(a) Show, nevertheless, that this is not a possible Minkowski force. [*Hint:* See Prob. 12.38d.]

(b) Find a correction term that, when added to the right side, removes the objection you raised in (a), without affecting the 4-vector character of the formula or its nonrelativistic limit.[21]

Problem 12.71 Generalize the laws of relativistic electrodynamics (Eqs. 12.126 and 12.127) to include magnetic charge. [Refer to Sect. 7.3.4.]

[21] For interesting commentary on the relativistic radiation reaction, see F. Rohrlich, *Am. J. Phys.* **65**, 1051 (1997).

Appendix A

Vector Calculus in Curvilinear Coordinates

A.1 Introduction

In this Appendix I sketch proofs of the three fundamental theorems of vector calculus. My aim is to convey the *essence* of the argument, not to track down every epsilon and delta. A much more elegant, modern, and unified—but necessarily also much longer—treatment will be found in M. Spivak's book, *Calculus on Manifolds* (New York: Benjamin, 1965).

For the sake of generality, I shall use arbitrary (orthogonal) curvilinear coordinates (u, v, w), developing formulas for the gradient, divergence, curl, and Laplacian in any such system. You can then specialize them to Cartesian, spherical, or cylindrical coordinates, or any other system you might wish to use. If the generality bothers you on a first reading, and you'd rather stick to Cartesian coordinates, just read (x, y, z) wherever you see (u, v, w), and make the associated simplifications as you go along.

A.2 Notation

We identify a point in space by its three *coordinates*, u, v, and w, (in the Cartesian system, (x, y, z); in the spherical system, (r, θ, ϕ); in the cylindrical system, (s, ϕ, z)). I shall assume the system is *orthogonal*, in the sense that the three *unit vectors*, $\hat{\mathbf{u}}$, $\hat{\mathbf{v}}$, and $\hat{\mathbf{w}}$, pointing in the direction of the increase of the corresponding coordinates, are mutually perpendicular. Note that the unit vectors are *functions of position*, since their *directions* (except in the Cartesian case) vary from point to point. Any vector can be expressed in terms of $\hat{\mathbf{u}}$, $\hat{\mathbf{v}}$, and $\hat{\mathbf{w}}$—in particular, the infinitesimal displacement vector from (u, v, w) to $(u + du, v + dv, w + dw)$ can be written

$$d\mathbf{l} = f\, du\, \hat{\mathbf{u}} + g\, dv\, \hat{\mathbf{v}} + h\, dw\, \hat{\mathbf{w}}, \tag{A.1}$$

where f, g, and h are functions of position characteristic of the particular coordinate system (in Cartesian coordinates $f = g = h = 1$; in spherical coordinates $f = 1$, $g = r$, $h = r\sin\theta$; and in cylindrical coordinates $f = h = 1$, $g = s$). As you'll soon see, these three functions tell you everything you need to know about a coordinate system.

A.3 Gradient

If you move from point (u, v, w) to point $(u + du, v + dv, w + dw)$, a scalar function $t(u, v, w)$ changes by an amount

$$dt = \frac{\partial t}{\partial u}du + \frac{\partial t}{\partial v}dv + \frac{\partial t}{\partial w}dw; \tag{A.2}$$

this is a standard theorem on partial differentiation.[1] We can write it as a dot product,

$$dt = \nabla t \cdot d\mathbf{l} = (\nabla t)_u\, f\,du + (\nabla t)_v\, g\,dv + (\nabla t)_w\, h\,dw, \tag{A.3}$$

provided we define

$$(\nabla t)_u \equiv \frac{1}{f}\frac{\partial t}{\partial u}, \quad (\nabla t)_v \equiv \frac{1}{g}\frac{\partial t}{\partial v}, \quad (\nabla t)_w \equiv \frac{1}{h}\frac{\partial t}{\partial w}.$$

The **gradient** of t, then, is

$$\boxed{\nabla t \equiv \frac{1}{f}\frac{\partial t}{\partial u}\hat{\mathbf{u}} + \frac{1}{g}\frac{\partial t}{\partial v}\hat{\mathbf{v}} + \frac{1}{h}\frac{\partial t}{\partial w}\hat{\mathbf{w}}.} \tag{A.4}$$

If you now pick the appropriate expressions for f, g, and h from Table A.1, you can easily generate the formulas for ∇t in Cartesian, spherical, and cylindrical coordinates, as they appear in the front cover of the book.

