Distortion of space

The speed of light is supposed to be the same in all frames of reference, and a speed is a distance divided by the time. We can't change time without changing distance, since then the speed couldn't come out the same. A rigorous treatment requires some delicacy, but we postpone that to section 7.2 and state for now the apparently reasonable result that if time is distorted by a factor of γ, then lengths must also be distorted according to the same ratio. An object in motion appears longest to someone who is at rest with respect to it, and is shortened along the direction of motion as seen by other observers.

7.1.3 Applications

Nothing can go faster than the speed of light.

What happens if we want to send a rocket ship off at, say, twice the speed of light, $v = 2c$? Then γ will be $1/\sqrt{-3}$. But your math teacher has always cautioned you about the severe penalties for taking the square root of a negative number. The result would be physically meaningless, so we conclude that no object can travel faster than the speed of light. Even travel exactly at the speed of light appears to be ruled out for material objects, since then γ would be infinite.

Einstein had therefore found a solution to his original paradox about riding on a motorcycle alongside a beam of light, resulting in a violation of Maxwell's theory of electromagnetism. The paradox is resolved because it is impossible for the motorcycle to travel at the speed of light.

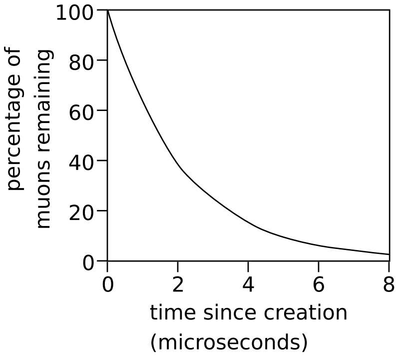
Most people, when told that nothing can go faster than the speed of light, immediately begin to imagine methods of violating the rule. For instance, it would seem that by applying a constant force to an object for a long time, we could give it a constant acceleration, which would eventually cause it to go faster than the speed of light. We will take up these issues in section 7.3.

Cosmic-ray muons

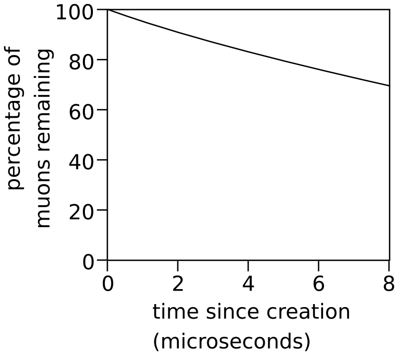
A classic experiment to demonstrate time distortion uses observations of cosmic rays. Cosmic rays are protons and other atomic nuclei from outer space. When a cosmic ray happens to come the way of our planet, the first earth-matter it encounters is an air molecule in the upper atmosphere. This collision then creates a shower of particles that cascade downward and can often be detected at the earth's surface. One of the more exotic particles created in these cosmic ray showers is the muon (named after the Greek letter mu, μ). The reason muons are not a normal part of our environment is that a muon is radioactive, lasting only 2.2 microseconds on the average before changing itself into an electron and two neutrinos. A muon can therefore be used as a sort of clock, albeit a self-destructing and somewhat random one! Figures h and i show the average rate at which a sample of muons decays, first for muons created at rest and then for high-velocity muons created in cosmic-ray showers. The second graph is found experimentally to be stretched out by a factor of about ten, which matches well with the prediction of relativity theory:

$$\begin{aligned}\gamma &= 1/\sqrt{1-(v/c)^2}\\ &= 1/\sqrt{1-(0.995)^2}\\ &\approx 10\end{aligned}$$

