WORKSHOP WEEK 4 CALCULUS U→LIATE

(19) $\displaystyle\int_{-\infty}^{0} x e^{-2x}\,dx = \lim_{b\to-\infty}\int_b^0 x e^{-2x}\,dx = \lim_{b\to-\infty} -\frac{x}{2}e^{-2x} - \frac{1}{4}e^{-2x}\Big|_0^b$

$u = x \quad dv = e^{-2x}dx$

$du = dx \quad v = \dfrac{e^{-2x}}{-2}$

$\displaystyle = \lim_{b\to-\infty}\frac{1}{4}\left[-1+(2b+1)e^{-2b}\right] = -\infty$ $\boxed{\text{diverges}}$

(21) $\displaystyle\int_0^{\infty} x^2 e^{-x}\,dx = \lim_{b\to\infty}\underbrace{\int_0^b x^2 e^{-x}\,dx}_{\text{integration by parts twice}} = \lim_{b\to\infty}\left[-x^2e^{-x} - 2xe^{-x} - 2e^{-x}\right|_0^b$

$\displaystyle = \lim_{b\to\infty}\Big(\underbrace{\frac{b^2+2b+2}{-e^b}}_{\phi} + 2\Big) = \boxed{2}$

(23) $\displaystyle\underbrace{\int_0^{\infty} e^{-x}\cos x\,dx}_{\text{integration by parts}} = \lim_{b\to\infty}\frac{1}{2}\left[e^{-x}(-\cos x + \sin x)\right|_0^b = \frac{1}{2}\left[0-(-1)\right] = \boxed{\frac{1}{2}}$

→ solved in class

(25) $\displaystyle\int_4^{\infty}\frac{1}{x(\ln x)^3}\,dx = \int_4^{\infty}\frac{du}{u^3} = \lim_{b\to\infty}\int_4^b u^{-3}\,du$

integration by substitution

$\displaystyle = \lim_{b\to\infty}\left[-\frac{1}{2}(\ln x)^{-2}\right|_4^b$

$u = \ln x$

$\displaystyle = \lim_{b\to\infty} -\frac{1}{2}(\ln b)^{-2} + \frac{1}{2}(\ln 4)^{-2}$

$\dfrac{du}{dx} = \dfrac{1}{x}$

$\displaystyle = 0 + \frac{1}{2}(\ln 4)^{-2} = \boxed{\frac{1}{2(\ln 4)^2}}$

$du = \dfrac{dx}{x}$

(29) $$\int_0^\infty \frac{1}{e^x+e^{-x}}\,dx = \int_0^\infty \frac{e^x\,dx}{e^{2x}+1} = \int_0^\infty \frac{du}{u^2+1^2}$$

multiplying by e^x both numerator and denominator

$u = e^x$

$\frac{du}{dx} = e^x \quad du = e^x dx$

$$= \lim_{b\to\infty}\left[\arctan(e^x)\right]_0^b$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \boxed{\frac{\pi}{4}}$$

(30) $$\int_0^\infty \frac{e^x}{1+e^x}\,dx = \int_0^\infty \frac{du}{u} = \lim_{b\to\infty}\left[\ln(1+e^x)\right]_0^b = \infty - \ln 2$$

$$= \infty$$

$u = 1+e^x$

$\frac{du}{dx} = e^x$

$du = e^x dx$

$\boxed{\text{diverges}}$