Chapter 7
Relativity

Complaining about the educational system is a national sport among professors in the U.S., and I, like my colleagues, am often tempted to imagine a golden age of education in our country's past, or to compare our system unfavorably with foreign ones. Reality intrudes, however, when my immigrant students recount the overemphasis on rote memorization in their native countries and the philosophy that what the teacher says is always right, even when it's wrong.

Albert Einstein's education in late-nineteenth-century Germany was neither modern nor liberal. He did well in the early grades (the myth that he failed his elementary-school classes comes from a misunderstanding based on a reversal of the German numerical grading scale), but in high school and college he began to get in trouble for what today's edspeak calls "critical thinking."

a / Albert Einstein.

Indeed, there was much that deserved criticism in the state of physics at that time. There was a subtle contradiction between Maxwell's theory of electromagnetism and Galileo's principle that all motion is relative. Einstein began thinking about this on an intuitive basis as a teenager, trying to imagine what a light beam would look like if you could ride along beside it on a motorcycle at the speed of light. Today we remember him most of all for his radical and far-reaching solution to this contradiction, his theory of relativity, but in his student years his insights were greeted with derision from his professors. One called him a "lazy dog." Einstein's distaste for authority was typified by his decision as a teenager to renounce his German citizenship and become a stateless person, based purely on his opposition to the militarism and repressiveness of German society. He spent his most productive scientific years in Switzerland and Berlin, first as a patent clerk but later as a university professor. He was an outspoken pacifist and a stubborn opponent of World War I, shielded from retribution by his eventual acquisition of Swiss citizenship.

As the epochal nature of his work began to become evident, some liberal Germans began to point to him as a model of the "new German," but with the Nazi coup d'etat, staged public meetings began to be held at which Nazi scientists criticized the work of this ethnically Jewish (but spiritually nonconformist) giant of science. Einstein was on a stint as a visiting professor at Caltech when Hitler was appointed chancellor, and never returned to the Nazi state. World War II convinced Einstein to soften his strict pacifist stance, and he signed a secret letter to President Roosevelt urging research into the building of a nuclear bomb, a device that could not have been imagined without his theory of relativity. He later wrote, however, that when Hiroshima and Nagasaki were bombed, it made him wish he could burn off his own fingers for having signed the letter.

This chapter and the next are specifically about Einstein's theory of relativity, but Einstein also began a second, parallel revolution in physics known as the quantum theory, which stated, among other things, that certain processes in nature are inescapably random. Ironically, Einstein was an outspoken doubter of the new quantum ideas, being convinced that "the Old One [God] does not play dice with the universe," but quantum and relativistic concepts are now thoroughly intertwined in physics. The remainder of this book beyond the present pair of chapters is an introduction to the quantum theory, but we will continually be led back to relativistic ideas.

The structure of this chapter

From the modern point of view, electricity and magnetism becomes much simpler and easier to understand if it is encountered after relativity. Most schools' curricula, however, place electricity and magnetism before relativity. In such a curriculum, section 7.1 should be covered before electricity and magnetism, and then later in the course one can go back and cover all of chapter 7. This chapter is also designed so that it can be read without having previously covered waves.

7.1 Basic Relativity

Absolute, true, and mathematical time ... flows at a constant rate without relation to anything external... Absolute space... without relation to anything external, remains always similar and immovable.

Isaac Newton (tr. Andrew Motte)

7.1.1 The principle of relativity

Galileo's most important physical discovery was that motion is relative. With modern hindsight, we restate this in a way that shows what made the teenage Einstein suspicious:

The principle of Galilean relativity: Matter obeys the same laws of physics in any inertial frame of reference, regardless of the frame's orientation, position, or constant-velocity motion.

Note that it only refers to matter, not light.

Einstein's professors taught that light waves obeyed an entirely different set of rules than material objects. They believed that light waves were a vibration of a mysterious medium called the ether, and that the speed of light should be interpreted as a speed relative to this ether. Thus although the cornerstone of the study of matter had for two centuries been the idea that motion is relative, the science of light seemed to contain a concept that a certain frame of reference was in an absolute state of rest with respect to the ether, and was therefore to be preferred over moving frames.