System	u	v	w	f	g	h
Cartesian	x	y	z	1	1	1
Spherical	r	θ	ϕ	1	r	$r\sin\theta$
Cylindrical	s	ϕ	z	1	s	1

Table A.1

From Eq. A.3 it follows that the *total* change in t, as you go from point $\mathbf{a}$ to point $\mathbf{b}$ (Fig. A.1), is

$$t(\mathbf{b}) - t(\mathbf{a}) = \int_{\mathbf{a}}^{\mathbf{b}} dt = \int_{\mathbf{a}}^{\mathbf{b}} (\nabla t)\cdot d\mathbf{l}, \tag{A.5}$$

which is the **fundamental theorem for gradients** (not much to prove, really, in this case). Notice that the integral is independent of the path taken from $\mathbf{a}$ to $\mathbf{b}$.

[1] M. Boas, *Mathematical Methods in the Physical Sciences*, 2nd ed., Chapter 4, Sect. 3 (New York: John Wiley, 1983).

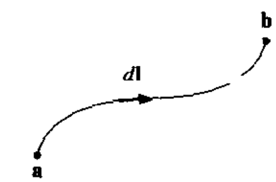

Figure A.1

A.4 Divergence

Suppose that we have a *vector* function,

$$\mathbf{A}(u, v, w) = A_u\,\hat{\mathbf{u}} + A_v\,\hat{\mathbf{v}} + A_w\,\hat{\mathbf{w}},$$

and we wish to evaluate the integral $\oint \mathbf{A}\cdot d\mathbf{a}$ over the surface of the infinitesimal volume generated by starting at the point (u, v, w) and increasing each of the coordinates in succession by an infinitesimal amount (Fig. A.2). Because the coordinates are orthogonal, this is (at least, in the infinitesimal limit) a rectangular solid, whose sides have lengths $dl_u = f\,du$, $dl_v = g\,dv$, and $dl_w = h\,dw$, and whose volume is therefore

$$d\tau = dl_u\,dl_v\,dl_w = (fgh)\,du\,dv\,dw. \tag{A.6}$$

(The sides are *not* just du, dv, dw—after all, v might be an *angle*, in which case dv doesn't even have the *dimensions* of length. The correct expressions follow from Eq. A.1.)

For the *front* surface,

$$d\mathbf{a} = -(gh)\,dv\,dw\,\hat{\mathbf{u}},$$

so that

$$\mathbf{A}\cdot d\mathbf{a} = -(ghA_u)\,dv\,dw.$$

The *back* surface is identical (except for the sign), *only this time the quantity ghA_u is to be evaluated at $(u + du)$, instead of u.* Since for any (differentiable) function $F(u)$,

$$F(u + du) - F(u) = \frac{dF}{du}du,$$

(in the limit), the front and back together amount to a contribution

$$\left[\frac{\partial}{\partial u}(ghA_u)\right] du\,dv\,dw = \frac{1}{fgh}\frac{\partial}{\partial u}(ghA_u)\,d\tau.$$

