Symmetry in Chemistry
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Table 6.1 The Six Crystal Systems

Point Group Symbol S^1	Point Group Symbol $H\text{-}M^2$	System	Unit Cell	Minimum Symmetry Requirements
C_1	1	Triclinic	$\alpha \neq \beta \neq \gamma \neq 90°$	None
S_2	$\bar{1}$		$a \neq b \neq c$	
C_2	2	Monoclinic	$\alpha = \beta = 90°$	One twofold
C_h	m		$\gamma \neq 90°$	axis or one
C_{2h}	$2/m$		$a \neq b \neq c$	mirror plane
D_2	222	Orthorhombic	$\alpha = \beta = \gamma = 90°$	Any combination
D_{2h}	mmm		$a \neq b \neq c$	of three mutually perpendicular twofold axes or mirror planes
C_{2v}	mm			
C_3	3	Hexagonal		
D_3	32		$\alpha = \beta = \gamma \neq 90°$	One threefold
S_6	$\bar{3}$		$a = b = c$	axis or one
D_{3d}	$\bar{3}m$	(Rhombedral		threefold inver-
C_{3v}	$3m$	Division)		sion axis
C_6	6	(Hexagonal	or	
C_{3h}	$\bar{6}$	Division)		
C_{6h}	$6/m$		$\alpha = \beta = 90°$	One sixfold axis
D_6	62		$\gamma = 120°$	or one sixfold
D_{3h}	$\bar{6}2m$		$a = b \neq c$	inversion axis
D_{6h}	$6/mmm$			
C_{6v}	$6mm$			
C_4	4	Tetragonal	$\alpha = \beta = \gamma = 90°$	One fourfold
C_{4h}	$4/m$		$a = b \neq c$	axis or one four-
D_4	42			fold inversion
D_{4h}	$4/mmm$			axis
S_4	$\bar{4}$			
D_{2d}	$\bar{4}2m$			
C_{4v}	$4mm$			
T	23	Cubic	$\alpha = \beta = \gamma = 90°$	Four threefold
O	43		$a = b = c$	axes at 109°28′
T_h	$m3$			to each other
O_h	$m3m$			
T_d	$\bar{4}3m$			

[1] Schönfliess notation.
[2] Hermann-Maugin notation.

Table 6.1 (Continued)

60° angles, is often convenient. (This rhombus is included in Fig. 6.19 by heavy lines.)

It may be worth pointing out that the axial inequalities given for the triclinic, monoclinic, and orthorhombic systems are *allowed* by symmetry. Equality of axes marked as unequal *may* occur accidentally.

This classification of the point groups into crystal classes reminds us of the classification of the point groups according to the possible degeneracy of their symmetry species. Thus, the point groups included in the isometric crystal class are all those permitting threefold degenerate species (with the exception of the point groups of types *I* and *K*, which cannot occur in crystallography because of their high-order axes). All the point groups in the tetragonal and hexagonal systems permit doubly degenerate species, as do all point groups with other axes higher than twofold which do not occur in crystallography. The point groups occurring in the other crystal classes have no axes higher than twofold, and hence no degenerate symmetry species. Molecules belonging to the point groups in the isometric system are spherical tops. Those belonging to the point groups in the tetragonal and hexagonal systems (and all others with higher-order axes) are symmetric tops; all other molecules are asymmetric tops.

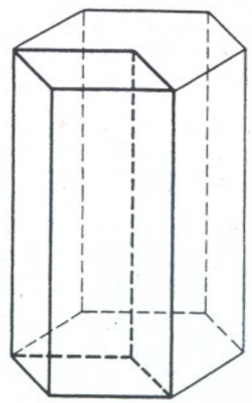

Fig. 6.19 The hexagonal unit cell.

Given thus the unit cells in the six systems, we can proceed to derive the possible space lattices. In the two-dimensional case we encountered primitive (p) and centered (c) lattices. In three dimensions the possibilities are considerably more varied. In each of the systems we can have a primitive lattice, each of which will be denoted by P, and the system to which any one belongs is determined