WORKSHOP WEEK 1 CALCULUS

PAGES 522-523

(43) $\displaystyle\int \frac{-1\,dt}{\sqrt{1-(2t-1)^2}}$ $\quad u = 2t-1$, $du = 2dt$

$$= -\frac{1}{2}\int \frac{2\,dt}{\sqrt{1-(2t-1)^2}} = -\frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = -\frac{1}{2}\arcsin(2t-1) + C$$

(47) $$\int \frac{3\,dx}{\sqrt{6x-x^2}} = 3\int \frac{dx}{\sqrt{-x^2+6x-9+9}} = 3\int \frac{dx}{\sqrt{-(x-3)^2+9}}$$

complete binomial

$$= 3\int \frac{dx}{\sqrt{3^2-(x-3)^2}} = 3\arcsin\left(\frac{x-3}{3}\right) + C$$

using table of integration page 520 #18

(59) $(4+\tan^2 x)\, y' = \sec^2 x$

$$\frac{dy}{dx} = \frac{\sec^2 x}{4+\tan^2 x}$$ $\quad u = \tan x$, $\frac{du}{dx} = \sec^2 x$

table page 520 #19

$$dy = \frac{du}{2^2+u^2} \qquad y = \frac{1}{2}\arctan\left(\frac{\tan x}{2}\right) + C$$

(60) $$y' = \frac{1}{x\sqrt{4x^2-1}}$$

$$\int dy = \int \frac{2\,dy}{2x\sqrt{(2x)^2-1}} \qquad y = \operatorname{arcsec}(2x) + C$$

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INTEGRATION BY PARTS

(50) $\int_0^{\pi} x \sin 2x \, dx$

$u = x \quad dv = \sin 2x \, dx$

$du = dx \quad v = -\frac{1}{2}\cos 2x$

$\int x \sin 2x \, dx = -\frac{1}{2} x \cos 2x + \frac{1}{2}\int \cos 2x \, dx = \frac{-1}{2} x \cos 2x + \frac{1}{4}\sin 2x + C$

$= \frac{1}{4}\left(\sin 2x - 2x\cos 2x\right)\Big|_0^{\pi} = -\pi/2$

(52) $\int_0^1 x \arcsin x^2 \, dx$

$u = \arcsin x^2 \quad dv = x \, dx$

$du = \frac{2x \, dx}{\sqrt{1-x^4}} \quad v = \frac{x^2}{2}$

$= \frac{x^2}{2}\arcsin x^2 - \int \frac{x^3 \, dx}{\sqrt{1-x^4}} = \frac{x^2}{2}\arcsin x^2 + \frac{1}{4}(2)(1-x^4)^{1/2} + C$

$\int_0^1 x \arcsin x^2 \, dx = \frac{1}{2}\left[x^2 \arcsin x^2 + \sqrt{1-x^4}\right]_0^1 = \frac{1}{4}(\pi - 2)$

(53) $\int_0^1 e^x \sin x \, dx$ use integration by parts twice

$u = \sin x \quad dv = e^x \, dx$

$du = \cos x \, dx \quad v = e^x$

$\int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx$

$u = \cos x \quad dv = e^x dx$

$du = -\sin x \, dx \quad v = e^x$

$\int e^x \sin x \, dx = e^x \sin x - \left[e^x \cos x - \int e^x (-\sin x \, dx)\right]$

$2\int e^x \sin x \, dx = e^x \sin x - e^x \cos x$

$\int_0^1 e^x \sin x \, dx = \frac{e^x \sin x - e^x \cos x}{2} = \frac{e}{2}(\sin 1 - \cos 1) = 0.909$

(54) $\int_0^2 e^{-x}\cos x\,dx$ $\quad u=\cos x \quad dv=e^{-x}$

$du=-\sin x\,dx \quad v=-e^{-x}$

$= -e^{-x}\cos x - \int e^{-x}\sin x\,dx \quad u=\sin x \quad dv=e^{-x}dx$

$du=\cos x\,dx \quad v=-e^{-x}$

$$\int e^{-x}\cos x\,dx = -e^{-x}\cos x - \left[-e^{-x}\sin x + \int e^{-x}\cos x\,dx\right]$$

$$2\int e^{-x}\cos x\,dx = e^{-x}\sin x - e^{-x}\cos x$$

$$\int_0^2 e^{-x}\cos x\,dx = \left[\frac{e^{-x}\sin x - e^{-x}\cos x}{2}\right]_0^2 = \frac{-e^{-2}}{2}\left[\sin 2-\cos 2\right]+\frac{1}{2}$$

(56) $\int_0^1 \ln(1+x^2)\,dx$ $\quad u=\ln(1+x^2) \quad dv=dx$

$du=\frac{2x\,dx}{1+x^2} \quad v=x$

$$= x\ln(1+x^2) - \int\frac{2x^2\,dx}{1+x^2} = x\ln(1+x^2) - 2\int\frac{(x^2+1-1)\,dx}{x^2+1}$$

$$= x\ln(1+x^2) - 2\int\left(1-\frac{1}{1+x^2}\right)dx = x\ln(1+x^2) - 2x + 2\arctan x\Big|_0^1$$

$$= \ln 2 - 2 + \pi/2$$

(58) $\int_0^{\pi/4} x\sec^2 x\,dx$ $\quad u=x \quad dv=\sec^2 x\,dx$

$du=dx \quad v=\tan x$

$$= x\tan x - \int\tan x\,dx = x\tan x + \ln(\cos x)\Big|_0^{\pi/4}$$

$$= \left(\frac{\pi}{4}+\ln\left(\sqrt{2}/2\right)\right) - 0 = \frac{\pi}{4} - \frac{1}{2}\ln 2$$

(64) $\int x^2(x-2)^{3/2}\,dx$

Using a tabular method

alternate signs	u and its derivatives	dv and its integrals
$+$	x^2	$(x-2)^{3/2}$
$-$	$2x$	$\frac{2}{5}(x-2)^{5/2}$
$+$	2	$\frac{4}{35}(x-2)^{7/2}$
$-$	0	$\frac{8}{315}(x-2)^{9/2}$

$$= \frac{2}{5}x^2(x-2)^{5/2} - \frac{8}{35}x(x-2)^{7/2} + \frac{16}{315}(x-2)^{9/2} + C$$

$$= \frac{2}{315}(x-2)^{5/2}(35x^2+40x+32) + C$$