By the same token, the right and left sides yield

$$\frac{1}{fgh}\frac{\partial}{\partial v}(fhA_v)\,d\tau,$$

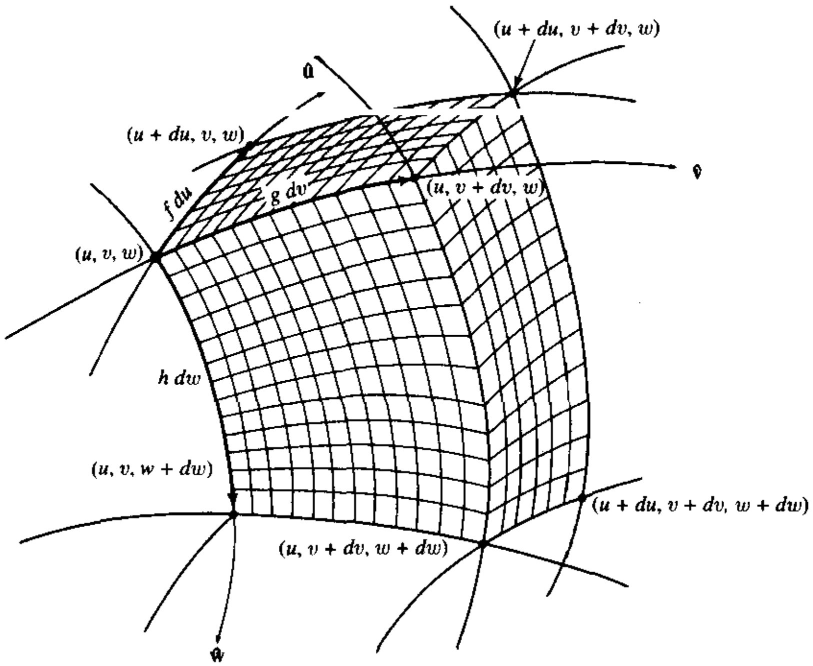

Figure A.2

and the top and bottom give

$$\frac{1}{fgh}\frac{\partial}{\partial w}(fgA_w)\,d\tau.$$

All told, then,

$$\oint \mathbf{A}\cdot d\mathbf{a} = \frac{1}{fgh}\left[\frac{\partial}{\partial u}(ghA_u) + \frac{\partial}{\partial v}(fhA_v) + \frac{\partial}{\partial w}(fgA_w)\right] d\tau. \tag{A.7}$$

The coefficient of $d\tau$ serves to define the **divergence** of **A** in curvilinear coordinates:

$$\boxed{\nabla\cdot\mathbf{A} \equiv \frac{1}{fgh}\left[\frac{\partial}{\partial u}(ghA_u) + \frac{\partial}{\partial v}(fhA_v) + \frac{\partial}{\partial w}(fgA_w)\right],} \tag{A.8}$$

and Eq. A.7 becomes

$$\oint \mathbf{A}\cdot d\mathbf{a} = (\nabla\cdot\mathbf{A})\,d\tau. \tag{A.9}$$

Using Table A.1, you can now derive the formulas for the divergence in Cartesian, spherical, and cylindrical coordinates, which appear in the front cover of the book.

As it stands, Eq. A.9 does not prove the divergence theorem, for it pertains only to *infinitesimal* volumes, and rather special infinitesimal volumes at that. Of course, a finite volume can be broken up into infinitesimal pieces, and Eq. A.9 can be applied to each one. The trouble is, when you then add up all the bits, the left-hand side is not just an integral over the *outer* surface, but over all those tiny *internal* surfaces as well. Luckily, however, these contributions cancel in pairs, for each internal surface occurs as the boundary of *two* adjacent infinitesimal volumes, and since $d\mathbf{a}$ always points *outward*, $\mathbf{A}\cdot d\mathbf{a}$ has the opposite sign for the two members of each pair (Fig. A.3). Only those surfaces that bound a *single* chunk—which is to say, only those at the outer boundary—survive when everything is added up. For *finite* regions, then,

$$\oint \mathbf{A}\cdot d\mathbf{a} = \int (\nabla\cdot\mathbf{A})\,d\tau, \tag{A.10}$$

and you need only integrate over the *external* surface.[2] This establishes the **divergence theorem**.

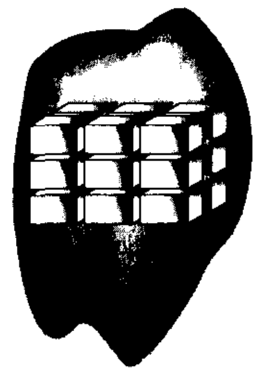

Figure A.3

[2]What about regions that cannot be fit perfectly by rectangular solids no matter *how* tiny they are—such as planes cut at an angle to the coordinate lines? It's not hard to dispose of this case; try thinking it out for yourself, or look at H. M. Schey's *Div, Grad, Curl and All That* (New York: W. W. Norton, 1973), starting with Prob. II-15.

A.5 Curl

To obtain the curl in curvilinear coordinates, we calculate the line integral,

$$\oint \mathbf{A} \cdot d\mathbf{l},$$

around the infinitesimal loop generated by starting at (u, v, w) and successively increasing u and v by infinitesimal amounts, holding w constant (Fig. A.4). The surface is a rectangle (at least, in the infinitesimal limit), of length $dl_u = f\,du$, width $dl_v = g\,dv$, and area

$$d\mathbf{a} = (fg) du\, dv\, \hat{\mathbf{w}}. \tag{A.11}$$

Assuming the coordinate system is right-handed, $\hat{\mathbf{w}}$ points out of the page in Fig. A.4. Having chosen this as the positive direction for $d\mathbf{a}$, we are obliged by the right-hand rule to run the line integral counterclockwise, as shown.

