e^{-hc/\lambda kT} \longrightarrow 0$$

as $\lambda \rightarrow 0$. The value of the constant in Wien's displacement law also follows from Planck's law, as you will show in Problem 3-23.

The value of Planck's constant, h, can be determined by fitting the function given by Equation 3-33 to the experimental data, although direct measurement (see Section 3-3) is better, but more difficult. The presently accepted value is

$$\begin{aligned} h &= 6.626 \times 10^{-34}\,\text{J}\cdot\text{s} \\ &= 4.136 \times 10^{-15}\,\text{eV}\cdot\text{s} \end{aligned} \qquad 3\text{-}34$$

Planck tried at length to reconcile his treatment with classical physics but was unable to do so. The fundamental importance of the quantization assumption implied by Equation 3-28 was suspected by Planck and others but was not generally appreciated

until 1905. In that year Einstein applied the same ideas to explain the photoelectric effect and suggested that, rather than being merely a mysterious property of oscillators in the cavity walls and blackbody radiation, quantization is a fundamental characteristic of light energy.

The electromagnetic spectrum emitted by incandescent bulbs is a common example of blackbody radiation, the amount of visible light being dependent on the temperature of the filament. Another application is the pyrometer, a device that measures the temperature of a glowing object, such as molten metal in a steel mill.

EXAMPLE 3-3 **Peak of the Solar Spectrum** The surface temperature of the sun is about 5800 K, and measurements of the sun's spectral distribution show that it radiates very nearly like a blackbody, deviating mainly at very short wavelengths. Assuming that the sun radiates like a perfect blackbody, at what wavelength does the peak of the solar spectrum occur?

Solution

1. The wavelength at the peak, or maximum intensity, of a perfect blackbody spectrum is given by Equation 3-20:

$$\lambda_m T = 2.898 \times 10^{-3}\,\text{m}\cdot\text{K}$$

2. Rearranging and substituting the sun's surface temperature yields:

$$\lambda_m = (2.898 \times 10^{-3}\,\text{m}\cdot\text{K})/T = \frac{2.898 \times 10^{-3}\,\text{m}\cdot\text{K}}{5800\ \text{K}}$$
$$= \frac{2.898 \times 10^{6}\,\text{nm}\cdot\text{K}}{5800\ \text{K}} = 499.7\ \text{nm}$$

where

$$1\ \text{nm} = 10^{-9}\,\text{m}$$

Remarks: *This value is near the middle of the visible spectrum.*

EXAMPLE 3-4 **Average Energy of an Oscillator** What is the average energy $\overline{E}$ for an oscillator that has a frequency given by $hf = kT$ according to Planck's calculation?

Solution

From Equation 3-32 with $\epsilon = hf = kT$, we have

$$\overline{E} = \frac{\epsilon}{e^{\epsilon/kT} - 1} = \frac{kT}{e^{1} - 1} = 0.582\ kT$$

Recall that, according to classical theory, $\overline{E} = kT$ regardless of the frequency.

EXAMPLE 3-5 **Stefan-Boltzmann from Planck** Show that the total energy density in a blackbody cavity is proportional to T^4 in accordance with the Stefan-Boltzmann law.

Solution

The total energy density is obtained from the distribution function (Equation 3-33) by integrating over all wavelengths:

$$U = \int_0^\infty u(\lambda)d\lambda = \int_0^\infty \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1} d\lambda$$

Define the dimensionless variable $x = hc/\lambda kT$. Then $dx = -hc\, d\lambda/\lambda^2 kT$ or $d\lambda = -\lambda^2(kT/hc)dx$. Then

$$U = -\int_0^\infty \frac{8\pi hc\lambda^{-3}}{e^x - 1}\left(\frac{kT}{hc}\right)dx$$
$$= 8\pi hc\left(\frac{kT}{hc}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1}dx$$

Since the integral is now dimensionless, this shows that U is proportional to T^4. The value of the integral can be obtained from tables; it is $\pi^4/15$. Then $U = (8\pi^5 k^4/15\, h^3 c^3)\, T^4$. This result can be combined with Equations 3-19 and 3-21 to express Stefan's constant σ in terms of π, k, h, and c (see Problem 3-13).

A dramatic example of an application of Planck's law on the current frontier of physics is in tests of the predictions of the so-called Big Bang theory of the formation and present expansion of the universe. Current cosmological theory suggests that the universe originated in an extremely high-temperature explosion, one consequence of which was to fill the infant universe with radiation whose spectral distribution must surely have been that of a blackbody. Since that time, the universe has expanded to its present size and cooled to its present temperature T_{now}. However, it should still be filled with radiation whose spectral distribution should be that characteristic of a blackbody at T_{now}.

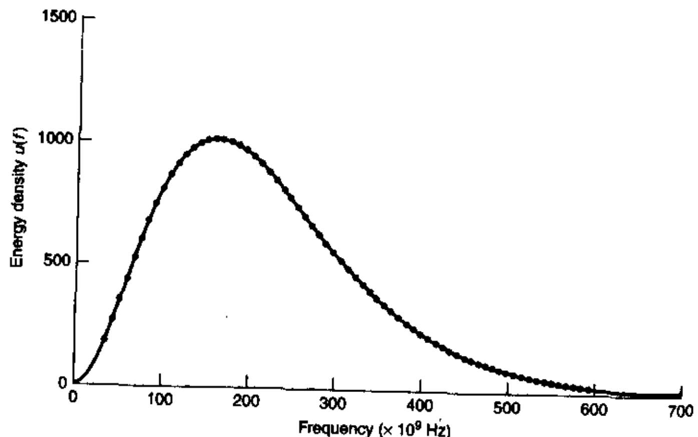


Fig. 3-10 The energy density spectral distribution of the cosmic microwave background radiation. The solid line is Planck's law with $T = 2.735$ K. The measurements were made by the COBE satellite.

In 1965, Arno Penzias and Robert Wilson discovered radiation of wavelength 7.35 cm reaching Earth with the same intensity from all directions in space. It was soon recognized that this radiation could be a remnant of the Big Bang fireball, and measurements were subsequently made at other wavelengths in order to construct an experimental energy density $u(\lambda)$ versus λ graph. The most recent data, collected by the Cosmic Background Explorer (COBE) satellite and shown in Figure 3-10, fit the Planck law for a blackbody at 2.735 K. The excellent agreement of the data with Planck's equation, indeed, the best fit that has ever been measured, is considered to be very strong support for the Big Bang theory (see Chapter 14).

3-3 The Photoelectric Effect

It is one of the ironies in the history of science that in the famous experiment of Heinrich Hertz[12] in 1887 in which he produced and detected electromagnetic waves, thus confirming James Clerk Maxwell's wave theory of light, he also discovered the photoelectric effect that led directly to the particle description of light.

Hertz was using a spark gap in a tuned circuit to generate the waves and another similar circuit to detect them. He noticed accidentally that when the light from the generating gap was shielded from the receiving gap, the receiving gap had to be made shorter to allow the sparks to pass. Light from any spark that fell on the terminals of the gap facilitated the passage of the sparks. He described the discovery with these words:

> In a series of experiments on the effects of resonance between very rapid electric oscillations that I carried out and recently published, two electric sparks were produced by the same discharge of an induction coil, and therefore simultaneously. One of these sparks, spark A, was the discharge spark of the induction coil, and served to excite the primary oscillation. The second, spark B, belonged to the induced or secondary oscillation. I occasionally enclosed spark B in a dark case so as to make observations more easily, and in so doing I observed that the maximum spark length became decidedly smaller inside the case than it was before.[13]

Albert A. Michelson, Albert Einstein, and Robert A. Millikan at a meeting in Pasadena, California, in 1931. [*AP/Wide World Photos.*]

The unexpected discovery of the photoelectric effect annoyed Hertz because it interfered with his primary research, but he recognized its importance immediately and interrupted his other work for six months in order to study it in detail. His results, published later that year, were then extended by others. It was found that negative particles were emitted from a clean surface when exposed to light. P. Lenard in 1900 deflected them in a magnetic field and found that they had a charge-to-mass ratio of the same magnitude as that measured by Thomson for cathode rays: the particles being emitted were electrons.

