

Week 8 homework

vector calculus: 1.43 & 1.44

Problem 1.43 Evaluate the following integrals:

(a) $\int_2^6 (3x^2 - 2x - 1) \delta(x-3) dx$. ^{includes 3 ✓} $x=3$ $= 3 \cdot 3^2 - 2 \cdot 3 - 1 = 3 \cdot 9 - 6 - 1 = 27 - 7 = 20$ ✓

(b) $\int_0^5 \cos x \delta(x-\pi) dx$. ^{includes π ✓} $x=\pi$ $= (\cos \pi) = -1$ ✓

(c) $\int_0^3 x^3 \delta(x+1) dx$. ^{does not} $x=-1$ $0-3$ does not include -1 so $\int = 0$ ✓

(d) $\int_{-\infty}^{\infty} \ln(x+3) \delta(x+2) dx$. $x=-2$ $= \ln(-2+3) = \ln 1 = 0$ ✓

Problem 1.44 Evaluate the following integrals:

(a) $\int_{-2}^2 (2x+3) \delta(3x) dx$. $= \int (2x+3) \frac{1}{3} \delta(x) dx = (2x+3) \frac{1}{3} \Big|_{x=0} = 0+1 = 1$ ✓

(b) $\int_0^2 (x^3 + 3x + 2) \delta(1-x) dx$. ^{includes 1} $x=1$ $= (1^3 + 3 \cdot 1 + 2) \Big|_{x=1} = 1+3+2 = 6$ ✓

(c) $\int_{-1}^1 9x^2 \delta(3x+1) dx$. $x=-\frac{1}{3}$ $= \int 9x^2 \frac{1}{3} \delta(x+\frac{1}{3}) dx = 3x^2 \Big|_{x=-\frac{1}{3}} = 3 \left(\frac{1}{3}\right)^2 = \frac{1}{3}$ ✓

(d) $\int_{-\infty}^{\infty} \delta(x-b) dx$. $x=b$ $= 1$ if $b < a$ ✓ $= 0$ if $b > a$ ✓

2M Ch.5 # 6 (p.214), 12(p.220), 13(p.231), 16(p.231), (extra credit: 17, p.232)

1.44 @ : $\delta(3x) = \frac{1}{3} \delta(x)$

1.44 @ $\delta(3x+1) = \frac{1}{3} \delta\left(x+\frac{1}{3}\right)$