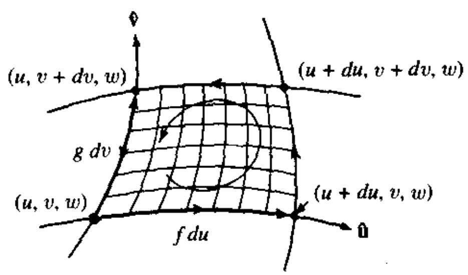

Figure A.4

Along the bottom segment,

$$d\mathbf{l} = f\,du\,\hat{\mathbf{u}},$$

so

$$\mathbf{A} \cdot d\mathbf{l} = (f A_u)\, du.$$

Along the top leg, the sign is reversed, and $f A_u$ is evaluated at $(v + dv)$ rather than v. Taken together, these two edges give

$$\left[-(f A_u)\big|_{v+dv} + (f A_u)\big|_{v}\right] du = -\left[\frac{\partial}{\partial v}(f A_u)\right] du\, dv.$$

Similarly, the right and left sides yield

$$\left[\frac{\partial}{\partial u}(g A_v)\right] du\, dv,$$

so the total is

$$\begin{aligned}\oint \mathbf{A} \cdot d\mathbf{l} &= \left[\frac{\partial}{\partial u}(g A_v) - \frac{\partial}{\partial v}(f A_u)\right] du\, dv \\ &= \frac{1}{fg}\left[\frac{\partial}{\partial u}(g A_v) - \frac{\partial}{\partial v}(f A_u)\right] \hat{\mathbf{w}} \cdot d\mathbf{a}.\end{aligned} \tag{A.12}$$

The coefficient of $d\mathbf{a}$ on the right serves to define the w-component of the **curl**. Constructing the u and v components in the same way, we have

$$\boxed{\begin{aligned}\nabla \times \mathbf{A} \equiv\ & \frac{1}{gh}\left[\frac{\partial}{\partial v}(h A_w) - \frac{\partial}{\partial w}(g A_v)\right]\hat{\mathbf{u}} + \frac{1}{fh}\left[\frac{\partial}{\partial w}(f A_u) - \frac{\partial}{\partial u}(h A_w)\right]\hat{\mathbf{v}} \\ & + \frac{1}{fg}\left[\frac{\partial}{\partial u}(g A_v) - \frac{\partial}{\partial v}(f A_u)\right]\hat{\mathbf{w}},\end{aligned}} \tag{A.13}$$

and Eq. A.11 generalizes to

$$\oint \mathbf{A} \cdot d\mathbf{l} = (\nabla \times \mathbf{A}) \cdot d\mathbf{a}. \tag{A.14}$$

Using Table A.1, you can now derive the formulas for the curl in Cartesian, spherical, and cylindrical coordinates.

Equation A.14 does not by itself prove Stokes' theorem, however, because at this point it pertains only to very special infinitesimal surfaces. Again, we can chop any *finite* surface into infinitesimal pieces and apply Eq. A.14 to each one (Fig. A.5). When we add them up, though, we obtain (on the left) not only a line integral around the outer boundary, but a lot of tiny line integrals around the internal loops as well. Fortunately, as before, the internal

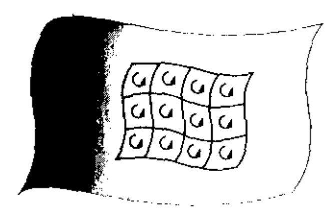

Figure A.5

contributions cancel in pairs, because every internal line is the edge of *two* adjacent loops running in opposite directions. Consequently, Eq. A.14 can be extended to finite surfaces,

$$\oint \mathbf{A} \cdot d\mathbf{l} = \int (\nabla \times \mathbf{A}) \cdot d\mathbf{a}, \tag{A.15}$$

and the line integral is to be taken over the external boundary only.[3] This establishes **Stokes' theorem**.