Figure 3-11 shows a schematic diagram of the basic apparatus used by Lenard. When light is incident on a clean metal surface (cathode *C*), electrons are emitted. If some of these electrons that reach the anode *A* pass through the small hole, a current results in the external electrometer circuit connected to α. The number of the emitted electrons reaching the anode can be increased or decreased by making the anode positive or negative with respect to the cathode. Letting *V* be the potential difference between cathode and anode, Figure 3-12*a* shows the current versus *V* for two values of the intensity of light incident on the cathode. If cathode C and anode A in Fig. 3-11 are different metals, V must be corrected for the contact potential (see Chap. 10). When *V* is positive, the electrons are attracted to the anode. At sufficiently large *V* all the emitted electrons reach the anode and the current reaches its maximum value. Lenard observed that the maximum current is proportional to the light intensity, an expected result since doubling the energy per unit time incident on the cathode should double the number of electrons emitted. Intensities too low to provide electrons with the energy necessary to escape from the metal should result in no emission of electrons. However, in contrast with the classical expectation, there was no minimum intensity below which the current was absent. When *V* is negative, the electrons are repelled from the anode. Then, only electrons with initial kinetic energy $\frac{1}{2}mv^2$ greater than $e|V|$ can reach the anode. From Figure 3-12*a* we see that if *V* is less than $-V_0$ no electrons reach the anode. The potential V_0 is called the *stopping potential.* It is related to the maximum kinetic energy of the emitted electrons by

$$\left(\tfrac{1}{2}mv^2\right)_{max} = eV_0 \qquad \text{3-35}$$

The experimental result, illustrated by Figure 3-12*a*, that V_0 is independent of the incident light intensity was surprising. Apparently, increasing the rate of energy

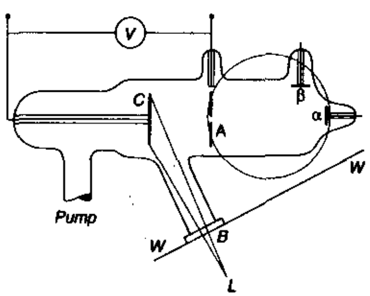


Fig. 3-11 Schematic diagram of the apparatus used by P. Lenard to demonstrate the photoelectric effect and to show that the particles emitted in the process were electrons. Light from the source *L* strikes the cathode *C*. Photoelectrons going through the hole in anode *A* are recorded by the electrometer connected to α. A magnetic field, indicated by the circular pole piece, could deflect the particles to an electrometer connected to β, enabling the establishment of the sign of their charge and their *q/m* ratio. [*P. Lenard,* Annalen der Physik, *2, 359 (1900).*]

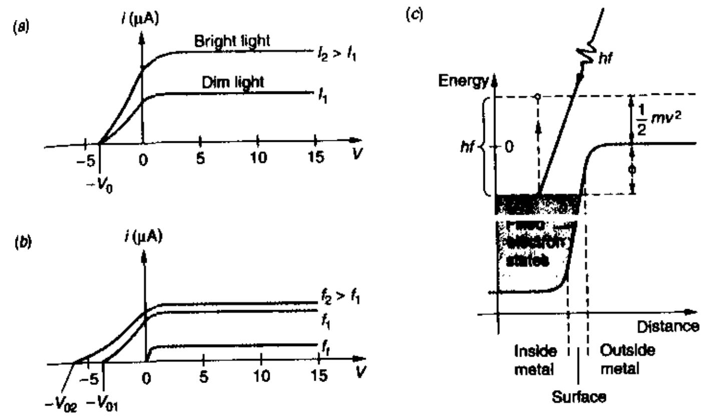


Fig. 3-12 (*a*) Photocurrent *i* versus anode voltage *V* for light of frequency *f* with two intensities I_1 and I_2, where $I_2 > I_1$. The stopping voltage V_0 is the same for both. (*b*) For constant *I*, Einstein's explanation of the photoelectric effect indicates that the magnitude of the stopping voltage should be greater for f_2 than f_1, as observed, and that there should be a threshold frequency f_t below which no photoelectrons were seen, also in agreement with experiment. (*c*) Electron potential energy curve across the metal surface. An electron with the highest energy in the metal absorbs a photon of energy *hf.* Conservation of energy requires that its kinetic energy after leaving the surface will be $hf - \phi$.

falling on the cathode does not increase the maximum kinetic energy of the emitted electrons, contrary to classical expectations. In 1905, Einstein offered an explanation of this result in a remarkable paper in the same volume of *Annalen der Physik* that contained his papers on special relativity and Brownian motion.

Einstein assumed that energy quantization used by Planck in the blackbody problem was a universal characteristic of light. Rather than being distributed evenly in the space through which it is propagated, light energy consists of discrete quanta of energy *hf.* When one of these quanta, called a *photon,* penetrates the surface of the cathode, all of its energy may be given completely to an electron. If ϕ is the energy necessary to remove an electron from the surface (ϕ is called the *work function* and is a characteristic of the metal), the maximum kinetic energy of the electrons leaving the surface will be $hf - \phi$ as a consequence of energy conservation; see Figure 3-12*c*. (Some electrons will have less than this amount because of energy lost in traversing the metal.) Thus the stopping potential V_0 should be given by

Among the many applications of the photoelectric effect is the photomultiplier, a device for enabling the accurate measurement of the energy of the light absorbed by the photosensitive surface. The SNO neutrino detector (see Figure 13-7) uses nearly 10,000 photomultipliers.

$$eV_0 = \left(\tfrac{1}{2}mv^2\right)_{max} = hf - \phi \qquad \text{3-36}$$

Equation 3-36 is referred to as the photoelectric-effect equation. As Einstein noted,

> If the derived formula is correct, then V_0, when represented in cartesian coordinates as a function of the frequency of the incident light, must be a straight line whose slope is independent of the nature of the emitting substance.[14]

As can be seen from Equation 3-36, the slope of V_0 versus *f* should equal *h/e*. At the time of this prediction, there was no evidence that Planck's constant had anything to do with the photoelectric effect. There was also no evidence for the dependence of

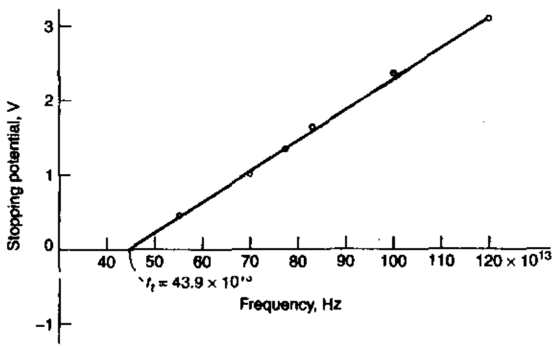


Fig. 3-13 Millikan's data for stopping potential versus frequency for the photoelectric effect. The data fall on a straight line of slope h/e, as predicted by Einstein a decade before the experiment. The intercept on the stopping potential axis is $-\phi/e$. [*R. A. Millikan,* Physical Review, *7, 362 (1915).*]

the stopping potential V_0 on frequency. Careful experiments by Millikan, reported in 1914 and in more detail in 1916, showed that Equation 3-36 was correct, and measurements of h from it agreed with the value obtained by Planck. A plot taken from this work is shown in Figure 3-13.

The minimum, or threshold, frequency for photoelectric effect, labeled f_t in this plot and in Figure 3-12*b*, and the corresponding threshold wavelength λ_t are related to the work function ϕ by setting $V_0 = 0$ in Equation 3-36:

$$\phi = hf_t = \frac{hc}{\lambda_t} \qquad 3\text{-}37$$

Photons of frequency lower than f_t (and therefore having wavelengths greater than λ_t) do not have enough energy to eject an electron from the metal. Work functions for metals are typically on the order of a few electron volts. The work functions for several elements are given in Table 3-1.

EXAMPLE 3-6 Photoelectric Effect in Potassium The threshold wavelength of potassium is 558 nm. What is the work function for potassium? What is the stopping potential when light of wavelength 400 nm is used?

