HOMEWORK WEEK 2 CALCULUS TRIGONOMETRIC INTEGRALS

pg 540 #5, 17, 19, 21 pg 559 #1, 3, 5, 7, 17 *PARTIAL FRACTIONS

(5) $\int \cos^3 x \sin x \, dx \qquad u = \cos x, \; du = -\sin x \, dx$

$= \int u^3 (-du) = -\frac{u^4}{4} = -\frac{1}{4}\cos^4 x + C$

(17) $\int x \sin^2 x \, dx \qquad \begin{matrix} u = x \\ du = dx \end{matrix} \qquad dv = \sin^2 x \, dx = \left(\frac{1-\cos 2x}{2}\right) dx$

$v = \frac{x}{2} - \frac{\sin 2x}{4} = \frac{1}{4}(2x - \sin 2x)$

$= \frac{1}{4} x (2x - \sin 2x) - \frac{1}{4}\int (2x - \sin 2x) \, dx$

$= \frac{1}{4} x (2x - \sin 2x) - \frac{1}{4}\left(x^2 + \frac{1}{2}\cos 2x\right) + C$

(19) $\int_0^{\pi/2} \cos^3 x \, dx = \int \underbrace{(1-\sin^2 x)}_{\cos^2 x} \cos x \, dx = \int \cos x \, dx - \int \sin^3 x \cos x \, dx$

$\uparrow u^2 \, du$

$= \left[\sin x - \frac{1}{3}\sin^3 x\right]_0^{\pi/2} = \frac{2}{3}$

(21) $\int_0^{\pi/2} \cos^7 x \, dx = \int \underbrace{(1-\sin^2 x)^3}_{(\cos^2)^3} \cos x \, dx$

$= \int_0^{\pi/2} \left(1 - 3\sin^2 x + 3\sin^4 x - \sin^6 x\right) \cos x \, dx$

$= \sin x - \sin^3 x + \frac{3}{5}\sin^5 x - \frac{1}{7}\sin^7 x \Big|_0^{\pi/2} = \frac{16}{35}$

p559

#1 $\frac{5}{x^2-10x} = \frac{5}{x(x-10)} = \frac{A}{x} + \frac{B}{x-10}$

3 $\frac{2x-3}{x^3+10x} = \frac{2x-3}{x(x^2+10)} = \frac{A}{x} + \frac{Bx+C}{x^2+10}$

5 $\frac{16}{x^2-10x} = \frac{16}{x(x-10)} = \frac{A}{x} + \frac{B}{x-10}$

7 $\int \frac{1}{x^2-1}\,dx \Rightarrow$ $\frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$

$= -\frac{1}{2}\int \frac{1}{x+1}\,dx + \frac{1}{2}\int \frac{1}{x-1}\,dx$

$= -\frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + C$

$= \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$

$\frac{1}{x^2-1} \rightarrow \frac{A(x-1)+B(x+1)}{(x+1)(x-1)}$

→ when $x=-1 \rightarrow 1 = -2A \rightarrow A = -1/2$

when $x=1 \rightarrow 1 = 2B \rightarrow B = 1/2$

17 $\int \frac{4x^2+2x-1}{x^3+x^2}\,dx$ $\frac{4x^2+2x-1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

$= \int \left(\frac{3}{x} - \frac{1}{x^2} + \frac{1}{x+1}\right)dx$

$= 3\ln x + \frac{1}{x} + \ln(x+1) + C$

$= \frac{1}{x} + \ln(x^4+x^3) + C$

$\frac{4x^2+2x-1}{x^2(x+1)} = \frac{Ax(x+1)}{x^2(x+1)} + \frac{B(x+1)}{x^2(x+1)} + \frac{Cx^2}{x^2(x+1)}$

→ when $x=0 \rightarrow B=-1$

→ when $x=-1 \rightarrow C=1$

→ when $x=1 \rightarrow A=3$