A.6 Laplacian

Since the **Laplacian** of a scalar is by definition the divergence of the gradient, we can read off from Eqs. A.4 and A.8 the general formula

$$\boxed{\nabla^2 t \equiv \frac{1}{fgh}\left[\frac{\partial}{\partial u}\left(\frac{gh}{f}\frac{\partial t}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{fh}{g}\frac{\partial t}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{fg}{h}\frac{\partial t}{\partial w}\right)\right].} \tag{A.16}$$

Once again, you are invited to use Table A.1 to derive the Laplacian in Cartesian, spherical, and cylindrical coordinates, and thus to confirm the formulas inside the front cover.

[3] What about surfaces that cannot be fit perfectly by tiny rectangles, no matter how small they are (such as triangles) or surfaces that do not correspond to holding one coordinate fixed? If such cases trouble you, and you cannot resolve them for yourself, look at H. M. Schey's *Div, Grad, Curl, and All That*, Prob. III-2 (New York: W. W. Norton, 1973).

Appendix B

The Helmholtz Theorem

Suppose we are told that the divergence of a vector function $\mathbf{F}(\mathbf{r})$ is a specified scalar function $D(\mathbf{r})$:

$$\nabla \cdot \mathbf{F} = D, \tag{B.1}$$

and the curl of $\mathbf{F}(\mathbf{r})$ is a specified vector function $\mathbf{C}(\mathbf{r})$:

$$\nabla \times \mathbf{F} = \mathbf{C}. \tag{B.2}$$

For consistency, $\mathbf{C}$ must be divergenceless,

$$\nabla \cdot \mathbf{C} = 0, \tag{B.3}$$

because the divergence of a curl is always zero. *Question:* can we, on the basis of this information, determine the function $\mathbf{F}$? If $D(\mathbf{r})$ and $\mathbf{C}(\mathbf{r})$ go to zero sufficiently rapidly at infinity, the answer is *yes*, as I will show by explicit construction.

I claim that

$$\mathbf{F} = -\nabla U + \nabla \times \mathbf{W}, \tag{B.4}$$

where

$$U(\mathbf{r}) \equiv \frac{1}{4\pi}\int \frac{D(\mathbf{r}')}{\mathscr{r}}\, d\tau', \tag{B.5}$$

and

$$\mathbf{W}(\mathbf{r}) \equiv \frac{1}{4\pi}\int \frac{\mathbf{C}(\mathbf{r}')}{\mathscr{r}}\, d\tau'; \tag{B.6}$$

the integrals are over all of space, and, as always, $\mathscr{r} = |\mathbf{r} - \mathbf{r}'|$. For if $\mathbf{F}$ is given by Eq. B.4, then its divergence (using Eq. 1.102) is

$$\nabla \cdot \mathbf{F} = -\nabla^2 U = -\frac{1}{4\pi}\int D\, \nabla^2\left(\frac{1}{\mathscr{r}}\right) d\tau' = \int D(\mathbf{r}')\, \delta^3(\mathbf{r} - \mathbf{r}')\, d\tau' = D(\mathbf{r}).$$

(Remember that the divergence of a curl is zero, so the $\mathbf{W}$ term drops out, and note that the differentiation is with respect to $\mathbf{r}$, which is contained in $\mathscr{r}$.)

So the divergence is right; how about the curl?

$$\nabla \times \mathbf{F} = \nabla \times (\nabla \times \mathbf{W}) = -\nabla^2 \mathbf{W} + \nabla(\nabla \cdot \mathbf{W}). \tag{B.7}$$

(Since the curl of a gradient is zero, the U term drops out.) Now:

$$-\nabla^2 \mathbf{W} = -\frac{1}{4\pi} \int \mathbf{C}\, \nabla^2 \left(\frac{1}{\mathscr{r}}\right) d\tau' = \int \mathbf{C}(\mathbf{r}') \delta^3(\mathbf{r} - \mathbf{r}')\, d\tau' = \mathbf{C}(\mathbf{r}),$$

which is perfect—I'll be done if I can just persuade you that the *second* term on the right side of Eq. B.7 vanishes. Using integration by parts (Eq. 1.59), and noting that derivatives of $\mathscr{r}$ with respect to *primed* coordinates differ by a sign from those with respect to *unprimed* coordinates, we have

$$\begin{aligned} 4\pi \nabla \cdot \mathbf{W} &= \int \mathbf{C} \cdot \nabla \left(\frac{1}{\mathscr{r}}\right) d\tau' = -\int \mathbf{C} \cdot \nabla' \left(\frac{1}{\mathscr{r}}\right) d\tau' \\ &= \int \frac{1}{\mathscr{r}} \nabla' \cdot \mathbf{C}\, d\tau - \oint \frac{1}{\mathscr{r}} \mathbf{C} \cdot d\mathbf{a}. \end{aligned} \tag{B.8}$$

