Length Contraction

Name ____________________

Two sets of students go to the hardware store and buy identical metersticks. One set of students (call them Lab A) then gets on a spaceship with their meterstick and goes into space to prepare to fly by Earth. The other set of students (call them Lab B) gets on a spaceship which simply goes into orbit around Earth.

The trajectories of the two spaceships are planned so that for two seconds they are traveling near each other, parallel to each other, at a relative speed equal to 60% the speed of light. During those two seconds, both sets of students hold their metersticks up along the window on the side of their spaceship so that the metersticks can be seen by both sets of students.

What do they see?

You should be able to predict the length of each meterstick as viewed from each frame.

There are two ways to do it. You can use the interval, but it is not the easiest way. Once you have used the interval over and over, you will find that there is an easier way to do special relativity problems. There is a special factor which comes up and which is dependent only on the relative speed of the two frames of reference.

This factor is usually called gamma and is calculated according to this formula:

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

where β is the relative speed of the two frames of reference expressed as a fraction of the speed of light.

Gamma can be used to predict relative lengths in two frames as well as the relative time dilation in two frames. If you look carefully, you will see that it is involved in the formula used in the [**?**] text.

In the case of this example, you should be able to calculate gamma easily given that $\beta = 0.6$.

$\gamma =$ __________

Then all you have to do is multiply or divide by this factor to predict distances and durations. What numbers make the most sense for the table below?

	as measured in frame 1	as measured in frame 2
length of meterstick in frame 1		
length of meterstick in frame 2		

If you have done things right, you should sense a paradox, or at least an unwinnable argument. What do the students in Lab A say to the students in Lab B?

"Your meterstick is ______________!" (Fill in the blank with either "short" or "long."

And what do the students in Lab B say to the students in Lab A?

"Your meterstick is ______________!" (Fill in the blank with either "short" or "long."

But shouldn't the metersticks overlap exactly at some point? To check this out, we need to be a little more specific about what happens.

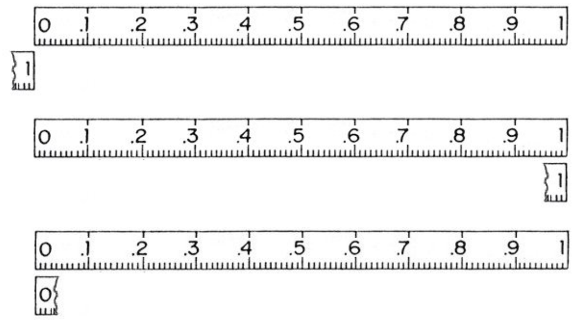
(over for more!)

There are two events which seem to be important here, but to make it clear what happens, we are going to look at three events.

Event 0 happens at the first point where any parts of the two metersticks overlap. Both frames of reference use this event as the origin.

Event 1 happens when the end of one meterstick is right next to the end of the other.

Event 2 happens when the beginning of one meterstick is right next to the beginning of the other.



What are the spacetime coordinates for these three events in each frame?

	Lab A		Lab B	
	t	x	t	x
Event 0				
Event 1				
Event 2				

Hints:

- Assume that Lab A's meterstick is the one on top in each of the diagrams above.
- Event 0 is the origin, so you can fill in all its coordinates.
- Notice (by looking at Event 0) that Lab A's origin is at zero on their own meterstick, but Lab B's origin is at the one meter mark of their meterstick.
- So you should now be able to fill in the x-coordinates of the other events in each frame.
- To get the t-coordinates for Event 1 in Lab A, you can use the relative speed of the two frames and the known distance that the front of Lab B's meterstick has traveled.
- You cannot get Lab A's t-coordinate for Event 2 in the same way. Why not?
- But you can get Lab B's t-coordinate for Event 2. You may need to draw your own images of events 0 and 2 from the perspective of Lab B. Then you should see that the relative speed of the frames can be used again.
- Finally, the remaining two blanks can be filled in by using the spacetime interval.

Final Questions:

1. What order do the events occur in each frame?
2. So why do the Lab A students think Lab B's meterstick is shorter than theirs?
3. And why do the Lab B students think Lab A's meterstick is the short one?