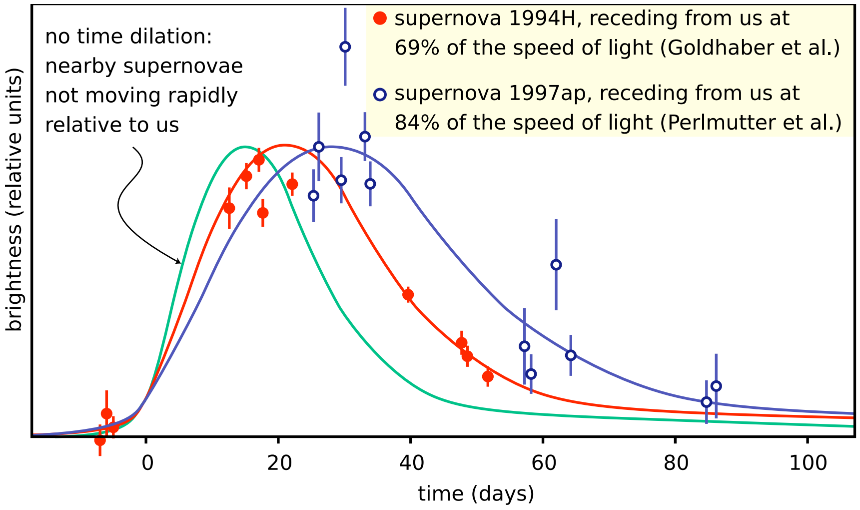
Since a muon takes many microseconds to pass through the atmosphere, the result is a marked increase in the number of muons that reach the surface.



h / Decay of muons created at rest with respect to the observer.



i / Decay of muons moving at a speed of 0.995c with respect to the observer.



j / Light curves of supernovae, showing a time-dilation effect for supernovae that are in motion relative to us.

Time dilation for objects larger than the atomic scale

Our world is (fortunately) not full of human-scale objects moving at significant speeds compared to the speed of light. For this reason, it took over 80 years after Einstein's theory was published before anyone could come up with a conclusive example of drastic time dilation that wasn't confined to cosmic rays or particle accelerators. Recently, however, astronomers have found definitive proof that entire stars undergo time dilation. The universe is expanding in the aftermath of the Big Bang, so in general everything in the universe is getting farther away from everything else. One need only find an astronomical process that takes a standard amount of time, and then observe how long it appears to take when it occurs in a part of the universe that is receding from us rapidly. A type of exploding star called a type Ia supernova fills the bill, and technology is now sufficiently advanced to allow them to be detected across vast distances. Figure j shows convincing evidence for time dilation in the brightening and dimming of two distant supernovae.

The twin paradox

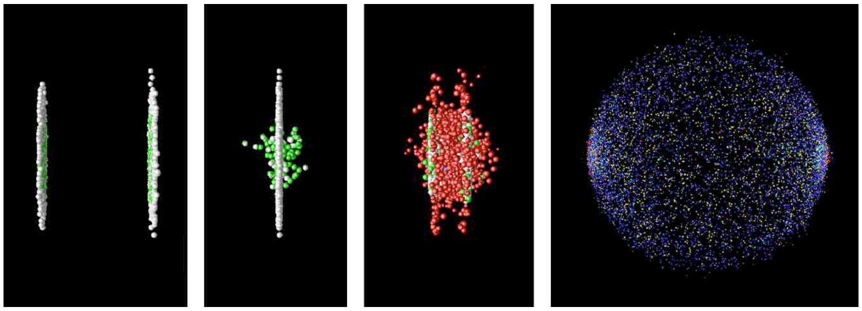
A natural source of confusion in understanding the time-dilation effect is summed up in the so-called twin paradox, which is not really a paradox. Suppose there are two teenaged twins, and one stays at home on earth while the other goes on a round trip in a spaceship at relativistic speeds (i.e. speeds comparable to the speed of light, for which the effects predicted by the theory of relativity are important). When the traveling twin gets home, he has aged only a few years, while his brother is now old and gray. (Robert Heinlein even wrote a science fiction novel on this topic, although it is not one of his better stories.)