But the divergence of $\mathbf{C}$ is zero, by assumption (Eq. B.3), and the surface integral (way out at infinity) will vanish, as long as $\mathbf{C}$ goes to zero sufficiently rapidly.

Of course, that proof tacitly assumes that the integrals in Eqs. B.5 and B.6 *converge*—otherwise U and $\mathbf{W}$ don't exist at all. At the large r' limit, where $\mathscr{r} \approx r'$, the integrals have the form

$$\int^{\infty} \frac{X(r')}{r'} r'^2\, dr' = \int^{\infty} r' X(r')\, dr'. \tag{B.9}$$

(Here X stands for D or $\mathbf{C}$, as the case may be). Obviously, $X(r')$ must go to zero at large r'—but that's not enough: if $X \sim 1/r'$, the integrand is constant, so the integral blows up, and even if $X \sim 1/r'^2$, the integral is a logarithm, which is still no good at $r' \to \infty$. Evidently the divergence and curl of $\mathbf{F}$ must go to zero *more rapidly than* $1/r^2$ for the proof to hold. (Incidentally, this is *more* than enough to ensure that the surface integral in Eq. B.8 vanishes.)

Now, assuming these conditions on $D(\mathbf{r})$ and $\mathbf{C}(\mathbf{r})$ are met, is the solution in Eq. B.4 *unique?* The answer is clearly *no,* for we can add to $\mathbf{F}$ any vector function whose divergence and curl both vanish, and the result still has divergence D and curl $\mathbf{C}$. However, it so happens that there is *no* function that has zero divergence and zero curl everywhere *and* goes to zero at infinity (see Sect. 3.1.5). So if we include a requirement that $\mathbf{F}(\mathbf{r})$ goes to zero as $r \to \infty$, then solution B.4 is unique.[1]

[1] Typically we *do* expect the electric and magnetic fields to go to zero at large distances from the charges and currents that produce them, so this is not an unreasonable stipulation. Occasionally one encounters artificial problems in which the charge or current distribution itself extends to infinity—infinite wires, for instance, or infinite planes. In such cases other means must be found to establish the existence and uniqueness of solutions to Maxwell's equations.

Now that all the cards are on the table, I can state the **Helmholtz theorem** more rigorously:

> If the divergence $D(\mathbf{r})$ and the curl $\mathbf{C}(\mathbf{r})$ of a vector function $\mathbf{F}(\mathbf{r})$ are specified, and if they both go to zero faster than $1/r^2$ as $r \to \infty$, and if $\mathbf{F}(\mathbf{r})$ goes to zero as $r \to \infty$, then $\mathbf{F}$ is given uniquely by Eq. B.4.

The Helmholtz theorem has an interesting **corollary**:

> Any (differentiable) vector function $\mathbf{F}(\mathbf{r})$ that goes to zero faster than $1/r$ as $r \to \infty$ can be expressed as the gradient of a scalar plus the curl of a vector:[2]

$$\mathbf{F}(\mathbf{r}) = \nabla \left(\frac{-1}{4\pi} \int \frac{\nabla' \cdot \mathbf{F}(\mathbf{r}')}{\mathscr{r}} d\tau' \right) + \nabla \times \left(\frac{1}{4\pi} \int \frac{\nabla' \times \mathbf{F}(\mathbf{r}')}{\mathscr{r}} d\tau' \right). \tag{B.10}$$

For example, in electrostatics $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$ and $\nabla \times \mathbf{E} = 0$, so

$$\mathbf{E}(\mathbf{r}) = -\nabla \left(\frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}')}{\mathscr{r}} d\tau' \right) = -\nabla V, \tag{B.11}$$

where V is the scalar potential, while in magnetostatics $\nabla \cdot \mathbf{B} = 0$ and $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$, so

$$\mathbf{B}(\mathbf{r}) = \nabla \times \left(\frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}')}{\mathscr{r}} d\tau' \right) = \nabla \times \mathbf{A}, \tag{B.12}$$

where $\mathbf{A}$ is the vector potential.

