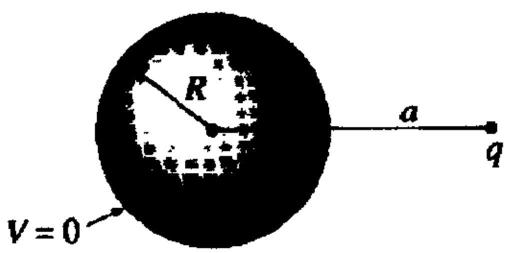
EM HW #3 - Physical Systems - due Feb 2007
3.2 (don't calculate), 3.8 (part a - only), 3.9 (finish - started in class)

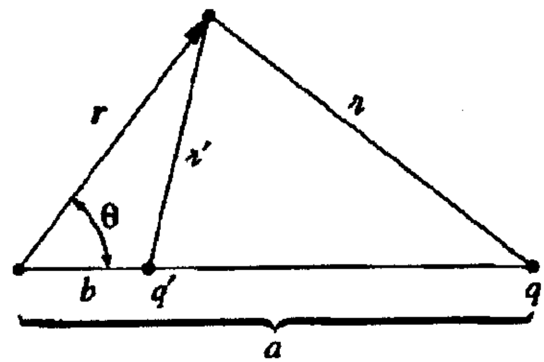
3.2 (don't calculate), 3.8 (part a), 3.9 (finish)

Problem 3.8 In Ex. 3.2 we assumed that the conducting sphere was grounded ($V = 0$). But with the addition of a second image charge, the same basic model will handle the case of a sphere at *any* potential V_0 (relative, of course, to infinity). What charge should you use, and where should you put it? Find the force of attraction between a point charge q and a *neutral* conducting sphere.

Example 3.2

A point charge q is situated a distance a from the center of a grounded conducting sphere of radius R (Fig. 3.12). Find the potential outside the sphere.





Solution: Examine the *completely different* configuration, consisting of the point charge q together with another point charge $q' = -\frac{R}{a}q$ placed a distance $b = \frac{R^2}{a}$ to the right of the center of the sphere (Fig. 3.13). No conductor, now—just the two point charges. The potential of this configuration is

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{\mathscr{r}} + \frac{q'}{\mathscr{r}'}\right). \tag{3.17}$$

where $\mathscr{r}$ and $\mathscr{r}'$ are the distances from q and q', respectively. Now, it happens (see Prob. 3.7) that this potential vanishes at all points on the sphere, and therefore fits the boundary conditions for our original problem, in the exterior region.

To increase the potential by V_0, recall that the potential of a point charge is $V = \frac{q_{pt}}{4\pi\epsilon_0 r}$, so just put a point charge $q_0 = V_0 4\pi\epsilon_0 R$ at the center of the sphere.

Physical Systems

fall EM HW #2 - Ch 3 - Griffiths - solutions - Zita

14 Oct 02

Ch 3 # $\frac{2}{115}$, $\frac{3}{116}$, [$\frac{5}{121}$, ($\frac{9}{126}$ + $\frac{2.22}{82}$) image line of charge] next week

Problem 3.2 In one sentence, justify **Earnshaw's Theorem:** *a charged particle cannot be held in stable equilibrium by electrostatic forces alone.* As an example, consider the cubical lattice of fixed charges in Fig. 3.4. It *looks*, offhand, as though a positive charge at the center would be suspended in midair—repelled away from each corner. Where is the leak in this "electrostatic bottle"? (To harness nuclear fusion as a practical energy source it is necessary to heat a plasma (soup of charged particles) to fantastic temperatures—so hot that contact would vaporize any ordinary pot. Earnshaw's Theorem says that electrostatic containment is also out of the question. Fortunately, it *is* possible to confine a hot plasma *magnetically*.)