The "paradox" arises from an incorrect application of the principle of relativity to a description of the story from the traveling twin's point of view. From his point of view, the argument goes, his homebody brother is the one who travels backward on the receding earth, and then returns as the earth approaches the spaceship again, while in the frame of reference fixed to the spaceship, the astronaut twin is not moving at all. It would then seem that the twin on earth is the one whose biological clock should tick more slowly, not the one on the spaceship. The flaw in the reasoning is that the principle of relativity only applies to frames that are in motion at constant velocity relative to one another, i.e., inertial frames of reference. The astronaut twin's frame of reference, however, is noninertial, because his spaceship must accelerate when it leaves, decelerate when it reaches its destination, and then repeat the whole process again on the way home. Their experiences are not equivalent, because the astronaut twin feels accelerations and decelerations. A correct treatment requires some mathematical complication to deal with the changing velocity of the astronaut twin, but the result is indeed that it's the traveling twin who is younger when they are reunited.

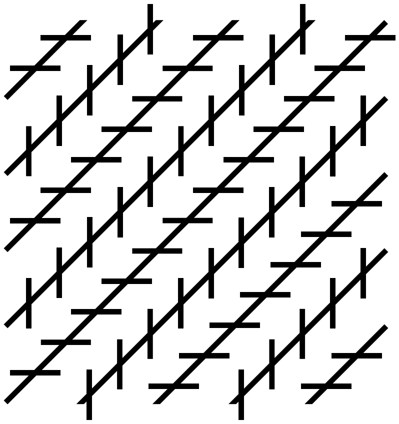
The twin "paradox" really isn't a paradox at all. It may even be a part of your ordinary life. The effect was first verified experimentally by synchronizing two atomic clocks in the same room, and then sending one for a round trip on a passenger jet. (They bought the clock its own ticket and put it in its own seat.) The clocks disagreed when the traveling one got back, and the discrepancy was exactly the amount predicted by relativity. The effects are strong enough to be important for making the global positioning system (GPS) work correctly. If you've ever taken a GPS receiver with you on a hiking trip, then you've used a device that has the twin "paradox" programmed into its calculations. Your handheld GPS box talks to a system onboard a satellite, and the satellite is moving fast enough that its time dilation is an important effect. So far no astronauts have gone fast enough to make time dilation a dramatic effect in terms of the human lifetime. The effect on the Apollo astronauts, for instance, was only a fraction of a second, since their speeds were still fairly small compared to the speed of light. (As far as I know, none of the astronauts had twin siblings back on earth!)

An example of length contraction

Figure k shows an artist's rendering of the length contraction for the collision of two gold nuclei at relativistic speeds in the RHIC accelerator in Long Island, New York, which went on line in 2000. The gold nuclei would appear nearly spherical (or just slightly lengthened like an American football) in frames moving along with them, but in the laboratory's frame, they both appear drastically foreshortened as they approach the point of collision. The later pictures show the nuclei merging to form a hot soup, in which experimenters hope to observe a new form of matter.



k / Colliding nuclei show relativistic length contraction.



Discussion question B

Discussion Questions

A On a spaceship moving at relativistic speeds, would a lecture seem even longer and more boring than normal?

B A question that students often struggle with is whether time and space can really be distorted, or whether it just seems that way. Compare with optical illusions or magic tricks. How could you verify, for instance, that the lines in the figure are actually parallel? Are relativistic effects the same or not?

C If you were in a spaceship traveling at the speed of light (or extremely close to the speed of light), would you be able to see yourself in a mirror?

D Mechanical clocks can be affected by motion. For example, it was a significant technological achievement to build a clock that could sail aboard a ship and still keep accurate time, allowing longitude to be determined. How is this similar to or different from relativistic time dilation?

E What would the shapes of the two nuclei in the RHIC experiment look like to a microscopic observer riding on the left-hand nucleus? To an observer riding on the right-hand one? Can they agree on what is happening? If not, why not — after all, shouldn't they see the same thing if they both compare the two nuclei side-by-side at the same instant in time?

F If you stick a piece of foam rubber out the window of your car while driving down the freeway, the wind may compress it a little. Does it make sense to interpret the relativistic length contraction as a type of strain that pushes an object's atoms together like this? How does this relate to discussion question E?