[2] As a matter of fact, any differentiable vector function *whatever* (regardless of its behavior at infinity) can be written as a gradient plus a curl, but this more general result does not follow directly from the Helmholtz theorem, nor does Eq. B.10 supply the explicit construction, since the integrals, in general, diverge.

Appendix C

Units

In our units (the **Système International**) Coulomb's law reads

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}} \quad \text{(SI)}. \tag{C.1}$$

Mechanical quantities are measured in meters, kilograms, seconds, and charge is in **coulombs** (Table C.1). In the **Gaussian system**, the constant in front is, in effect, absorbed into the unit of charge, so that

$$\mathbf{F} = \frac{q_1 q_2}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}} \quad \text{(Gaussian)}. \tag{C.2}$$

Mechanical quantities are measured in centimeters, grams, seconds, and charge is in **electrostatic units** (or **esu**). For what it's worth, an esu is evidently a $(\text{dyne})^{1/2}$-centimeter. Converting electrostatic equations from SI to Gaussian units is not difficult: just set

$$\epsilon_0 \rightarrow \frac{1}{4\pi}.$$

For example, the energy stored in an electric field (Eq. 2.45),

$$U = \frac{\epsilon_0}{2}\int E^2\, d\tau \quad \text{(SI)},$$

becomes

$$U = \frac{1}{8\pi}\int E^2\, d\tau \quad \text{(Gaussian)}.$$

(Formulas pertaining to fields inside dielectrics are not so easy to translate, because of differing definitions of displacement, susceptibility, and so on; see Table C.2.)

Quantity	SI	Factor	Gaussian
Length	meter (m)	10^2	centimeter
Mass	kilogram (kg)	10^3	gram
Time	second (s)	1	second
Force	newton (N)	10^5	dyne
Energy	joule (J)	10^7	erg
Power	watt (W)	10^7	erg/second
Charge	coulomb (C)	3×10^9	esu (statcoulomb)
Current	ampere (A)	3×10^9	esu/second (statampere)
Electric field	volt/meter	$(1/3) \times 10^{-4}$	statvolt/centimeter
Potential	volt (V)	$1/300$	statvolt
Displacement	coulomb/meter^2	$12\pi \times 10^5$	statcoulomb/centimeter^2
Resistance	ohm (Ω)	$(1/9) \times 10^{-11}$	second/centimeter
Capacitance	farad (F)	9×10^{11}	centimeter
Magnetic field	tesla (T)	10^4	gauss
Magnetic flux	weber (Wb)	10^8	maxwell
H	ampere/meter	$4\pi \times 10^{-3}$	oersted
Inductance	henry (H)	$(1/9) \times 10^{-11}$	second^2/centimeter

Table C.1 **Conversion Factors.** [*Note:* Except in exponents, every "3" is short for $\alpha \equiv 2.99792458$ (the numerical value of the speed of light), "9" means α^2, and "12" is 4α.]

The Biot-Savart law, which for us reads

$$\mathbf{B} = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l} \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} \quad \text{(SI)}, \tag{C.3}$$

becomes, in the Gaussian system,

$$\mathbf{B} = \frac{I}{c}\int \frac{d\mathbf{l} \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} \quad \text{(Gaussian)}, \tag{C.4}$$

where c is the speed of light, and current is measured in esu/s. The Gaussian unit of magnetic field (the **gauss**) is the one quantity from this system in everyday use: people speak of volts, amperes, henries, and so on (all SI units), but for some reason they tend to measure magnetic fields in gauss (the Gaussian unit); the correct SI unit is the **tesla** (10^4 gauss).