Solution

1. Both questions can be answered with the aid of Equation 3-36:

$$eV_0 = (\tfrac{1}{2}mv^2)_{max} = hf - \phi$$
$$V_0 = \frac{hf}{e} - \frac{\phi}{e}$$

2. At the threshold wavelength the photoelectrons have just enough energy to overcome the work function barrier, so $(\frac{1}{2}mv^2)_{max} = 0$, hence $V_0 = 0$, and:

$$\frac{\phi}{e} = \frac{hf_t}{e} = \frac{hc}{e\lambda_t}$$
$$= \frac{1240\ \text{eV}\cdot\text{nm}}{558\ \text{nm}}$$
$$= 2.22\ \text{eV}$$

3. When 400-nm light is used, V_0 is given by Equation 3-36:

$$V_0 = \frac{hc}{e\lambda} - \frac{\phi}{e}$$
$$= \frac{1240\ \text{eV}\cdot\text{nm}}{400\ \text{nm}} - 2.22\ \text{eV}$$
$$= 3.10\ \text{eV} - 2.22\ \text{eV}$$
$$= 0.88\ \text{V}$$

Another interesting feature of the photoelectric effect, which is contrary to classical physics but easily explained by the photon hypothesis, is the lack of any time lag between the turning on of the light source and the appearance of electrons. Classically, the incident energy is distributed uniformly over the illuminated surface; the time required for an area the size of an atom to acquire enough energy to allow the emission of an electron can be calculated from the intensity (power per unit area) of the incident radiation. Experimentally, the incident intensity can be adjusted so that this calculated time lag should be several minutes or even hours. But no time lag is ever observed. The photon explanation of this result is that although the rate at which

TABLE 3-1 Photoelectric work functions

Element	ϕ (eV)
Na	2.28
C	4.81
Cd	4.07
Al	4.08
Ag	4.73
Pt	6.35
Mg	3.68
Ni	5.01
Se	5.11
Pb	4.14

photons are incident upon the metal is very small when the intensity is low, *each* photon has enough energy to eject an electron, and there is some chance that a photon will be absorbed immediately. The classical calculation gives the correct *average* number of photons absorbed per unit time.

EXAMPLE 3-7 Classical Time Lag Light of wavelength 400 nm and intensity 10^{-2} W/m^2 is incident on potassium. Estimate the time lag for emission of photoelectrons that would be expected classically.

Solution

According to the previous example, the work function for potassium is 2.22 eV. If we take $r = 10^{-10}$ m as a typical radius of an atom, the total energy falling on the atom in time t is

$$E = (10^{-2}\ \text{W/m}^2)(\pi r^2)t = (10^{-2}\ \text{W/m}^2)(\pi 10^{-20}\ \text{m}^2)t$$
$$= (3.14 \times 10^{-22}\ \text{J/s})t$$

Setting this energy equal to 2.22 eV ($= 2.22 \times 1.6 \times 10^{-19}$ J) gives

$$(3.14 \times 10^{-22}\ \text{J/s})t = (2.22)(1.6 \times 10^{-19}\ \text{J})$$
$$t = 1.13 \times 10^3\ \text{s} = 18.8\ \text{min}$$

According to the classical prediction, no atom would be expected to emit an electron until 18.8 min after the light source was turned on. According to the photon model of light, each photon has enough energy to eject an electron immediately. Because of the low intensity, there are few photons incident per second, so that the chance of any particular atom absorbing a photon and emitting an electron in any given time interval is small. However, there are so many atoms in the cathode that some emit electrons immediately.

EXAMPLE 3-8 Incident Photon Intensity In the previous example, how many photons are incident per second per square meter?

Solution

The energy of each photon is $E = hf = hc/\lambda = (1240\ \text{eV}\cdot\text{nm})/400\ \text{nm} = 3.1$ eV $= (3.1\ \text{eV})(1.6 \times 10^{-19}\ \text{J/eV}) = 4.96 \times 10^{-19}$ J. Since the incident intensity is 10^{-2} W/m^2 $= 10^{-2}$ J/s·m^2, the number of photons per second per square meter is

$$N = \frac{10^{2}\ \text{J/s}\cdot\text{m}^2}{4.96 \times 10^{-19}\ \text{J/photon}}$$
$$= 2.02 \times 10^{16}\ \text{photons/s}\cdot\text{m}^2$$

This is, of course, a lot of photons, not a few; however, the number n per atom at the surface is quite small. $n = 2.02 \times 10^{16}\ \text{photons/s}\cdot\text{m}^2 \times \pi(10^{-10})^2\ \text{m}^2/\text{atom} = 6.3 \times 10^{-4}$ photons/s · atom, or about 1 photon for every 1000 atoms.

QUESTIONS

3. How is the result that the maximum photoelectric current is proportional to the intensity explained in the photon model of light?
4. What experimental features of the photoelectric effect can be explained by classical physics? What features cannot?

The photoemission of electrons has developed into a significant technique for investigating the detailed structure of molecules and solids, making possible discoveries far beyond anything that Hertz may have imagined. The use of x-ray sources (see Section 3-4) and precision detectors has made possible precise determination of valence electron configurations in chemical compounds, leading to detailed understanding of chemical bonding and the differences between the bulk and surface atoms of solids. Photoelectric-effect microscopes now in advanced development will show the chemical situation of each element in a specimen, a prospect of intriguing and crucial importance in molecular biology and microelectronics. And they are all based on a discovery that annoyed Hertz—at first.

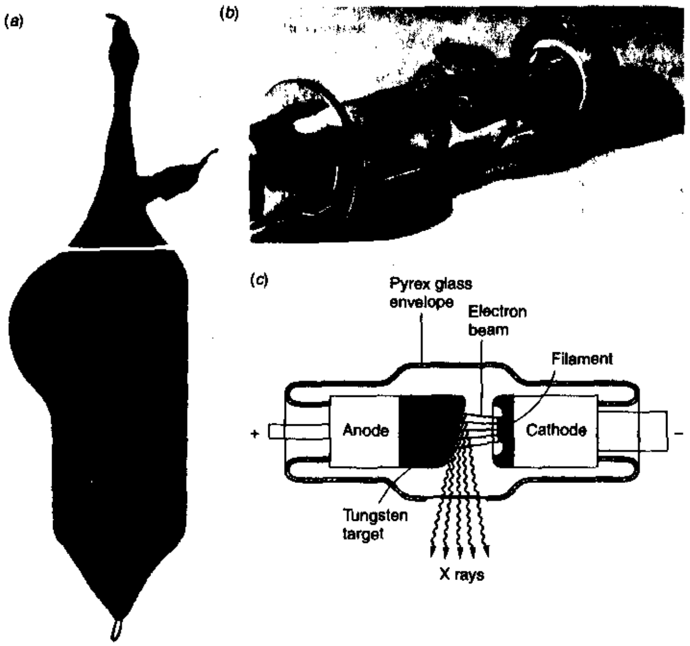
3-4 X Rays and the Compton Effect

Further evidence of the correctness of the photon concept was furnished by Arthur H. Compton, who measured the scattering of x rays by free electrons and, by his analysis of the data, resolved the last lingering doubts regarding special relativity (see Chapter 1). Before we examine Compton scattering in detail, we shall briefly describe some of the early work with x rays.

X Rays

The German physicist Wilhelm K. Roentgen discovered x rays in 1895 when he was working with a cathode-ray tube. His discovery turned out to be the first significant development in quantum physics. He found that "rays" originating from the point where the cathode rays (electrons) hit the glass tube, or a target within the tube, could pass through materials opaque to light and activate a fluorescent screen or photographic film. He investigated this phenomenon extensively and found that all materials were transparent to these rays to some degree and that the transparency decreased with increasing density. This fact led to the medical use of x rays within months after Roentgen's first paper.[15]

Roentgen was unable to deflect these rays in a magnetic field, nor was he able to observe refraction or the interference phenomena associated with waves. He thus gave the rays the somewhat mysterious name of x rays. Since classical electromagnetic theory predicts that charges will radiate electromagnetic waves when accelerated, it is natural to expect that x rays are electromagnetic waves produced by the acceleration of electrons when they are deflected and stopped by a target. Such radiation is called *bremsstrahlung*, the German for "braking radiation." The slight diffraction broadening of an x-ray beam after passing through slits a few thousandths of a millimeter wide indicated their wavelengths to be of the order of 10^{-10} m $= 0.1$ nm. In 1912, Laue suggested that since the wavelengths of x rays were of the same order of magnitude as the spacing of atoms in a crystal, the regular array of atoms in a crystal might act as a



(*a*) Early x-ray tube. [*Courtesy of Cavendish Laboratory.*] (*b*) X-ray tubes became more compact over time. This tube was a design typical of the mid-twentieth century. [*Courtesy of Schenectady Museum, Hall of Electrical History, Schenectady, NY.*] (*c*) Diagram of the components of a modern x-ray tube. Design technology has advanced enormously, enabling very high operating voltages, beam currents, and x-ray intensities, but the essential elements of the tubes remain unchanged.

three-dimensional grating for the diffraction of x rays. Experiment (see Figure 3-14) soon confirmed that x rays are a form of electromagnetic radiation with wavelengths of about 0.01 to 0.10 nm, and that atoms in crystals are arranged in regular arrays.