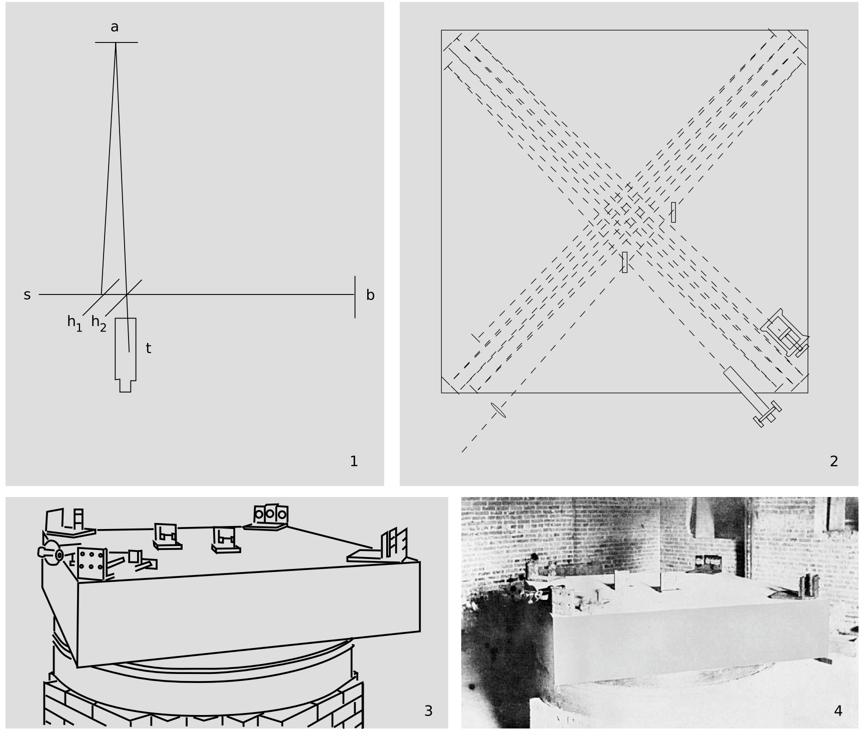
Now let's think about Albert Einstein's daydream of riding a motorcycle alongside a beam of light. In cyclist Albert's frame of reference, the light wave appears to be standing still. However, James Clerk Maxwell had already constructed a highly successful mathematical description of light waves as patterns of electric and magnetic fields. Einstein on his motorcycle can stick measuring instruments into the wave to monitor the electric and magnetic fields, and they will be constant at any given point. But an electromagnetic wave pattern standing frozen in space like this violates Maxwell's equations and cannot exist. Maxwell's equations say that light waves always move with the same velocity, notated c, equal to 3.0×10^8 m/s. Einstein could not tolerate this disagreement between the treatment of relative and absolute motion in the theories of matter on the one hand and light on the other. He decided to rebuild physics with a single guiding principle:

Einstein's principle of relativity: Both light and matter obey the same laws of physics in any inertial frame of reference, regardless of the frame's orientation, position, or constant-velocity motion.

7.1.2 Distortion of time and space

This is hard to swallow. If a dog is running away from me at 5 m/s relative to the sidewalk, and I run after it at 3 m/s, the dog's velocity in my frame of reference is 2 m/s. According to everything we have learned about motion, the dog must have different speeds in the two frames: 5 m/s in the sidewalk's frame and 2 m/s in mine. How, then, can a beam of light have the same speed as seen by someone who is chasing the beam?

In fact the strange constancy of the speed of light had shown up in the now-famous Michelson-Morley experiment of 1887. Michelson and Morley set up a clever apparatus to measure any difference in the speed of light beams traveling east-west and north-south. The motion of the earth around the sun at 110,000 km/hour (about 0.01% of the speed of light) is to our west during the day. Michelson and Morley believed in the ether hypothesis, so they expected that the speed of light would be a fixed value relative to the ether. As the earth moved through the ether, they thought they would observe an effect on the velocity of light along an east-west line. For instance, if they released a beam of light in a westward direction during the day, they expected that it would move away from them at less than the normal speed because the earth was chasing

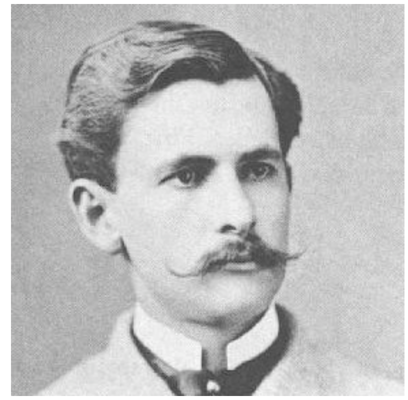


a / The Michelson-Morley experiment, shown in photographs, and drawings from the original 1887 paper. 1. A simplified drawing of the apparatus. A beam of light from the source, s, is partially reflected and partially transmitted by the half-silvered mirror h_1. The two half-intensity parts of the beam are reflected by the mirrors at a and b, reunited, and observed in the telescope, t. If the earth's surface was supposed to be moving through the ether, then the times taken by the two light waves to pass through the moving ether would be unequal, and the resulting time lag would be detectable by observing the interference between the waves when they were reunited. 2. In the real apparatus, the light beams were reflected multiple times. The effective length of each arm was increased to 11 meters, which greatly improved its sensitivity to the small expected difference in the speed of light. 3. In an earlier version of the experiment, they had run into problems with its "extreme sensitiveness to vibration," which was "so great that it was impossible to see the interference fringes except at brief intervals ... even at two o'clock in the morning." They therefore mounted the whole thing on a massive stone floating in a pool of mercury, which also made it possible to rotate it easily. 4. A photo of the apparatus. Note that it is underground, in a room with solid brick walls.

it through the ether. They were surprised when they found that the expected 0.01% change in the speed of light did not occur.