One major virtue of the Gaussian system is that electric and magnetic fields have the same dimensions (in principle, one could measure the electric fields in gauss too, though no one uses the term in this context). Thus the Lorentz force law, which we have written

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad \text{(SI)}, \tag{C.5}$$

	SI	Gaussian
Maxwell's equations		
In general:	$\nabla\cdot\mathbf{E}=\frac{1}{\epsilon_0}\rho$ $\nabla\times\mathbf{E}=-\partial\mathbf{B}/\partial t$ $\nabla\cdot\mathbf{B}=0$ $\nabla\times\mathbf{B}=\mu_0\mathbf{J}+\mu_0\epsilon_0\partial\mathbf{E}/\partial t$	$\nabla\cdot\mathbf{E}=4\pi\rho$ $\nabla\times\mathbf{E}=-\frac{1}{c}\partial\mathbf{B}/\partial t$ $\nabla\cdot\mathbf{B}=0$ $\nabla\times\mathbf{B}=\frac{4\pi}{c}\mathbf{J}+\frac{1}{c}\partial\mathbf{E}/\partial t$
In matter:	$\nabla\cdot\mathbf{D}=\rho_f$ $\nabla\times\mathbf{E}=-\partial\mathbf{B}/\partial t$ $\nabla\cdot\mathbf{B}=0$ $\nabla\times\mathbf{H}=\mathbf{J}_f+\partial\mathbf{D}/\partial t$	$\nabla\cdot\mathbf{D}=4\pi\rho_f$ $\nabla\times\mathbf{E}=-\frac{1}{c}\partial\mathbf{B}/\partial t$ $\nabla\cdot\mathbf{B}=0$ $\nabla\times\mathbf{H}=\frac{4\pi}{c}\mathbf{J}_f+\frac{1}{c}\partial\mathbf{D}/\partial t$
D and H		
Definitions:	$\mathbf{D}=\epsilon_0\mathbf{E}+\mathbf{P}$ $\mathbf{H}=\frac{1}{\mu_0}\mathbf{B}-\mathbf{M}$	$\mathbf{D}=\mathbf{E}+4\pi\mathbf{P}$ $\mathbf{H}=\mathbf{B}-4\pi\mathbf{M}$
Linear media:	$\mathbf{P}=\epsilon_0\chi_e\mathbf{E},\quad \mathbf{D}=\epsilon\mathbf{E}$ $\mathbf{M}=\chi_m\mathbf{H},\quad \mathbf{H}=\frac{1}{\mu}\mathbf{B}$	$\mathbf{P}=\chi_e\mathbf{E},\quad \mathbf{D}=\epsilon\mathbf{E}$ $\mathbf{M}=\chi_m\mathbf{H},\quad \mathbf{H}=\frac{1}{\mu}\mathbf{B}$
Lorentz force law	$\mathbf{F}=q(\mathbf{E}+\mathbf{v}\times\mathbf{B})$	$\mathbf{F}=q\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right)$
Energy and power		
Energy:	$U=\frac{1}{2}\int(\epsilon_0E^2+\frac{1}{\mu_0}B^2)\,d\tau$	$U=\frac{1}{8\pi}\int(E^2+B^2)\,d\tau$
Poynting vector:	$\mathbf{S}=\frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B})$	$\mathbf{S}=\frac{c}{4\pi}(\mathbf{E}\times\mathbf{B})$
Larmor formula:	$P=\frac{1}{4\pi\epsilon_0}\frac{2}{3}\frac{q^2a^2}{c^3}$	$P=\frac{2}{3}\frac{q^2a^2}{c^3}$

Table C.2 **Fundamental Equations in SI and Gaussian Units.**

(indicating that E/B has the dimensions of *velocity*), takes the form

$$\mathbf{F}=q\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right)\quad\text{(Gaussian)}.\qquad\text{(C.6)}$$

In effect, the magnetic field is "scaled up" by a factor of c. This reveals more starkly the parallel structure of electricity and magnetism. For instance, the total energy stored in electromagnetic fields is

$$U=\frac{1}{8\pi}\int(E^2+B^2)\,d\tau\quad\text{(Gaussian)},\qquad\text{(C.7)}$$

eliminating the ϵ_0 and μ_0 that spoil the symmetry in the SI formula,

$$U=\frac{1}{2}\int\left(\epsilon_0E^2+\frac{1}{\mu_0}B^2\right)d\tau\quad\text{(SI)}.\qquad\text{(C.8)}$$

Table C.2 lists some of the basic formulas of electrodynamics in both systems. For equations not found here, and for Heaviside-Lorentz units, I refer you to the appendix of J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (New York: John Wiley, 1999), where a more complete listing is to be found.[1]

[1] For an interesting "primer" on electrical SI units see N. M. Zimmerman, *Am. J. Phys.* **66**, 324 (1998).