REQUIREMENTS FOR AN ACCEPTABLE (WELL-BEHAVED) WAVEFUNCTION

1. The wave function **ψ must be continuous**. All its **partial derivatives must also be continuous** (partial derivatives are $\frac{\partial\psi}{\partial x}, \frac{\partial\psi}{\partial \mathrm{y}}$ etc.). This makes the wave function "smooth".

2. The wave function ψ must be **quadratically integrable**. This means that the integral $\int \psi^* \, \psi \, \mathrm{d}\tau$ must exist.

3. Since $\int \psi^* \, \psi \, d\tau$ is the probability density, **it must be single valued**.

4. The wave functions must form an **orthonormal set**. This means that
   - the wave functions must be **normalized.**

$$\int_{-\infty}^{\infty} \psi_i^* \ \ \psi_i \, d\tau \ = 1$$

   - the wave functions must be **orthogonal**.

$$\int_{-\infty}^{\infty} \psi_i^* \ \ \psi_j \, d\tau \ = 0$$

OR $\int_{-\infty}^{\infty} \psi_i^* \ \ \psi_j \, d\tau = \ \delta_{ij}$ where $\delta_{ij} = 1$ when i = j and $\delta_{ij} = 0$ when $i \neq j$

δ_{ij} is called Kronecker delta

5. The wave function must be **finite everywhere**.

6. The wave function must satisfy the **boundary conditions** of the quantum mechanical system it represents.

POSTULATES OF QUANTUM MECHANICS

1. The state of a quantum-mechanical system is completely specified by its wave function $\Psi(r)$. $\Psi^*(r)\Psi(r)$ dxdydz is the probability that the particle lies in the volume element dxdydz, located at r. (Note: we are considering the time independent wave function for all our work).

2. To every observable in classical mechanics, there corresponds a linear operator in quantum mechanics.

3. In any measurement of the observable associated with the operator $\hat{A}$, the only values that will ever be observed are the eigenvalues a, which satisfy the eigenvalue equation

$$\hat{A}\,\Psi(r) = a\,\Psi(r)$$

4. If a system is in a state described by a normalized wave function Ψ, then the average value (or the expectation value) of the observable corresponding to operator A is given by

$$< a > = \int_{-\infty}^{\infty} \Psi^* \hat{A}\, \Psi \, d\tau$$

5. The wave function of a system evolves in time according to the time dependent Schrodinger equation

$$\hat{H}\,\Psi(x,t) = i\hbar \frac{d\Psi}{dt}$$