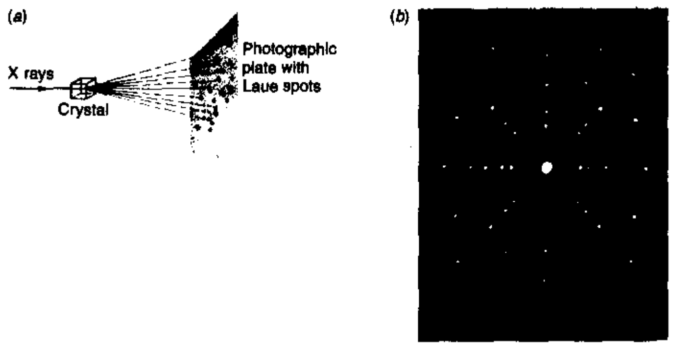
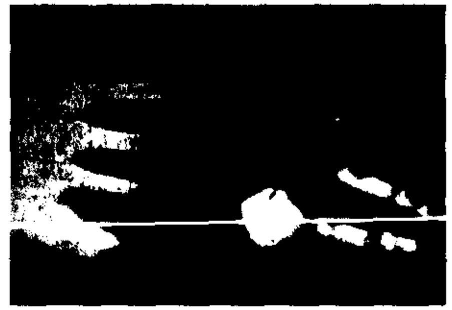


Fig. 3-14 (*a*) Schematic sketch of a Laue experiment. The crystal acts as a three-dimensional grating, which diffracts the x-ray beam and produces a regular array of spots, called a *Laue pattern*, on a photographic plate. (*b*) Modern Laue-type x-ray diffraction pattern using a niobium diboride crystal and 20-kV molybdenum x rays. [*General Electric Company.*]



An x ray of Mrs. Roentgen's hand taken by Roentgen shortly after his discovery.

W. L. Bragg, in 1912, proposed a simple and convenient way of analyzing the diffraction of x rays by crystals.[16] He examined the interference of x rays due to scattering from various sets of parallel planes of atoms, now called *Bragg planes*. Two sets of Bragg planes are illustrated in Figure 3-15 for NaCl, which has a simple crystal structure called *face-centered cubic*. Consider Figure 3-16. Waves scattered from the two successive atoms within a plane will be in phase and thus interfere constructively, independent of the wavelength, if the scattering angle equals the incident angle. (This condition is the same as for reflection.) Waves scattered at equal angles from atoms in two different planes will be in phase (constructive interference) if the difference in path length is an integral number of wavelengths. From Figure 3-16 we see that this condition is satisfied if

$$2\,d \sin\theta = m\,\lambda \qquad \text{where} \qquad m = \text{an integer} \qquad \textbf{3-38}$$

Equation 3-38 is called the *Bragg condition*.

Measurements of the spectral distribution of the intensity of x rays as a function of the wavelength using an experimental arrangement such as shown in Figure 3-17 produce the x-ray spectrum and, for classical physics, some surprises. Figure 3-18*a* (page 151) shows two typical x-ray spectra produced by accelerating electrons through two voltages V and bombarding a tungsten target mounted on the anode of the tube. In this figure $I(\lambda)$ is the intensity emitted with the wavelength interval $d\lambda$ at

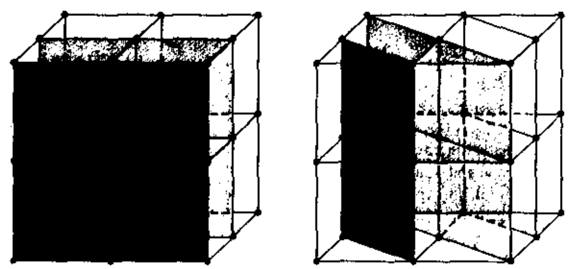


Fig. 3-15 A crystal of NaCl showing two sets of Bragg planes.

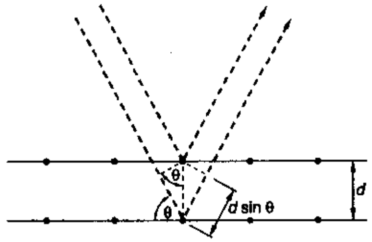


Fig. 3-16 Bragg scattering from two successive planes. The waves from the two atoms shown have a path difference of $2d \sin\theta$. They will be in phase if the Bragg condition $2d \sin\theta = m\lambda$ is met.

each value of λ. Figure 3-18*b* shows the short-wavelength lines produced with a molybdenum target and 35-keV electrons. Three features of the spectra are of immediate interest, only one of which could be explained by classical physics. (1) The spectrum consists of a series of sharp lines, called the ***characteristic spectrum***, superimposed on (2) the continuous bremsstrahlung spectrum. The line spectrum is characteristic of the target material and varies from element to element. (3) The continuous spectrum has a sharp cutoff wavelength, λ_m, which is independent of the target material but depends on the energy of the bombarding electrons. If the voltage of the x-ray tube is V in volts, the cutoff wavelength was found to be given empirically by

$$\lambda_m = \frac{1.24 \times 10^3}{V} \text{nm} \qquad \textbf{3-39}$$

Equation 3-39 is called the ***Duane-Hunt rule***, after its discoverers. It was pointed out rather quickly by Einstein that x-ray production by electron bombardment was an inverse photoelectric effect and that Equation 3-36 should apply. The Duane-Hunt λ_m

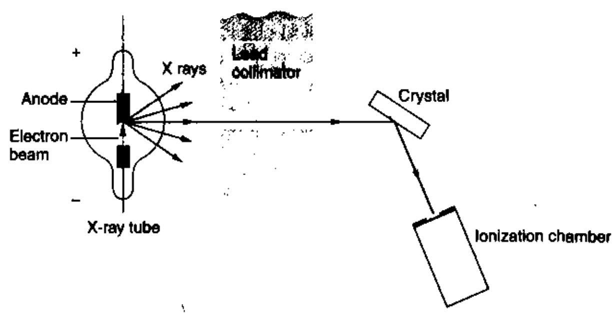


Fig. 3-17 Schematic diagram of Bragg crystal spectrometer. A collimated x-ray beam is incident on a crystal and scattered into an ionization chamber. The crystal and ionization chamber can be rotated to keep the angles of incidence and scattering equal as both are varied. By measuring the ionization in the chamber as a function of angle, the spectrum of the x rays can be determined using the Bragg condition $2d \sin\theta = m\lambda$, where d is the separation of the Bragg planes in the crystal. If the wavelength λ is known, the spacing d can be determined.