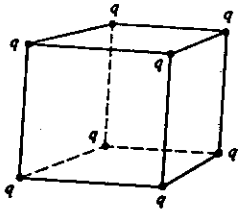


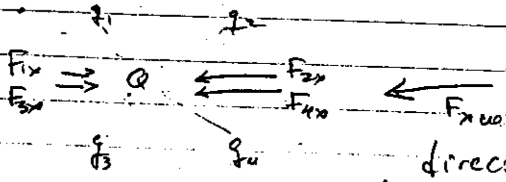
Figure 3.4

Stable equilibria occur at potential energy minima ($W \sim qV$), but solutions of $\nabla^2 V = 0$ (V) can have no local minimum, therefore any electrostatic equilibrium must be unstable.

$(\nabla \cdot E = 0)$

$\nabla \cdot (\nabla V) = \nabla^2 V = 0$

For example, at the center point above, all forces cancel — it is a point of equilibrium. But this equilibrium is unstable - a small displacement (of a test charge) from the center of the cube results in a force imbalance.



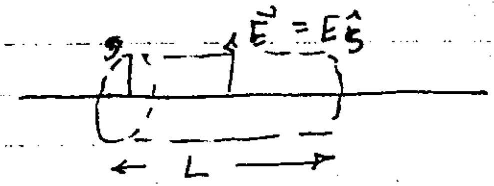
$F_{x\,net}$ is in the direction of the displacement $\therefore$ the displacement grows, out the sides of box

(V cannot have a minimum at center of cube - it must be some sort of saddle point)

UNSTABLE equilibrium

3.9 First, do

126 **Problem 2.22** Find the potential a distance r from an infinitely long straight wire that carries a uniform line charge λ. Compute the gradient of your potential, and check that it yields the correct field.



$\lambda = \frac{dq}{dl}$

Flux through Gaussian cylinder $\oint \bar{E}\cdot d\bar{a} = \frac{q}{\epsilon_0}$

$E(r)\, 2\pi s\, L = \lambda L/\epsilon_0$

$\bar{E}(s) = \frac{\lambda}{2\pi s \epsilon_0}\hat{s}$

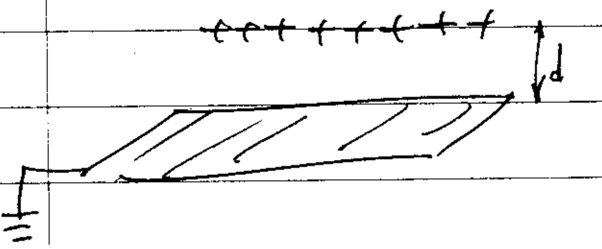
$V(r) = -\int_a^s \bar{E}\cdot d\bar{l} = -\frac{\lambda}{2\pi\epsilon_0}\int_a^s \frac{ds}{s} = -\frac{\lambda}{2\pi\epsilon_0}(\ln s - \ln a)$

BC: Fix Reference point at a: $V(a) = 0$

Can't choose $V(\infty) = 0$ because λ extends to ∞!

Check: $-\nabla V = \frac{\partial}{\partial s}\left(+\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{s}{a}\right)\right)\hat{s} = -\frac{\lambda}{2\pi\epsilon_0 s}\hat{s} = \bar{E}$ ✓

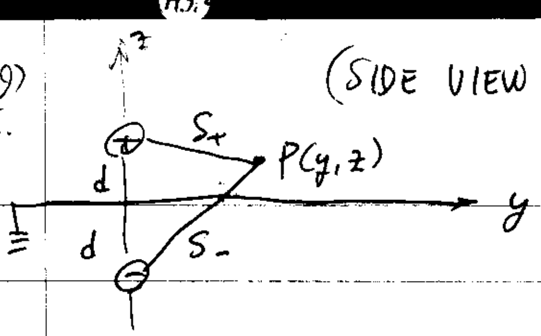
3.9 Now put this line of charge above a grounded conducting plane

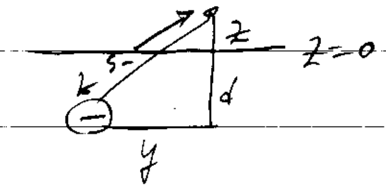


Guess that the induced charge on the plane has the same field as a line of ⊖ charge a distance d BELOW the sheet.