Although the Michelson-Morley experiment was nearly two decades in the past by the time Einstein published his first paper on relativity in 1905, it's unclear how much it influenced Einstein. Michelson and Morley themselves were uncertain about whether the result was to be trusted, or whether systematic and random errors were masking a real effect from the ether. There were a variety of competing theories, each of which could claim some support from the shaky data. Some physicists believed that the ether could be dragged along by matter moving through it, which inspired variations on the experiment that were conducted on mountaintops in thin-walled buildings, b, or with one arm of the appartus out in the open, and the other surrounded by massive lead walls. In the standard sanitized textbook version of the history of science, every scientist does his experiments without any preconceived notions about the truth, and any disagreement is quickly settled by a definitive experiment. In reality, this period of confusion about the Michelson-Morley experiment lasted for four decades, and a few reputable skeptics, including Miller, continued to believe that Einstein was wrong, and kept trying different variations of the experiment as late as the 1920's. Most of the remaining doubters were convinced by an extremely precise version of the experiment performed by Joos in 1930, although you can still find kooks on the internet who insist that Miller was right, and that there was a vast conspiracy to cover up his results.

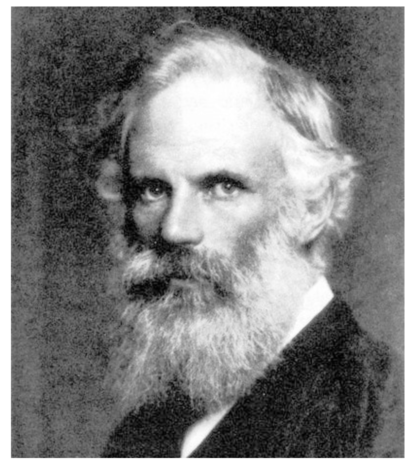
b / Dayton Miller thought that the result of the Michelson-Morley experiment could be explained because the ether had been pulled along by the dirt, and the walls of the laboratory. This motivated him to carry out a series of experiments at the top of Mount Wilson, in a building with thin walls.



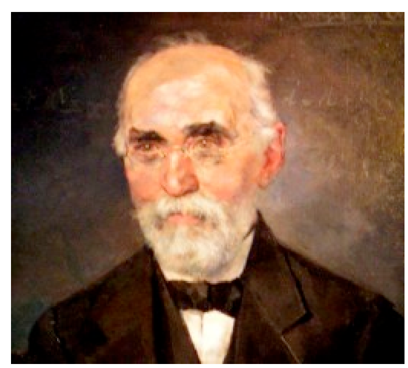
c / Albert Michelson, in 1887, the year of the Michelson-Morley experiment.

Before Einstein, some physicists who did believe the negative result of the Michelson-Morley experiment came up with explanations that preserved the ether. In the period from 1889 to 1895, Hendrik Lorentz and George FitzGerald suggested that the negative result of the Michelson-Morley experiment could be explained if the earth, and every physical object on its surface, was contracted slightly by the strain of the earth's motion through the ether.[1]

How did Einstein explain this strange refusal of light waves to obey the usual rules of addition and subtraction of velocities due to relative motion? He had the originality and bravery to suggest a radical solution. He decided that space and time must be stretched and compressed as seen by observers in different frames of reference. Since velocity equals distance divided by time, an appropriate distortion of time and space could cause the speed of light to come out the same in a moving frame. This conclusion could have been reached by the physicists of two generations before, on the day after Maxwell published his theory of light, but the attitudes about absolute space and time stated by Newton were so strongly ingrained that such a radical approach did not occur to anyone before Einstein.



d / George FitzGerald, 1851-1901.

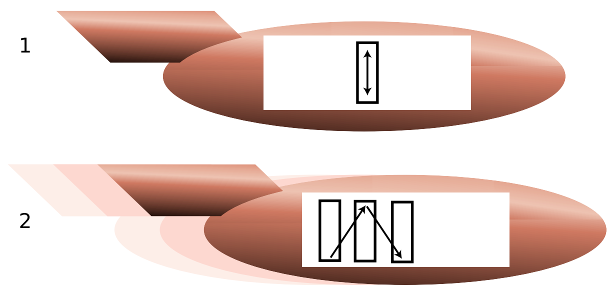


e / Hendrik Lorentz, 1853-1928.