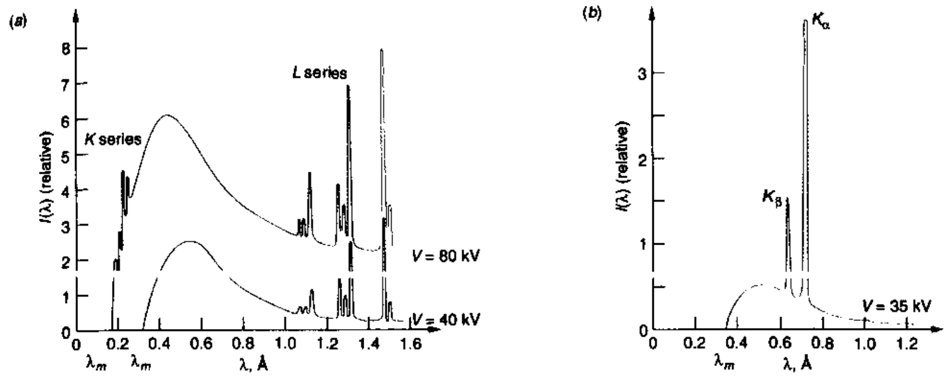


Fig. 3-18 X-ray spectra from tungsten at two accelerating voltages (*a*) and from molybdenum at one (*b*). The names of the line series (*K* and *L*) are historical and explained in Chapter 4. The *L*-series lines (not shown) for Mo are at about 0.5 nm. The cutoff wavelength λ_m is independent of the target element and is related to the voltage of the x-ray tube V by $\lambda_m = hc/eV$. The wavelengths of the lines are characteristic of the target element.

simply corresponds to a photon with the maximum energy of the electrons, that is, the photon emitted when the electron loses all of its kinetic energy in a single collision. Since the kinetic energy of the electrons in the x-ray tube is 20,000 eV or larger, the work function ϕ is negligible by comparison. That is, Equation 3-36 becomes $eV \approx hf = hc/\lambda$ or $\lambda = hc/eV = 1.2407 \times 10^{-6}\,\text{V}^{-1}\,\text{m} = 1.24 \times 10^3\,\text{V}^{-1}\,\text{nm}$. Thus, the Duane-Hunt rule is explained by Planck's quantum hypothesis. (Notice that λ_m can be used to determine h/e.)

The continuous spectrum was understood as the result of the acceleration (i.e., "braking") of the bombarding electrons in the strong electric fields of the target atoms. Maxwell's equations predicted the continuous radiation. The real problem for classical physics was the sharp lines. The wavelengths of the sharp lines were a function of the target element, the set for each element being always the same, but the sharp lines never appeared if V was such that λ_m was larger than the particular line, as can be seen in Figure 3-18*a*, where the shortest-wavelength group disappears when V is reduced from 80 kV to 40 kV, so that λ_m becomes larger. The origin of the sharp lines was a mystery that had to await the discovery of the nuclear atom. We will explain them in Chapter 4.

Well-known applications of x rays are medical and dental x rays (both diagnostic and treatment) and industrial x-ray inspection of welds and castings. Perhaps not so well known are their use in determining the structure of crystals, identifying black holes in space, and "seeing" the folded shape of proteins in biological materials.

Compton Effect

It had been observed that scattered x rays were "softer" than those in the incident beam, that is, were absorbed more readily. Compton[17] pointed out that if the scattering process were considered a "collision" between a photon of energy hf_1 (and momentum hf_1/c) and an electron, the recoiling electron would absorb part of the total energy, and the energy hf_2 of the scattered photon would therefore be less than the incident one and thus of lower frequency f_2 and momentum hf_2/c. (The fact that electromagnetic radiation of energy E carried momentum E/c was known from classical theory and from experiments of E. F. Nichols and G. F. Hull in 1903. This relation is also consistent with the relativistic expression $E^2 = p^2c^2 + (mc^2)^2$ for a particle with zero rest mass.) Compton applied the laws of conservation of momentum and energy in their relativistic form (see Chapter 2) to the collision of a photon with an isolated electron to obtain the change in the wavelength $\lambda_2 - \lambda_1$ of the photon as a function of the scattering angle θ. The result, called *Compton's equation* and derived on the home page, is

$$\lambda_2 - \lambda_1 = \frac{h}{mc}(1 - \cos\theta) \qquad \textbf{3-40}$$

The change in wavelength is thus predicted to be independent of the original wavelength. The quantity h/mc has dimensions of length and is called the *Compton wavelength of the electron.* Its value is

$$\lambda_c = \frac{h}{mc} = \frac{hc}{mc^2} = \frac{1.24 \times 10^3\,\text{eV}\cdot\text{nm}}{5.11 \times 10^5\,\text{eV}} = 0.00243\ \text{nm}$$

Because $\lambda_2 - \lambda_1$ is small, it is difficult to observe unless λ_1 is very small so that the fractional change $(\lambda_2 - \lambda_1)/\lambda_1$ is appreciable. For this reason the Compton effect is generally observed only for x rays and gamma radiation.

Compton verified his result experimentally using the characteristic x-ray line of wavelength 0.0711 nm from molybdenum for the incident monochromatic photons, and scattering these photons from electrons in graphite. The wavelength of the scattered photons was measured using a Bragg crystal spectrometer. His experimental arrangement is shown in Figure 3-19; Figure 3-20 shows his results. The first peak at each scattering angle corresponds to scattering with no shift in the wavelength due to scattering by the inner electrons of carbon. Since these are tightly bound to the atom, it is the whole atom that recoils rather than the individual electron. The expected shift for this case is given by Equation 3-40, with m being the mass of the atom, which is

Arthur Compton. After discovering the Compton effect, he became a world traveler seeking an explanation for cosmic rays. He ultimately showed that their intensity varied with latitude, indicating an interaction with Earth's magnetic field, and thus proved that they were charged particles. [*Courtesy of American Institute of Physics, Niels Bohr Library.*]

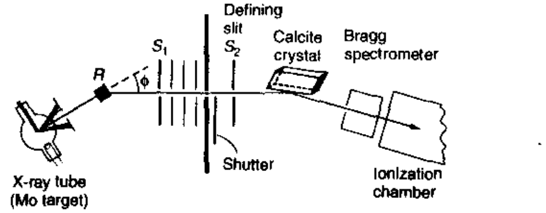


Fig. 3-19 Schematic sketch of Compton apparatus. X rays from the tube strike the carbon block R and are scattered into a Bragg-type crystal spectrometer. In this diagram, the scattering angle is 30°. The beam was defined by slits S_1 and S_2. Although the entire spectrum is being scattered by R, the spectrometer scanned the region around the K_α line of molybdenum.

about 10^4 times that of the electron; thus this shift is negligible. The variation of $\Delta\lambda$ with θ was found to be that predicted by Equation 3-40.

We have seen in this and the preceding two sections that the interaction of electromagnetic radiation with matter is a discrete interaction which occurs at the atomic level. It is perhaps curious that after so many years of debate about the nature of light, we now find that we must have both a particle (i.e., quantum) theory to describe in detail the energy exchange between electromagnetic radiation and matter, and a wave theory to describe the interference and diffraction of electromagnetic radiation. We shall discuss this so-called particle-wave duality in more detail in Chapter 5.

More

Derivation of Compton's Equation, applying conservation of energy and momentum to the relativistic collision of a photon and an electron, is included on the home page: www.whfreeman.com/modphysics4e See also Equations 3-41 and 3-42 and Figure 3-21 here.

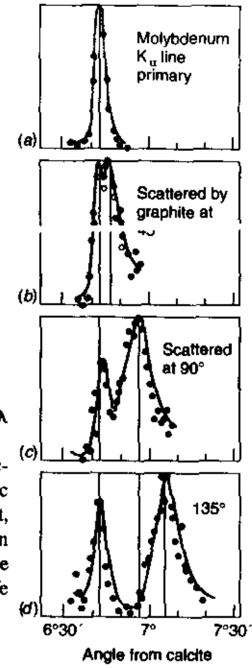


Fig. 3-20 Intensity versus wavelength for Compton scattering at several angles. The left peak in each case results from photons of the original wavelength that are scattered by tightly bound electrons, which have an effective mass equal to that of the atom. The separation in wavelength of the peaks is given by Equation 3-40. The horizontal scale used by Compton "angle from calcite" refers to the calcite analyzing crystal in Figure 3-19.

Questions

5. Why is it extremely difficult to observe the Compton effect using visible light?
6. Why is the Compton effect unimportant in the transmission of television and radio waves? How many Compton scatterings could a typical FM signal have before its wavelengths were shifted by 0.01 percent?

EXAMPLE 3-9 X Rays from TV The accelerating voltage of the electrons in a typical color television picture tube is 25 kV. What is the minimum-wavelength x ray produced when these electrons strike surfaces within the tube?