By symmetry, $V=0$ at sheet ∴ this solution works.

3.9) (SIDE VIEW)





$S_+^2 = (z-d)^2 + y^2 \qquad S_-^2 = (z+d)^2 + y^2$

We found $V_+(S_+) = \frac{-\lambda}{2\pi\epsilon_0} \ln\left(\frac{S_+}{a}\right)$. Also $V_-(S_-) = \frac{-\lambda}{2\pi\epsilon_0} \ln\left(\frac{S_-}{a}\right)$

Potential at $P(y,z) = V_+(S_+) - V_-(S_-)$

$$= \frac{-\lambda}{2\pi\epsilon_0}\left[(\ln S_+ - \ln a) - (\ln S_- - \ln a)\right]$$

$$= \frac{-\lambda}{2\pi\epsilon_0}\left[\ln S_+ - \ln S_-\right] = \frac{+\lambda}{2\pi\epsilon_0}\left[\ln S_- - \ln S_+\right] = \frac{\lambda}{2\pi\epsilon_0} \ln\frac{S_-}{S_+}$$

NB: $\ln x = \ln(x^2)^{\frac{1}{2}} = \frac{1}{2}\ln x^2$

So $\ln\frac{S_-}{S_+} = \frac{1}{2}\ln\left(\frac{S_-^2}{S_+^2}\right)$

and $V(P) = \frac{\lambda}{2\pi\epsilon_0}\frac{1}{2}\ln\left[\frac{(z+d)^2+y^2}{(z-d)^2+y^2}\right] = \frac{\lambda}{4\pi\epsilon_0}\ln\left(\frac{f(z)}{g(z)}\right)$

(b) Find charge density σ induced on the conducting plane:

$$\sigma = -\epsilon_0\frac{\partial V}{\partial n} = -\epsilon_0\frac{\partial V}{\partial z}\bigg|_{z=0} = \frac{-\lambda}{4\pi}\frac{\partial}{\partial z}\ln\left(\frac{f(z)}{g(z)}\right)$$

$$\frac{\partial}{\partial z}\ln\frac{f(z)}{g(z)} = \frac{1}{f/g}\frac{\partial}{\partial z}\left(\frac{f}{g}\right) = \frac{g}{f}\frac{1}{g^2}(gf' - fg') = \frac{1}{fg}(gf' - fg')$$

$$f' = \frac{\partial}{\partial z}\left[(z+d)^2+y^2\right] = 2(z+d), \quad g' = \frac{\partial}{\partial z}\left[(z-d)^2+y^2\right] = 2(z-d)$$

3.9 (continued)

$$\sigma = -\epsilon_0 \frac{\partial V}{\partial n} = \frac{-\lambda}{4\pi}\frac{\partial}{\partial z} \ln\left[\frac{(z+d)^2+y^2}{(z-d)^2+y^2}\right] = \frac{-\lambda}{4\pi}\frac{\partial}{\partial z}\ln\left(\frac{f}{g}\right)$$

at $z=0$!

$$\sigma = -\frac{\lambda}{4\pi}\frac{(f'g - fg')}{fg} \qquad f' = 2(z+d) \qquad g' = 2(z-d)$$

$$(f'g - fg') = 2(z+d)\left[(z-d)^2+y^2\right] - 2(z-d)\left[(z+d)^2+y^2\right]$$
$$fg = \left[(z+d)^2+y^2)\right]\left[(z-d)^2+y^2\right]$$

$$\left.\frac{f'g - fg'}{fg}\right|_{z=0} = \frac{2d\left[d^2+y^2\right] - 2(-d)\left[d^2+y^2\right]}{\left[d^2+y^2\right]\left[d^2+y^2\right]} = \frac{4d}{d^2+y^2}$$

$$\sigma = -\frac{\lambda}{4\pi}\frac{\partial}{\partial z}\left(\frac{(f'g-fg')}{fg}\right)_{z=0} = \frac{-\lambda}{4\pi}\frac{4d}{(d^2+y^2)}$$

$$\sigma = -\frac{d\lambda}{\pi(d^2+y^2)}$$