[1]See discussion question F on page 316, and homework problem 22

Time distortion

Consider the situation shown in figure f. Aboard a rocket ship we have a tube with mirrors at the ends. If we let off a flash of light at the bottom of the tube, it will be reflected back and forth between the top and bottom. It can be used as a clock: by counting the number of times the light goes back and forth we get an indication of how much time has passed. (This may not seem very practical, but a real atomic clock does work on essentially the same principle.) Now imagine that the rocket is cruising at a significant fraction of the speed of light relative to the earth. Motion is relative, so for a person inside the rocket, f/1, there is no detectable change in the behavior of the clock, just as a person on a jet plane can toss a ball up and down without noticing anything unusual. But to an observer in the earth's frame of reference, the light appears to take a zigzag path through space, f/2, increasing the distance the light has to travel.



f / A light beam bounces between two mirrors in a spaceship.

If we didn't believe in the principle of relativity, we could say that the light just goes faster according to the earthbound observer. Indeed, this would be correct if the speeds were not close to the speed of light, and if the thing traveling back and forth was, say, a ping-pong ball. But according to the principle of relativity, the speed of light must be the same in both frames of reference. We are forced to conclude that time is distorted, and the light-clock appears to run more slowly than normal as seen by the earthbound observer. In general, a clock appears to run most quickly for observers who are in the same state of motion as the clock, and runs more slowly as perceived by observers who are moving relative to the clock.

We can easily calculate the size of this time-distortion effect. In the frame of reference shown in figure f/1, moving with the spaceship, let t_1 be the time required for the beam of light to move from the bottom to the top. An observer on the earth, who sees the situation shown in figure f/2, disagrees, and says this motion took a longer time t_2. Let v be the velocity of the spaceship relative to the earth. In frame 2, the light beam travels along the hypotenuse of a right triangle whose base has length

$$\text{base} = vt_2 \quad .$$

Observers in the two frames of reference agree on the vertical distance traveled by the beam, i.e. the height of the triangle perceived in frame 2, and an observer in frame 1 says that this height is the distance covered by a light beam in time t_1, so the height is

$$\text{height} = ct_1 \qquad .$$

The hypotenuse of this triangle is the distance the light travels in frame 2,

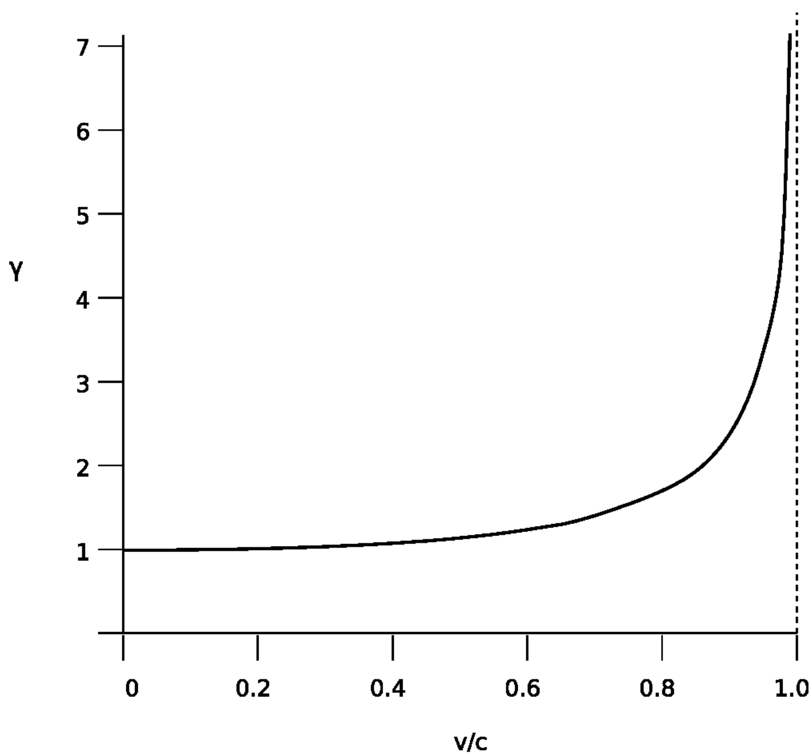
$$\text{hypotenuse} = ct_2 \qquad .$$

Using the Pythagorean theorem, we can relate these three quantities, and solving for t_2 we find

$$t_2 = \frac{t_1}{\sqrt{1-(v/c)^2}} \qquad .$$

The amount of distortion is given by the factor $1/\sqrt{1-(v/c)^2}$, and this quantity appears so often that we give it a special name, γ (Greek letter gamma),

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \qquad . \qquad \text{[definition of the } \gamma \text{ factor]}$$



g / The behavior of the γ factor.

Self-Check
What is γ when v=0? What does this mean? ▷ Answer, p. 708