Solution

From Equation 3-41, we have

$$\lambda_m = \frac{1.24 \times 10^3}{V}\text{ nm} = \frac{1.24 \times 10^3}{25{,}000} = 0.050\text{ nm}$$

These x rays penetrate matter very effectively. Manufacturers provide essential shields to protect against the hazard.

EXAMPLE 3-10 Compton Effect In a particular Compton scattering experiment it is found that the incident wavelength λ_1 is shifted by 1.5% when the scattering angle $\theta = 120°$. (*a*) What is the value of λ_1? (*b*) What will be the wavelength λ_2 of the shifted photon when the scattering angle is 75°?

Solution

1. Considering question (*a*), the value of λ_1 is contained in Equation 3-40:

$$\lambda_2 - \lambda_1 = \Delta\lambda = \frac{h}{mc}(1 - \cos\theta)$$
$$= 0.00243\,(1 - \cos 120°)\text{ nm}$$

2. That the scattered wavelength λ_2 is shifted by 1.5 percent from λ_1 means that:

$$\frac{\Delta\lambda}{\lambda_1} = 0.015$$

3. Combining these yields:

$$\lambda_1 = \frac{\Delta\lambda}{0.015} = \frac{0.00243(1 - \cos 120°)}{0.015}$$
$$= 0.243\text{ nm}$$

4. Question (*b*) is also solved using Equation 3-40, rearranged as:

$$\lambda_2 = \lambda_1 + 0.00243\,(1 - \cos\theta)$$

5. Substituting $\theta = 75°$ and λ_1 from above yields:

$$\lambda_2 = 0.243 + 0.00243(1 - \cos 75°)$$
$$= 0.243 + 0.002$$
$$= 0.245\text{ nm}$$

Summary

TOPIC	RELEVANT EQUATIONS AND REMARKS	
1. J. J. Thomson's experiment	Thomson's measurements with cathode rays showed that the same particle (the electron), with *e/m* about 2000 times that of ionized hydrogen, exists in all elements.	
2. Quantization of electric charge	$e = 1.60217733 \times 10^{-19}$ C	
3. Blackbody radiation		
Stefan-Boltzmann law	$R = \sigma T^4$	3-19
Wien's displacement law	$\lambda_m T = 2.898 \times 10^{-3}\text{ m}\cdot\text{K}$	3-20
Planck's radiation law	$u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$	3-33
Planck's constant	$h = 6.626 \times 10^{-34}\text{ J}\cdot\text{s}$	3-34
4. Photoelectric effect	$eV_0 = hf - \phi$	3-36
5. Compton effect	$\lambda_2 - \lambda_1 = \frac{h}{mc}(1 - \cos\theta)$	3-40

General References

The following references are written at a level appropriate for the readers of this book.

Millikan, R. A., *Electrons (+ and −), Protons, Photons, Neutrons, Mesotrons, and Cosmic Rays,* 2d ed., University of Chicago Press, Chicago, 1947. This book on modern physics by one of the great experimentalists of his time contains fascinating, detailed descriptions of Millikan's oil-drop experiment and his verification of the Einstein photoelectric-effect equation.

Mohr, P. J., and B. N. Taylor, "The Fundamental Physical Constants," *Physics Today* (August 2000).

Richtmyer, F. K., E. H. Kennard, and J. N. Cooper, *Introduction to Modern Physics,* 6th ed., McGraw-Hill, New York, 1969. This excellent text was originally published in 1928, intended as a survey course for graduate students.

Shamos, M. H. (ed.), *Great Experiments in Physics,* Holt, Rinehart & Winston, New York, 1962. This book contains 25 original papers and extensive editorial comment. Of particular interest for this chapter are papers by Faraday, Hertz, Roentgen, J. J. Thomson, Einstein (photoelectric effect), Millikan, Planck, and Compton.

Thomson, G. P., *J. J. Thomson, Discoverer of the Electron,* Doubleday/Anchor, Garden City, N.Y., 1964. An interesting study of J. J. Thomson by his son, G. P. Thomson, also a physicist.

Virtual Laboratory (PEARL), Physics Academic Software, North Carolina State University, Raleigh, 1996. Computer simulation software allows the user to analyze blackbody radiation emitted over a wide range of temperatures and investigate the Compton effect in detail.

Weart, S. R. (ed.), *Selected Papers of Great American Physicists,* American Institute of Physics, New York, 1976. The bicentennial commemorative volume of the American Physical Society.

Notes

1. Democritus (about 470 B.C. to about 380 B.C.). Among his other modern-sounding ideas were the suggestions that the Milky Way was a vast conglomeration of stars and that the moon, like Earth, had mountains and valleys.

2. 1826–1911. Irish physicist who first called the fundamental unit of charge the electron. After Thomson discovered the particle that carried the charge, the name was transferred from the quantity of charge to the particle itself by Lorentz.

3. Hermann von Helmholtz (1821–1894). German physician and physicist who first proposed the law of conservation of energy in 1847 on the basis of his analysis of a meticulous set of experiments conducted some years earlier by James Joule.

4. Joseph J. Thomson (1856–1940). English physicist and director, for more than 30 years, of the Cavendish Laboratory, the first laboratory in the world established expressly for research in physics. He was awarded the Nobel Prize in 1906 for his work on the electron. Seven of his research assistants also won Nobel Prizes.

5. There had been much early confusion about the nature of cathode rays due to the failure of Heinrich Hertz in 1883 to observe any deflection of the rays in an electric field. This failure was later found to be the result of ionization of the gas in the tube; the ions quickly neutralized the charges on the deflecting plates so that there was actually no electric field between the plates. With better vacuum technology in 1897, Thomson was able to work at lower pressure and observe electrostatic deflection.

6. R. A. Millikan, *Philosophical Magazine (6)*, **19**, 209 (1910). Millikan, who held the first physics Ph.D. awarded by Columbia University, was one of the most accomplished experimentalists of his time. He received the Nobel Prize in 1923 for the measurement of the electron's charge. Also among his many contributions, he coined the term *cosmic rays* to describe radiation produced in outer space.

7. R. A. Millikan, *Physical Review*, **32**, 349 (1911).

8. P. J. Mohr and B. N. Taylor, "The Fundamental Physical Constants," *Physics Today* (August 2000).

9. See pp. 135–137 of F. K. Richtmyer, E. H. Kennard, and J. N. Cooper (1969).

10. John W. S. Rayleigh (1842–1919), English physicist, almost invariably referred to by the title that he inherited from his father. He was Maxwell's successor and Thomson's predecessor as director of the Cavendish Laboratory.

11. Max K. E. L. Planck (1858–1947). Most of his career was spent at the University of Berlin. In his later years his renown in the world of science was probably second only to that of Einstein.

12. Heinrich R. Hertz (1857–1894), German physicist, student of Helmholtz. He was the discoverer of electromagnetic "radio" waves, later developed for practical communication by Marconi.

13. H. Hertz, *Annalen der Physik*, **31**, 983 (1887).

14. A. Einstein, *Annalen der Physik*, **17**, 144 (1905).

15. A translation of this paper can be found in E. C. Watson, *American Journal of Physics*, **13**, 284 (1945), and in M. H. Shamos, ed., *Great Experiments in Physics* (New York: Holt, Rinehart & Winston, 1962). Roentgen (1845–1923) was honored in 1901 with the first Nobel Prize in physics for his discovery of x rays.

16. William Lawrence Bragg (1890–1971), Australian-English physicist, an infant prodigy. His work on x-ray diffraction performed with his father, William Henry Bragg (1862–1942), earned for them both the Nobel Prize in physics for 1915, the only father-son team to be so honored thus far. In 1938 W. L. Bragg became director of the Cavendish Laboratory, succeeding Rutherford.

17. Arthur H. Compton (1892–1962), American physicist. It was Compton who suggested the name *photon* for the light quantum. His discovery and explanation of the Compton effect earned him a share of the Nobel Prize in physics in 1927.

Problems

Level I

Section 3-1 Quantization of Electric Charge

3-1. A beam of charged particles consisting of protons, electrons, deuterons, and singly ionized helium atoms and H_2 molecules all pass through a velocity selector, all emerging with speeds of 2.5×10^6 m/s. The beam then enters a region of uniform magnetic field $B = 0.40$ T directed perpendicular to their velocity. Compute the radius of curvature of the path of each type of particle.

