ADVANCED CHEMISTRY - 2008

QUANTUM MECHANICS - WINTER - HOMEWORK - WEEK ②

Chapter 9

(10) $\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$ $\qquad \Delta t = 10^{-10}\,s$

$$\Delta E \geq \frac{\hbar}{2(\Delta t)} \geq \frac{1.055 \times 10^{-34}\,J\,s}{2(10^{-10})\,s}$$

$$\Delta E \geq 5.275 \times 10^{-25}\,J$$

$$5.275 \times 10^{-25}\,J \times \left(\frac{6.02 \times 10^{23}}{1\,mol}\right) = 3.176 \times 10^{-1}\,J\,mol^{-1}$$

(11) $$\frac{d}{dx}(f) = \frac{d}{dx}(8e^{5x}) = 8.5.e^{5x}$$

$$= 5(8e^{5x}) = 5f$$

$\frac{d}{dx}(f) = 5(f)$ $\quad \therefore f$ is an eigen function of the operator $\frac{d}{dx}$ with the eigen value $= 5$

(12) (a) $\frac{d}{dx}\left(e^{-ax^2}\right) = (-2ax)\left(e^{-ax^2}\right)$

$\neq$ not an eigen function

$$\frac{d^2}{dx^2}\left(e^{-ax^2}\right) = \frac{d}{dx}\left[\frac{d}{dx}\left(e^{-ax^2}\right)\right] = \frac{d}{dx}\left[-2ax\,e^{-ax^2}\right]$$

$$= -2a\,\frac{d}{dx}\left[x \cdot e^{-ax^2}\right]$$

$$= -2a\left[x\left(-2ax\, e^{-ax^2}\right) + e^{-ax^2}(1)\right]$$

$$= -2a\left[-2ax^2 + 1\right] e^{-ax^2}$$

$$= \underline{2a(2ax^2 - 1)\, e^{-ax^2}}$$ not an eigen function

(b) $\frac{d}{dx}(\cos bx) = \underline{-b(\sin bx)}$ not an eigen function

$$\frac{d^2}{dx^2}(\cos bx) = \frac{d}{dx}\left[\frac{d}{dx}(\cos bx)\right] = \frac{d}{dx}\left[-b\sin bx\right]$$

$$= -b.\ b\cos bx$$

$$= (-b^2)(\cos bx)$$ eigen function with the eigen value $\underline{-b^2}$

(c) $\frac{d}{dx}\, e^{ikx} = (ik)\, e^{ikx}$ eigen function with the eigen value (ik)

$$\frac{d^2}{dx^2}\left(e^{ikx}\right) = \frac{d}{dx}\left[\frac{d}{dx}\left(e^{ikx}\right)\right] = \frac{d}{dx}\left[ik\, e^{ikx}\right]$$

$$= ik.\ ik\, e^{ikx} = -k^2 e^{ikx}$$

eigen function with the eigen value $-k^2$