Calculus Homework Week 1 Integration by Parts

pg 531-532 #51, 55, 57, 61, 63

(51) $\int_0^{1/2} \arccos x \, dx$ $\quad u = \arccos x \quad dv = dx$

$du = -\frac{1}{\sqrt{1-x^2}}\,dx \quad v = x$

$= x\arccos x + \int \frac{x}{\sqrt{1-x^2}}\,dx = x\arccos x - \sqrt{1-x^2}\Big|_0^{1/2}$

$= \frac{1}{2}\arccos\left(\frac{1}{2}\right) - \sqrt{\frac{3}{4}} + 1 = .0658$

(55) $\int_1^2 x^2 \ln x \, dx$ $\quad u = \ln x \quad dv = x^2\,dx$

$du = \frac{1}{x}\,dx \quad v = \frac{x^3}{3}$

$= \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\left(\frac{1}{x}\right)dx = \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2\,dx = \frac{x^3}{3}\ln x - \frac{1}{9}x^3\Big|_1^2$

$= \frac{8}{3}\ln 2 - \frac{8}{9} + \frac{1}{9} = 1.071$

(57) $\int_2^4 x \operatorname{arcsec} x \, dx$ $\quad u = \operatorname{arcsec} x \quad dv = x\,dx$

$du = \frac{1\,dx}{x\sqrt{x^2-1}} \quad v = \frac{x^2}{2}$

$= \frac{x^2}{2}\operatorname{arcsec} x - \int \frac{x^2/2\,dx}{x\sqrt{x^2-1}} = \frac{x^2}{2}\operatorname{arcsec} x - \frac{1}{4}\int \frac{2x\,dx}{\sqrt{x^2-1}}$

$= \frac{x^2}{2}\operatorname{arcsec} x - \frac{1}{2}\sqrt{x^2-1}\Big|_2^4 = \left(8\operatorname{arcsec} 4 - \frac{\sqrt{15}}{2}\right) - \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$= 7.38$

(61) $\int x^3 \sin x \, dx =$

alternate signs	u and its derivatives	v' and its integrals
+	x^3	$\sin x$
−	$3x^2$	$-\cos x$
+	$6x$	$-\sin x$
−	6	$\cos x$
+	0	$\sin x$

$= x^3(-\cos x) - 3x^2(-\sin x) + 6x\cos x - 6\sin x + C$

$= (3x^2 - 6)\sin x - (x^3 - 6x)\cos x + C$

(63) $\int x \sec^2 x \, dx$

alternate signs	u and its derivatives	v' and its integrals
+	x	$\sec^2 x$
−	1	$\tan x$
+	0	$-\ln(\cos x)$

$= x\tan x + \ln(\cos x) + C$