3-2. We wish to use a mass spectrometer to separate ^{197}Au from ^{198}Hg in a sample of material. Using the isotopic masses listed in Appendix A and assuming that each singly ionized atom enters the spectrometer at a speed of 1.5×10^5 m/s, answer the following: (*a*) What uniform magnetic field perpendicular to the ion's velocity is needed in order for the orbits to have a radius of 1 m (approximately)? (*b*) What will be the separation ΔR of the two impact points after the ions have covered half a complete circle? (See Figure 3-3.) (*c*) What would your answers to (*a*) and (*b*) change to if the atoms were all doubly ionized?

3-3. Equation 3-4 suggests how a velocity selector for particles or mixtures of different particles all having the same charge can be made. Suppose you wish to make a velocity selector that allows undeflected passage for electrons whose kinetic energy is 5.0×10^4 eV. The electric field available to you is 2.0×10^5 V/m. What magnetic field will be needed?

3-4. A cosmic-ray proton approaches Earth vertically at the equator, where the horizontal component of Earth's magnetic field is 3.5×10^{-5} T. If the proton is moving at 3.0×10^6 m/s, what is the ratio of the magnetic force to the gravitational force on the proton?

3-5. An electron of kinetic energy 45 keV moves in a circular orbit perpendicular to a magnetic field of 0.325 T. (*a*) Compute the radius of the orbit. (*b*) Find the frequency and period of the motion.

3-6. If electrons have kinetic energy of 2000 eV, find (*a*) their speed, (*b*) the time needed to traverse a distance of 5 cm between plates *D* and *E* in Figure 3-1, and (*c*) the vertical component of their velocity after passing between the plates if the electric field is $3.33 \times 10^{[illegible]}$ V/m.

3-7. In J. J. Thomson's first method, the heat capacity of the beam stopper was about 5×10^{-3} cal/°C and the temperature increase was about 2°C. How many 2000-eV electrons struck the beam stopper?

3-8. On drop #16, Millikan measured the following total charges, among others, at different times:

25.41×10^{-19} C	17.47×10^{-19} C	12.70×10^{-19} C
20.64×10^{-19} C	19.06×10^{-19} C	14.29×10^{-19} C

What value of the fundamental quantized charge *e* do these numbers imply?

3-9. Show that the electric field needed to make the rise time of the oil drop equal to its field-free fall time is $\mathscr{E} = 2\,mg/q$.

3-10. One variation of the Millikan oil-drop apparatus arranges the electric field horizontally, rather than vertically, giving the charged droplets an acceleration in the horizontal direction. The result is that the droplet falls in a straight line which makes an angle θ with the vertical. Show that

$$\sin\theta = q\mathscr{E}/bv_t'$$

where v_t' is the terminal speed along the angled path.

3-11. A charged oil droplet falls 5.0 mm in 20.0 s at terminal speed in the absence of an electric field. The specific gravity of air is 1.35×10^{-3}, and that of the oil is 0.75. The viscosity of air is $1.80 \times 10^{-5}\ \mathrm{N \cdot s/m^2}$. (*a*) What are the mass and radius of the drop? (*b*) If the droplet carries 2 units of electric charge and is in an electric field of 2.5×10^5 V/m, what is the ratio of the electric force to the gravitational force on the droplet?

Section 3-2 Blackbody Radiation

3-12. Find λ_m for blackbody radiation at (*a*) $T = 3$ K, (*b*) $T = 300$ K, and (*c*) $T = 3000$ K.

3-13. Use the result of Example 3-5 and Equations 3-19 and 3-21 to express Stefan's constant in terms of *h*, *c*, and *k*. Using the known values of these constants, calculate Stefan's constant.

3-14. Show that Planck's law, Equation 3-33, expressed in terms of the frequency *f*, is

$$u(f) = \frac{8\pi f^2}{c^3}\frac{hf}{e^{hf/kT} - 1}$$

3-15. As noted in the chapter, the cosmic microwave background radiation fits the Planck equations for a blackbody at 2.7 K. (*a*) What is the wavelength at the maximum intensity of the spectrum of the background radiation? (*b*) What is the frequency of the radiation at the maximum? (*c*) What is the total power incident on Earth from the background radiation?

3-16. Find the temperature of a blackbody if its spectrum has its peak at (*a*) $\lambda_m = 700$ nm, (*b*) $\lambda_m = 3$ cm (microwave region), and (*c*) $\lambda_m = 3$ m (FM radio waves).

3-17. If the absolute temperature of a blackbody is doubled, by what factor is the total emitted power increased?

3-18. Calculate the average energy $\bar{E}$ per mode of oscillation for (*a*) a long wavelength $\lambda = 10\, hc/kT$, (*b*) a short wavelength $\lambda = 0.1\, hc/kT$, and compare your results with the classical prediction kT (see Equation 3-24). (The classical value comes from the equipartition theorem discussed in Chapter 8.)

3-19. A particular radiating cavity has the maximum of its spectral distribution of radiated power at a wavelength of 27.0 μm (in the infrared region of the spectrum). The temperature is then changed so that the total power radiated by the cavity doubles. (*a*) Compute the new temperature. (*b*) At what wavelength does the new spectral distribution have its maximum value?

3-20. A certain very bright star has an effective surface temperature of 20,000 K. Assuming that it radiates as a blackbody, what is the wavelength at which $u(\lambda)$ is maximum?

3-21. The energy reaching Earth from the sun at the top of the atmosphere is 1.36×10^3 W/m^2, called the *solar constant*. Assuming that Earth radiates like a blackbody at uniform temperature, what do you conclude is the equilibrium temperature of Earth?

3-22. A 40-W incandescent bulb radiates from a tungsten filament operating at 3300 K. Assuming that the bulb radiates like a blackbody, (*a*) what are the frequency f_m and the wavelength λ_m at the maximum of the spectral distribution? (*b*) If f_m is a good approximation of the average frequency of the photons emitted by the bulb, about how many photons is the bulb radiating per second? (*c*) If you are looking at the bulb from 5 m away, how many photons enter your eye per second? (The diameter of your pupil is about 5.0 mm.)

3-23. Use Planck's law, Equation 3-33, to derive the constant in Wien's law, Equation 3-20.

Section 3-3 The Photoelectric Effect

3-24. Black-and-white photographic film is exposed by light that has sufficient energy to dissociate the AgBr molecules contained in the photosensitive emulsion. The minimum energy necessary is 0.68 eV. What is the maximum wavelength beyond which this film will not record light? In what region of the spectrum does this light fall?

3-25. The orbiting space shuttle moves around Earth well above 99 percent of the atmosphere, yet it still accumulates an electric charge on its skin due, in part, to the loss of electrons caused by the photoelectric effect with sunlight. Suppose the skin of the shuttle is coated with Ni, which has a relatively large work function $\phi = 4.87$ eV at the temperatures encountered in orbit. (*a*) What is the maximum wavelength in the solar spectrum that can result in the emission of photoelectrons from the shuttle's skin? (*b*) What is the maximum fraction of the total power falling on the shuttle that could potentially produce photoelectrons?

3-26. The work function for cesium is 1.9 eV. (*a*) Find the threshold frequency and wavelength for the photoelectric effect. Find the stopping potential if the wavelength of the incident light is (*b*) 300 nm, and (*c*) 400 nm.

3-27. (*a*) If 5 percent of the power of a 100-W bulb is radiated in the visible spectrum, how many visible photons are radiated per second? (*b*) If the bulb is a point source radiating equally in all directions, what is the flux of photons (number per unit time per unit area) at a distance of 2 m?

3-28. The work function of molybdenum is 4.22 eV. (*a*) What is the threshold frequency for the photoelectric effect in molybdenum? (*b*) Will yellow light of wavelength 560 nm cause ejection of photoelectrons from molybdenum? Prove your answer.

3-29. Find the photon energy corresponding to (*a*) a wavelength of 0.1 nm (about 1 atomic diameter), (*b*) a wavelength of 1 fm ($= 10^{-15}$ m, about 1 nuclear diameter), and (*c*) a frequency of 90.7 MHz in the FM radio band.

3-30. A photoelectric-effect experiment with cesium yields stopping potentials for $\lambda = 435.8$ nm and $\lambda = 546.1$ nm to be 0.95 V and 0.38 V, respectively. Using these data only, find the threshold frequency and work function for cesium and the value of h.

3-31. Under optimum conditions, the eye will perceive a flash if about 60 photons arrive at the cornea. How much energy is this in joules if the wavelength is 550 nm?

3-32. The longest wavelength of light that will cause the emission of electrons from cesium is 653 nm. (*a*) What is the work function for cesium? (*b*) If light of 300 nm (ultraviolet) were to shine on cesium, what would be the energy of the ejected electrons?

Section 3-4 X Rays and the Compton Effect

3-33. Use Compton's equation (Equation 3-40) to compute the value of $\Delta\lambda$ in Figure 3-20*d*. To what percent shift in the wavelength does this correspond?

3-34. X-ray tubes currently used by dentists often have accelerating voltages of 80 kV. What is the minimum wavelength of the x rays that they produce?

3-35. Find the momentum of a photon in eV/*c* and in kg · m/s if the wavelength is (*a*) 400 nm, (*b*) 1 Å = 0.1 nm, (*c*) 3 cm, and (*d*) 2 nm.

3-36. Gamma rays emitted by radioactive nuclei also exhibit measurable Compton scattering. Suppose a 0.511-MeV photon from a positron-electron annihilation scatters at 110° from a free electron. What are the energies of the scattered photon and the recoiling electron? Relative to the initial direction of the 0.511-MeV photon, what is the direction of the recoiling electron?

3-37. The wavelength of Compton-scattered photons is measured at $\theta = 90°$. If $\Delta\lambda/\lambda$ is to be 1 percent, what should the wavelength of the incident photon be?

3-38. Compton used photons of wavelength 0.0711 nm. (*a*) What is the energy of these photons? (*b*) What is the wavelength of the photons scattered at $\theta = 180°$? (*c*) What is the energy of the photons scattered at $\theta = 180°$? (*d*) What is the recoil energy of the electrons if $\theta = 180°$?

3-39. When photons are scattered by electrons in carbon, the shift in wavelength is 0.29 pm. Compute the scattering angle.

3-40. Compton's equation (Equation 3-40) indicates that a graph of λ_2 versus $(1 - \cos\theta)$ should be a straight line whose slope h/mc allows a determination of h. Given that the wavelength of λ in Figure 3-20 is 0.0711 nm, compute λ_2 for each scattering angle in the figure, and graph the results versus $(1 - \cos\theta)$. What is the slope of the line?

3-41. (*a*) Compute the Compton wavelength of an electron and a proton. (*b*) What is the energy of a photon whose wavelength is equal to the Compton wavelength of (1) the electron and (2) the proton?

Level II

3-42. When light of wavelength 450 nm is shone on potassium, photoelectrons with stopping potential of 0.52 V are emitted. If the wavelength of the incident light is changed to 300 nm, the stopping potential is 1.90 V. Using *only* these numbers together with the values of the speed of light and the electron charge, (*a*) find the work function of potassium and (*b*) compute a value for Planck's constant.

3-43. Referring to Figure 3-2, show that the angle of deflection θ is given by

$$\theta = \frac{e}{m}\frac{\mathscr{E} x_1}{u_x^2}$$

3-44. Assuming that the difference between Thomson's calculated e/m in his second experiment and the currently accepted value was due entirely to his neglecting the horizontal component of Earth's magnetic field outside the deflection plates, what value for that component does the difference imply?
3-45. Data for stopping potential versus wavelength for the photoelectric effect using sodium are

λ(nm) 200	300	400	500	600
V_0(V) 4.20	2.06	1.05	0.41	0.03

Plot these data in such a way as to be able to obtain (*a*) the work function, (*b*) the threshold frequency, and (*c*) the ratio h/e.
3-46. Prove that the photoelectric effect cannot occur with a completely free electron, i.e., one not bound to an atom. (*Hint:* Consider the reference frame in which the total momentum of the electron and incident photon is zero.)
3-47. When a beam of monochromatic x rays is incident on a particular NaCl crystal, Bragg reflection in the first order (i.e., with $m = 1$) occurs at $\theta = 20°$. The value of $d = 0.28$ nm. What is the minimum voltage at which the x-ray tube can be operating?
3-48. A 100-W beam of light is shone onto a blackbody of mass 2×10^{-3} kg for 10^4 s. The blackbody is initially at rest in a frictionless space. (*a*) Compute the total energy and momentum absorbed by the blackbody from the light beam, (*b*) calculate the blackbody's velocity at the end of the period of illumination, and (*c*) compute the final kinetic energy of the blackbody. Why is the latter less than the total energy of the absorbed photons?
3-49. Show that the maximum kinetic energy E_k that a recoiling electron can carry away from a Compton scattering event is given by

$$E_k = \frac{hf}{1 + mc^2/2hf}$$

3-50. The x-ray spectrometer on board a satellite measures the wavelength at the maximum intensity emitted by a particular star to be $\lambda_m = 82.8$ nm. Assuming that the star radiates like a blackbody, (*a*) compute its surface temperature. (*b*) What is the ratio of the intensity radiated at $\lambda = 70$ nm and at $\lambda = 100$ nm to λ_m.
3-51. Determine the fraction of the energy radiated by the sun in the visible region of the spectrum (350 nm to 700 nm). (Assume the sun's surface temperature is 5800 K.)
3-52. Millikan's data for the photoelectric effect in lithium are shown in the table:

Incident λ (nm)	253.5	312.5	365.0	404.7	433.9
Stopping voltage V_0 (V)	2.57	1.67	1.09	0.73	0.55

(*a*) Graph these data and determine the work function for lithium. (*b*) Find a value of Planck's constant from the graph in (*a*). (*c*) The work function for lead is 4.14 eV. Which of the wavelengths in the table would not cause emission of photoelectrons from lead?

Level III

3-53. This problem is to derive the Wien displacement law, Equation 3-20. (*a*) Show that the energy density distribution function can be written $u = C\lambda^{-5}(e^{a/\lambda} - 1)^{-1}$, where C is a constant and $a = hc/kT$. (*b*) Show that the value of λ for which $du/d\lambda = 0$ satisfies the equation $5\lambda(1 - e^{-a/\lambda}) = a$. (*c*) This equation can be solved with a calculator by the trial-and-error method. Try $\lambda = \alpha a$ for various values of α until λ/a is determined to four significant figures. (*d*) Show that your solution in (*c*) implies $\lambda_m T =$ constant and calculate the value of the constant.

3-54. This problem is one of *estimating* the time lag (expected classically but not observed) for the photoelectric effect. Assume that a point light source gives 1 W = 1 J/s of light energy. (*a*) Assuming uniform radiation in all directions, find the light intensity in $eV/m^2 \cdot s$ at a distance of 1 m from the light source. (*b*) Assuming some reasonable size for an atom, find the energy per unit time incident on the atom for this intensity. (*c*) If the work function is 2 eV, how long does it take for this much energy to be absorbed, assuming that all the energy hitting the atom is absorbed?
3-55. A photon can be absorbed by a system that can have internal energy. Assume that a 15-MeV photon is absorbed by a carbon nucleus initially at rest. The momentum of the carbon nucleus must be 15 MeV/c. (*a*) Calculate the kinetic energy of the carbon nucleus. What is the internal energy of this nucleus? (*b*) The carbon nucleus comes to rest and then loses its internal energy by emitting a photon. What is the energy of the photon?
3-56. The maximum kinetic energy given to the electron in a Compton scattering event plays a role in the measurement of gamma-ray spectra using scintillation detectors. The maximum is referred to as the *Compton edge.* Suppose the Compton edge in a particular experiment is found to be 520 keV. What were the wavelength and energy of the incident gamma rays?
3-57. An electron accelerated to 50 keV in an x-ray tube has two successive collisions in being brought to rest in the target, emitting two bremsstrahlung photons in the process. The second photon emitted has a wavelength 0.095 nm longer than the first. (*a*) What are the wavelengths of the two photons? (*b*) What was the energy of the electron after emission of the first photon?
3-58. Derive Equation 3-32 from Equations 3-30 and 3-31.